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WEIBULL DIAMETER DISTRIBUTION AND MAXIMUM LIKELIHOOD ESTIMATORS (MLE) IN PINUS CARIBAEA PLANTATION, ENUGU NGWO, NIGERIA

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ABSTRACT

Maximum likelihood estimators (MLE) method was used to generate the parameters for the Weibull diameter distribution functions for data collected from Temporary Sample Plots (TSPs) of *Pinus caribaea* plantation at Enugu Ngwo, Nigeria. Major growth characteristics variables such as mean diameter at breast height (MDBH), mean basal area (MBA) and quadratic mean diameter (QMD) exhibited a strong linear relationship with all the Weibull parameters and also emerged suitable as better predictors for Weibull probability distribution models at 5% level of significance with reasonable values of fit indices. The diameter Weibull distribution models showed consistent predictability potential and were recommended as parsimonious functions for diameter distribution of the *Pinus caribaea* stand.

Keywords: *Pinus caribaea*, maximum likelihood estimators, weibull distribution function, parameter estimation.

INTRODUCTION

Weibull function has been the most widely used probability distribution function (PDF) for expressing diameter distributions following its unfussiness and relative simplicity. Importantly, the introduction of Weibull distribution by Bailey and Dell (1973) as illustrated by Lei (2008) justified its significant contribution and application for diameter distributions in Forestry resulting from its ability to perfectly describe an extensive range of unimodal distribution and its closed cumulated density functional approach. Weibull PDF has also recorded several successes in describing diameter frequency distribution among the boreal (Liu *et al*., 2004: Newton *et al*., 2004, 2005).

Several methods abound for the parameters of Weibull distribution either from parameter prediction method or parameter recovery perspective. Gorgoso *et al.* (2007) reported that parameter prediction method expresses the parameters of a Weibull distribution function with stand variables using regression models while the parameters recovery mode predicts percentiles or moments from which the distribution parameters are recovered.

As concomitant as this PDF, several methods have been proposed to estimate the parameters of Weibull PDF distribution in Forestry (Lei, 2008); and many of these include the maximum likelihood estimation (MLE), the percentile estimation (PCE) and the method of moment (MOM) estimation. MLE is notably considered the best; as it is asymptotically the most efficient method and has thus remains the widely used method in estimating parameters of the distribution (Lei, 2008; Kelejian and Prucha, 1999; Anselin, 1990; Jiang and Brooks, 2009; Lee and Coble, 2006). The Weibull probability distribution function often used for this diameter distribution is a three-parameter distribution. Mathematically, the probability and cumulative distribution of the three-parameter Weibull distribution for a random variable diameter (D) are given as:

$$f(D;a,b,c) = \frac{c}{b}\left(\frac{D-a}{b}\right)^{c-1}\exp\left(-\left(\frac{D-a}{b}\right)^{c}\right) = 0 \qquad (1)$$

$(a \leq D \leq \alpha)$
$(D<a)$

$$f(D;a,b,c) = 1 - \exp\left(-\left(\frac{D-a}{b}\right)^{c}\right) = 0 \qquad (2)$$

where

D = diameter at breast height (cm)
a = location parameter
b = scale parameter
c = shape parameter

The objective of the study therefore is to model the diameter distribution of trees with absolute dependent on data obtained from *Pinus caribaea* plantation in Enugu Ngwo, Nigeria. Maximum likelihood estimator (MLE) method was however used to generate Weibull parameters using appropriate statistical procedures that would be significantly valuable in predicting the diameter distribution of the future *Pinus caribaea*.

MATERIALS AND METHODS

Study area and data description

The data was collected from Enugu Ngwo *Pinus caribaea* plantation under the management of Forestry Research Institute of Nigeria (FRIN). Enugu Ngwo is located in the south-eastern State of Enugu, Nigeria. The plantation is situated within the Udi's escarpment at an altitude of about 396.24m on Longitude 07° 27' and

Latitude 06^{o} 26' of the equator. Diameter at breast height (dbh) measurements was carried out on the *Pinus caribaea* Temporary Sample Plots (TSPs) in three age series; 1974, 1975 and 1976. Twenty (20) Temporary Sample Plots (TSPs) of size 0.04 ha each were randomly sampled from the three age series where 532 individual trees were measured across the sampled TSPs.

Maximum likelihood estimator

The concept of maximum likelihood estimation (MLE) is a regularly used procedure for the Weibull distribution in forest management studies subsequent to its high useful properties. Essentially, the statistical evaluation of the parameters by maximum likelihood has been discovered to produce better goodness-of-fit statistics when compared to the earlier methods, except that rigorous iterative computations are pertinent when considering it usage (Bailey and Dell, 1973; Cao and McCarty, 2005). Also the study of Lu and Zhang (2010) reported that MLE is more efficient than any other estimators in that it produces lowest variance when the assumptions of normality and homoscedasticity hold for any sample data. Thus, the MLE function used for this study was derived from the empirical equation of the Weibull PDF given in equation (1), and then became the likelihood function (L) of order:

$$L(D_1,\ldots,D_n;b,c) = \prod_{i=1}^{n}\frac{c}{b}\left(\frac{D}{b}\right)^{c-1}\exp\left(-\left(\frac{D}{b}\right)^{c}\right) \quad (3)$$

Essentially, the principle of simple integration was applied to equation (3) above through direct differentiation with respect to b and c respectively and to fulfill the order of the equation to arrive at:

$$b = [n^{-1}\sum_{i=1}^{n} D_i^c]^{1/c} \quad (4)$$

Then the Newton-Raphson method of standard iterative procedure was thus used to obtain value of c from equation (5) and by substitution method in equation (4), the value of which was obtained as used by Lei (2008).

$$c = [(\sum_{i=1}^{n} D_i^c \ln D_i)(\sum_{i=1}^{n} D_i^c)^{-1} - n^{-1}\sum_{i=1}^{n} D_i]^{-1} \quad (5)$$

RESULTS AND DISCUSSIONS

The result of diameter frequency distribution measurements from twenty sample plots used for the assessment of the study area is shown in Figure 1. The summary statistics for both stand and tree variables in *Pinus caribaea* is shown in Table 1 while the values of the three parameters Weibull probability distribution function are shown in Table 2. There were significant variations between the three parameters Weibull probability distribution function used in the assessment of the study area. The location parameter ranged across the study periods from 0.058 to 3.963 while the scale parameter ranged from 4.187 to 27.551. The shape parameter which measures the element of skewness of the diameter distribution ranged from 0.117 to 22.251. These trends of variability among the three parameters Weibull distribution function was similar to past investigation of Weibull diameter distribution (Akinnagbe and Akindele, 2003; Tang *et al*., 2006; Cao and Mccarty, 2005).

Correlation analysis for the determination of association between the major growth characteristics variables and Weibull parameters is shown in Table 3. There was strong correlation between the scale Weibull parameter and the mean diameter, mean basal area and quadratic mean diameter at 5% level of significance. Similarly, the same association existed between the shape Weibull parameter and the major growth parameters. However, location parameter exhibited weak and negative association between the growth attribute of the *Pinus caribaea* data.

The fit indices used for the evaluation of the study indicated high relationship and better predictability diameter distribution models. This shows that the Weibull parameters are good predictors of the probability distribution models. Though the values of coefficient of determination (R^2) of the relationships between the growth attributes and Weibull parameters used here were relatively lower comparatively to the works of Alder (1977) and Akindele and Abayomi (1993) on their studies on diameter Weibull distribution at different plantation locations in Africa; it is apparent that Weibull diameter distribution function could be conveniently be predicted for any plantation grown exotic species in the tropics. The results also confirmed the viability and robustness of maximum likelihood estimation (MLE) method in estimating the three parameters estimates for the diameter distribution (Kilkki *et al*., 1989; Shiver, 1988; Rennolls *et al*., 1985).

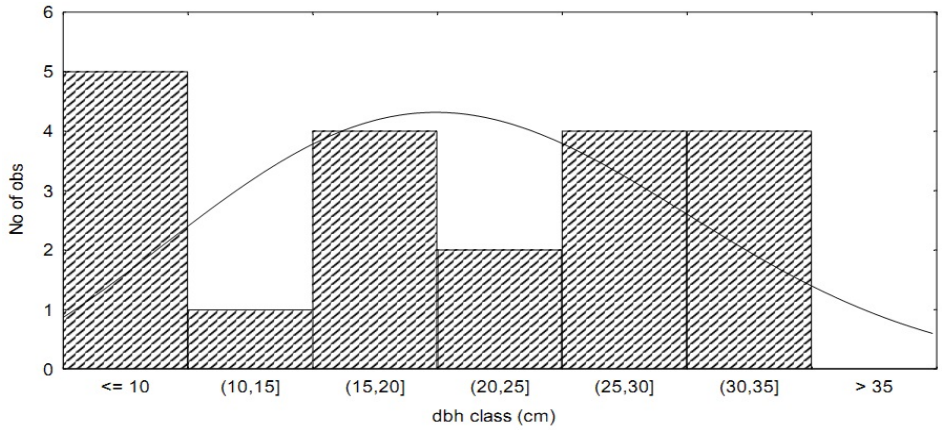

Figure 1. Weibull distribution curve in *Pinus caribaea* at Enugu, Ngwo, Nigeria.

Table 1. Descriptive statistics of stand and tree variables in *Pinus caribaea.*

	Stand variables (20 plots)				**Tree variable** (n=532)	
	dbh (cm)	**Age** (years)	**N** (trees/ha)	**H** (m)	**Dbh** (cm)	**BA** (m^2/tree)
Mean	21.30	36.15	665	11.05	21.78	0.04650
Standard deviation	9.40	0.88	251	0.98	10.86	0.04360
Min.	7.99	35	350	8.76	2.89	0.00066
Max.	33.78	37	1275	12.98	58.20	0.26611

Table 2. Values of the maximum likelihood estimators (MLE) of Weibull distribution.

Year of assessment	**Weibull parameters** (MLE)		
	a	**b**	**c**
1974	2.6497	12.706	7.8109
1974	1.8653	14.447	6.2268
1974	1.823	14.54	6.8934
1974	1.5092	17.903	16.624
1974	2.2665	27.551	3.8784
1974	1.4997	12.988	7.4892
1974	2.3531	23.235	12.795
1974	2.2665	27.551	3.8784
1974	0.0578	4.1869	22.251
1975	3.963	9.6308	2.6756
1975	1.9295	14.038	16.992
1975	1.5342	13.477	21.618
1975	3.6124	8.7059	1.3919
1975	2.9995	9.3702	0.1168
1976	1.9376	5.0762	4.4459
1976	2.1998	5.8333	3.2545
1976	1.7158	4.667	3.81
1976	2.3121	21.029	11.728
1976	1.6817	15.178	16.083
1976	1.6009	13.229	17.495

a = location parameter; b = scale parameter; c = shape parameter

Table 3. Correlation matrix for Weibull parameters and stand growth characteristics in *Pinus caribaea.*

	Dbh	QMD	Mean basal area	A	b	c
Dbh	1					
QMD	0.9487	1				
Mean basal area	0.9869	0.9526	1			
A	-0.4089	-0.4419	-0.3709	1		
B	0.6775	0.5468	0.6447	0.0893	1	
C	0.6204	0.6775	0.6345	-0.6217	-0.1481	1

CONCLUSION

The study has explored the predictability potential of the three parameter Weibull diameter distribution models of which maximum likelihood estimators (MLE) method was adopted in estimating the Weibul parameters. The estimated parameters were good predictors of the diameter Weibull probability distribution functions for *Pinus caribaea* in the study area. The study therefore revealed that diameter distribution of *Pinus caribaea* stand in the study area and other pine plantations in Nigeria could be predicted favourably using Weibull distribution function from maximum likelihood estimators (MLE).

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