AN ELEMENTARY MODEL TO STUDY SENSITIVITY OF THE DEPARTURE FROM NUCLEATE BOILING RATIO (DNBR) TO PERTURBATIONS IN NUCLEAR REACTOR SYSTEMS

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ABSTRACT
The heat flux in a water-cooled nuclear reactor system is used to estimate the Departure from Nucleate Boiling Ratio (DNBR) of the system which is an important engineering design parameter for nuclear reactors. The DNBR signifies an operational safety limit. During normal operation, there may be fluctuations in, for example, the power of the reactor, which can affect the temperature distribution along a coolant channel which may subsequently result in boiling leading to a reduction in heat transfer. This may further cause variations in the DNBR, which may reduce the safety margin requiring immediate remedial action. This work quantifies the sensitivity of the DNBR, and the location of the minimum DNBR due to power fluctuations obtained from a steady-state energy balance using radial heat conduction and convection. Results are presented for a reference PWR of 1893 MW(th). A future study will use the same analysis for the 300 MW(e) Chashma Nuclear Power Plant (Chasnupp).

Keywords: Nuclear Safety, DNBR

INTRODUCTION
A reactor flow channel is considered, with fuel assemblies between which the coolant flows vertically upwards at a given flow rate \( w \). The heat generated by nuclear fission in the fuel is removed by forced convection during which the coolant is sub-cooled. A thermal analysis is carried out on the central assembly. All non-central assemblies can be analyzed by including the Bessel function radial dependence of the heat flux. The temperature distributions found in the system are used to estimate the maximum temperature in the central fuel rod. Power and flow rate variations can be made to keep the DNBR within acceptable bounds.

ANALYSIS
An energy balance [1-4] on radial heat conduction from the fuel, to the cladding, and by convection from the cladding to the coolant, is carried out. For conduction in the fuel and cladding, the temperature is found from:

\[
\nabla^2 T(r, z) + \frac{q''(r, z)}{k} = 0 \quad \text{Eqn (1)}
\]

The volumetric fission heat source \( q'' \) in the fuel region given by

\[
q''(r, z) = \alpha j_s \left( \frac{2.405r}{R} \right) \cos \left( \frac{\pi z}{L} \right) \quad \text{Eqn (2)}
\]

where

\[
\alpha = \frac{1.16 P_{n_c}}{L_j (D_j/2)^2 N_j E_j} \quad \text{Eqn (2a)}.
\]

The heat in a section of volume \( dV = A_j dz \) is found from the equation:

\[
q''(r, z) A_j dz = w c_p dT_{b} \quad \text{Eqn (3)}
\]

Using Eqn (2) in Eqn (3) and integrating, gives [1] the coolant temperature \( T_b \)

\[
T_b(z) = T_{b,c} + \frac{q''_{max} A_j}{\pi c_p} \left( 1 + \sin \frac{\pi z}{L} \right) \quad \text{Eqn (3a)}
\]

This can be used to find the cladding temperature \( T_c \) from

\[
q''(r, z) A_j dz = 2\pi h R_c (T_c - T_b) \quad \text{Eqn (3b)}
\]

which in turn, is used to find the fuel temperature \( T_f \) for the central rod \( r=0 \)

\[
T_f(z) = T_b + \frac{q''_{max} A_j \cos \frac{\pi z}{L}}{2\pi h R_f} \quad \text{Eqn (3a)}
\]

The resistances \( R_i \) are given as:

\[
R_f = \frac{1}{4\pi h k f} \quad \text{Eqn (4a)}
\]

\[
R_c = \frac{\ln \left( 1 + \frac{b}{R_f} \right)}{2\pi h k c} \approx \frac{h}{2\pi h k c R_f} \quad \text{Eqn (4b)}
\]

\[
R_b = \frac{1}{2\pi h k b L b} \quad \text{Eqn (4c)}.
\]

The locations of the maxima of temperatures can be readily determined from the above equations. The departure from nucleate boiling ratio, DNBR, is an important design parameter for water-cooled reactors, and represents an operational safety limit. For PWR systems, a minimum value of 1.3 is usually specified. The DNBR is defined as the ratio of critical heat flux to actual heat flux

\[
DNBR(P, z) = \frac{q_c}{q_{actual}} \quad \text{Eqn (5)}.
\]
This work focuses on an analysis that quantifies the sensitivity of the DNBR to perturbations in the power $P$ of the system, and can be extended to include the fluid flow rate $w$. For the above, the actual heat flux can be found from the energy balance or more simply, from the heat generation, as

$$q^*_{\text{actual}}(z) = \frac{q^*(z)A_f}{2\pi R_c} \ldots \text{Eqn}(6).$$

The critical heat flux is determined from one of many correlations. This work uses the Bernath correlations for sub-cooled boiling:

$$T_{we} = 102.6 \ln P - \frac{972.2P}{P + 15} - 0.45V_c + 32 \ldots \text{Eqn}(7a)$$

$$h_c = 10890 \left( \frac{D_e}{D_c + D_f} \right) + 48V_c \ldots \text{Eqn}(7b)$$

$$q_c^* = h_c (T_{we} - T_{th}) \ldots \text{Eqn}(7c).$$

The validity of the above correlations: pressures between 23 and 3000 psia, fluid velocities between 4.0 and 54 ft/s, and $D_c$ between 0.143 and 0.66 inches. The burnout heat flux $q_c^*$, also called the critical heat flux (CHF), corresponds to the departure from nucleate boiling, which is also referred to as the boiling crisis on the heat flux vs T graph. This can be determined from the above correlations. For the heat transfer coefficient, we use the Dittus-Boelter equation for turbulent flow of water

$$Nu = CRe^m Pr^n \ldots \text{Eqn}(8)$$

with $C=0.023$, $m=0.8$, $n=0.4$ for water, non-metallic liquids and gases flowing through long, straight, and circular tubes.

**Variation of DNBR with Axial Dependence**

The DNBR, for a given set of operating parameters, varies along the length of the flow channel. At the coolant entrance, it is high indicating low heat flux, and correspondingly high critical heat flux, thus a system well within safety limits. As fuel temperature increases, the DNBR decreases until it reaches a minimum at a location $z_o$ which is readily determined from

$$\frac{\partial(DNBR)}{\partial z} = \frac{2\pi R_c h_c}{A_f} \frac{\partial}{\partial z} \left( \frac{T_{we} - T_{th}(P,z)}{q^*(P,z)} \right) = 0$$

$$\ldots \text{Eqn}(9)$$

$$z_o = \frac{L}{\pi} \sin^{-1} \left( \frac{\gamma (T_{we} - T_{ht,o})}{\gamma} \right) \ldots \text{Eqn}(10)$$

where $\gamma = \frac{q_{max}LA_f}{\pi wC_p}$. Thus the minimum DNBR is

$$DNBR_{\min}(P,z) = \frac{q_c^*}{q^*_{\text{actual}}} \bigg|_{z_o} \ldots \text{Eqn}(11)$$

**Variation of minimum DNBR with Power**

Using a first-order perturbation, the change in $DNBR$ with power can be expressed as

$$\delta (DNBR) \equiv \Gamma (P^\omega_{th},z) \delta P_{th} \ldots \text{Eqn}(12)$$

where

$$\Gamma (P^\omega_{th},z) = \frac{-2h_cR_cL}{(P_{th}^\omega wC_p)} \left(1 + \frac{\pi z}{L} \right) \cos \frac{\pi z}{L} \ldots \text{Eqn}(12a)$$

The change in $DNBR_{\min}$ is thus $\Gamma (P^\omega_{th},z_o)$.

**RESULTS**

The temperature profiles in cladding, coolant and fuel for a reference system [1] described in Tables 1-2, are shown in Figures 1-2. Peak temperatures (locations and magnitudes) as the power varies, and coolant velocity remains the same, are shown in Table-3.

**Table-1. Material Data**

<table>
<thead>
<tr>
<th>S.No</th>
<th>Material</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fuel</td>
<td>$\text{UO}_2$</td>
</tr>
<tr>
<td>2</td>
<td>Cladding</td>
<td>$\text{Zircalloy}$</td>
</tr>
<tr>
<td>3</td>
<td>Coolant</td>
<td>Water</td>
</tr>
</tbody>
</table>

**Table-2. Physical and Operating Data**

<table>
<thead>
<tr>
<th>S. #</th>
<th>Quantity</th>
<th>Units</th>
<th>Description</th>
<th>Ref Design</th>
<th>Chasnupp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Radius</td>
<td>$m$</td>
<td>Cylinder radius</td>
<td>1.7018</td>
<td>1.243</td>
</tr>
<tr>
<td>2</td>
<td>Height</td>
<td>$m$</td>
<td>Cylinder height</td>
<td>3.6576</td>
<td>2.900</td>
</tr>
<tr>
<td>3</td>
<td>$P_{th}$</td>
<td>$MW$</td>
<td>Thermal Power</td>
<td>1893</td>
<td>998.6</td>
</tr>
<tr>
<td>4</td>
<td>NFA</td>
<td>-</td>
<td>No. of fuel assemblies</td>
<td>193</td>
<td>121</td>
</tr>
<tr>
<td>5</td>
<td>RPA</td>
<td>-</td>
<td>Rods per assembly</td>
<td>204</td>
<td>204</td>
</tr>
<tr>
<td>6</td>
<td>$D_f$</td>
<td>$m$</td>
<td>Fuel rod diameter</td>
<td>0.0107</td>
<td>0.01</td>
</tr>
<tr>
<td>7</td>
<td>$L_f$</td>
<td>$m$</td>
<td>Fuel rod length</td>
<td>3.6576</td>
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</tr>
<tr>
<td>8</td>
<td>$E_d$</td>
<td>$MeV$</td>
<td>Energy deposited in fuel</td>
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<td>180</td>
</tr>
<tr>
<td>9</td>
<td>$E_r$</td>
<td>$MeV$</td>
<td>Energy given in reaction</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>10</td>
<td>$\Delta t$</td>
<td>$mm$</td>
<td>Cladding Thickness</td>
<td>0.6096</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Table-3. Peak Temperature vs Thermal Power

<table>
<thead>
<tr>
<th>S. #</th>
<th>Power Ratio (P/P₀)</th>
<th>Fuel Temperature</th>
<th>Cladding Temperature</th>
<th>DNBR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Temp (K)</td>
<td>Location (m)</td>
<td>Temp (K)</td>
</tr>
<tr>
<td>1</td>
<td>0.90</td>
<td>2305</td>
<td>0.0142</td>
<td>647</td>
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<tr>
<td>2</td>
<td>0.95</td>
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<td>3</td>
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<td>2495</td>
<td>0.0142</td>
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<td>0.0142</td>
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<tr>
<td>5</td>
<td>1.10</td>
<td>2686</td>
<td>0.0142</td>
<td>661</td>
</tr>
</tbody>
</table>

Figure-1. Coolant and cladding axial temperature profiles.

Figure-2. Central fuel rod axial temperature profile.

Figure-3. DNBR vs axial position in flow channel.

Figure-4 shows the variation of DNBRmin with Power. This occurs at a point that moves upwards, as can be seen from Figure-5. Similar curves can be found for Chashma Nuclear Power Plant (Chasnupp) [5] by using an appropriate cell.
CONCLUSION

A first-order analysis can be used to accurately estimate the change in DNBR, and the subsequent shift of the location of the most critical point along the flow channel, for power variations as large as 25% of normal operating power. An increase in the power level must be countered by an increase in the coolant velocity to keep the DNBR from exceeding the safety limit. For example, a 5% increase in power level (reference condition: 1893 MWth) reduces the minimum DNBR by approximately 13% (from 1.3785 to 1.205) which can be countered by a 5% increase in the coolant velocity. Other factors that need to be checked include the maximum fuel temperature which should also remain well below melting.

An important conclusion that emerges from this elementary model is that the location of the minimum DNBR shifts. This could be due to the constant pressure assumed in Bernath’s correlation.

Further work can be carried out in this field by incorporating neural networks as has been done by Kim et al [6], to predict the DNBR. It will also be useful to estimate the coolant flow changes required to offset the effect on the DNBR due to power fluctuations.

NOMENCLATURE

\( b \) = cladding thickness
\( h \) = heat transfer coefficient
\( k \) = thermal conductivity
\( q' \) = actual heat flux
\( q_c \) = critical heat flux
\( w \) = fluid flow rate
\( A_f \) = fuel cross-section area \( \pi R_f^2 \)
\( D_e \) = equivalent diameter
\( D_c \) = channel heated perimeter (feet)/\( \pi \)
\( N_f \) = number of fuel rods
\( Nu \) = Nusselt Number \( hD_e/k \)
\( P \) = pressure (psia)

\( Pr \) = Prandtl Number \( \mu C_p/k \)
\( Re \) = Reynolds Number \( \rho VD_e/\mu \)
\( R_c \) = cladding radius
\( R_f \) = fuel radius
\( T_{wc} \) = wall cladding temperature at the onset of the boiling crisis
\( V_c \) = coolant velocity (ft/s)

\( \rho \) = fluid density
\( \mu \) = fluid viscosity
\( C_p \) = specific heat capacity

REFERENCES


