SOME EXACT SOLUTIONS OF EQUATIONS OF MOTION OF A FINITELY CONDUCTING INCOMPRESSIBLE FLUID OF VARIABLE VISCOSITY IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD BY TRANSFORMATION METHOD

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ABSTRACT

Using transformation method some exact solutions of equations of motion of a finitely conducting incompressible fluid of variable viscosity in the presence of a transverse magnetic field are determined. These solutions consist of flows for which the vorticity distribution is proportional to the stream function perturbed by a uniform stream. Streamline patterns for some of the solutions are also presented.

Keywords: equation, navier stocks, MFD, transverse, flow, viscous, fluid, magnetic field.

1. INTRODUCTION

Various methods have been used by the researchers to determine exact solutions of the Navier-Stocks equations. These methods and the exact solutions determined through these methods can be found in references [1-20] and the references their in.

Recently Naeem RK and Muhammad Jamil[21] applied transformation method to determine a class of exact solutions to flow equations of an incompressible fluid of variable viscosity in which the vorticity distribution is proportional to the stream function perturbed by a uniform stream parallel to the x-axis.

In the present work, we extend Naeem RK and Muhammad Jamil approach to determine some exact solutions of equations of motion of a finitely conducting incompressible fluid of variable viscosity in the presence of a transverse magnetic field for which the vorticity distribution is proportional to the stream function perturbed by a uniform stream inclined to the x-axis.

The plan of this paper is as follows: In section 2 basic flow equations are considered and are transformed into a new system of equations using the transformation method. In section 3, some exact solutions of the new system of equations are determined. The transformation method used in determining the exact solutions to these equations is straightforward.

2. BASIC FLOW EQUATIONS

The non-dimensional equations governing the steady motion of a finitely conducting incompressible fluid of variable viscosity, in the presence of a magnetic field from Naeem RK and Najma Tayyab[22] are:

$$u_x + v_y + w_z = 0$$

$$u\,u_x + v\,u_y + w\,u_z = -p_x + \frac{1}{Re}\left[(2\mu u_x)_x + (\mu(u_y + v_x))_y + (\mu(w_x + u_z))_z\right] + R_H\left[H_3(H_{1z} - H_{3x}) - H_2(H_{2x} - H_{1y})\right]$$

$$u\,v_x + v\,v_y + w\,v_z = -p_y + \frac{1}{Re}\left[(2\mu v_y)_y + (\mu(v_z + w_y))_z + (\mu(u_y + v_x))_x\right] + R_H\left[H_1(H_{2x} - H_{1y}) - H_3(H_{3y} - H_{2z})\right]$$

$$u\,w_x + v\,w_y + w\,w_z = -p_z + \frac{1}{Re}\left[(2\mu\,w_z)_z + (\mu(w_x + u_z))_x + (\mu(v_z + w_y))_y\right]$$

$$+ R_H\left[H_2(H_{3y} - H_{2z}) - H_1(H_{1z} - H_{3x})\right]$$

$$(v\,H_1 - u\,H_2)_y - (u\,H_3 - w\,H_1)_z = \frac{1}{R_\sigma}\left(H_{1xx} + H_{1yy} + H_{1zz}\right)$$

$$(w\,H_2 - v\,H_3)_z - (v\,H_1 - u\,H_2)_x = \frac{1}{R_\sigma}\left(H_{2xx} + H_{2yy} + H_{2zz}\right)$$

$$(u\,H_3 - w\,H_1)_x - (w\,H_2 - v\,H_3)_y = \frac{1}{R_\sigma}\left(H_{3xx} + H_{3yy} + H_{3zz}\right)$$

$$u\,T_x + v\,T_y + w\,T_z = \frac{1}{Re\;Pr}\left(T_{xx} + T_{yy} + T_{zz}\right) + \frac{Ec}{Re}\Big[2\mu\left(u_x^2 + v_y^2 + w_z^2\right) + \mu\{(v_x + u_y)^2$$

$$+ (w_y + v_z)^2 + (u_z + w_x)^2\}\Big] + \frac{R_H\,Ec}{R_\sigma}\left[(H_{3y} - H_{2z})^2 + (H_{1z} - H_{3x})^2 + (H_{2x} - H_{1y})^2\right]$$

$$H_{1x} + H_{2y} + H_{3z} = 0 \tag{1}$$

The meanings of various symbols used here are given in the Nomenclature.
Considering the flow to be plane transverse flow, we have

$$(u, v, w) = (u, v, 0)$$
$$(H_1, H_2, H_3) = (0, 0, H) \tag{2}$$
$$(\bullet)_z = 0$$

The system of Eqs.(1), utilizing Eqs.(2), becomes

$$u_x + v_y = 0 \tag{3}$$

$$u\,u_x + v\,u_y = -P_x + \frac{1}{Re}\left[(2\mu\,u_x)_x + (\mu(u_y + v_x))_y\right] \tag{4}$$

$$u\,v_x + v\,v_y = -P_y + \frac{1}{Re}\left[(2\mu\,v_y)_y + (\mu(u_y + v_x))_x\right] \tag{5}$$

$$u\,H_x + v\,H_y = \frac{1}{R_\sigma}\left(H_{xx} + H_{yy}\right) \tag{6}$$

$$u\,T_x + v\,T_y = \frac{1}{Re\,Pr}\left(T_{xx} + T_{yy}\right) + \frac{Ec\,\mu}{Re}\left[2\left(u_x^2 + v_y^2\right) + \left(u_y + v_x\right)^2\right] + \frac{R_H\,Ec}{R_\sigma}\left(H_x^2 + H_y^2\right) \tag{7}$$

Where

$$P = p + R_H \frac{H^2}{2} \tag{8}$$

Equation (3) implies the existence of the stream function ψ such that

$$u = \psi_y \;, \quad v = -\psi_x \tag{9}$$

The system of Eqs.(3 – 7), on utilizing Eq. (9), transform into the following system of equations

$$\psi_x \, \omega = -J_x + \frac{1}{Re}\left[\mu\left(\psi_{yy} - \psi_{xx}\right)\right]_y \tag{10}$$

$$\psi_y \, \omega = -J_y - \frac{4}{Re}\left(\mu\,\psi_{xy}\right)_y + \frac{1}{Re}\left[\mu\left(\psi_{yy} - \psi_{xx}\right)\right]_x \tag{11}$$

$$\psi_y H_x - \psi_x H_y = \frac{1}{R_\sigma}\left(H_{xx} + H_{yy}\right) \tag{12}$$

$$\psi_y T_x - \psi_x T_y = \frac{1}{Re\,Pr}\left(T_{xx} + T_{yy}\right) + \frac{Ec\;\mu}{Re}\left[4\left(\psi_{xy}\right)^2 + \left(\psi_{yy} - \psi_{xx}\right)^2\right] + \frac{R_H\,Ec}{R_\sigma}\left(H_x^{\,2} + H_y^{\,2}\right) \tag{13}$$

Where the vorticity function ω and the generalized energy function J are defined by

$$\omega = -\left(\psi_{xx} + \psi_{yy}\right) \tag{14}$$

$$J = P + \frac{1}{2}\left(\psi_x^{\,2} + \psi_y^{\,2}\right) - \frac{2\,\mu\,\psi_{xy}}{Re} \tag{15}$$

The generalized energy function J, on using Eq.(8), becomes

$$J = p + R_H \frac{H^2}{2} + \frac{1}{2}\left(\psi_x^{\,2} + \psi_y^{\,2}\right) - \frac{2\,\mu\,\psi_{xy}}{Re} \tag{16}$$

Once a solution of system of Eqs. (10-13) is determined, the pressure p is obtained from Eq. (16).
Since we are interested in the solution of the system of Eqs. (10-13) when the vorticity distribution is proportional to the stream function, perturbed by a uniform stream, we set

$$\psi_{xx} + \psi_{yy} = K\left(\psi - U x - U y\right) \tag{17}$$

Where $K \neq 0$ and U are real constants.
On substituting

$$\Psi = \psi - U x - U y \tag{18}$$

and employing Eq.(17), the Eq. (14) becomes

$$\omega = -K\,\Psi \tag{19}$$

Equations (10) and (11), utilizing Eqs. (18) and (19), become

$$L_x = K\,U\,\Psi + \frac{1}{Re}\left[\mu\left(\Psi_{yy} - \Psi_{xx}\right)\right]_y \tag{20}$$

$$L_y = K\,U\,\Psi - \frac{4}{Re}\left(\mu\,\Psi_{xy}\right)_y + \frac{1}{Re}\left[\mu\left(\Psi_{yy} - \Psi_{xx}\right)\right]_x \tag{21}$$

Where

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$$L = J - \frac{K\Psi^2}{2}$$

Equations (20) and (21), on using the integrability condition $L_{xy} = L_{yx}$, provide

$$M_{xx} - M_{yy} - \frac{4}{Re}\left(\mu\Psi_{xy}\right)_{xy} + K\,U\,(\Psi_x - \Psi_y) = 0 \tag{22}$$

Where

$$M = \frac{\mu\left(\Psi_{yy} - \Psi_{xx}\right)}{Re}$$

Equation (22) is the equation that must be satisfied by the function Ψ and the viscosity μ for the motion of a finitely conducting incompressible fluid of variable viscosity in the presence of a transverse magnetic field in which the vorticity distribution is proportional to the stream function ψ perturbed by a uniform stream inclined to the x-axis.
Equation (12) and (13), employing Eq. (18), become

$$\left(\Psi_y + U\right)H_x - \left(\Psi_x + U\right)H_y = \frac{1}{R_\sigma}\left(H_{xx} + H_{yy}\right) \tag{23}$$

$$\left(\Psi_y + U\right)T_x - \left(\Psi_x + U\right)T_y = \frac{1}{Re\,Pr}\left(T_{xx} + T_{yy}\right) + \frac{Ec\,\mu}{Re}\left[4\left(\psi_{xy}\right)^2 + \left(\psi_{yy} - \psi_{xx}\right)^2\right] + \frac{R_H\,Ec}{R_\sigma}\left(H_x^{\,2} + H_y^{\,2}\right) \tag{24}$$

Let us now determine the solutions of the system of Eqs. (22 – 24) in the next section.

3. SOLUTIONS

In this section we determine some exact solutions of the system of Eqs. (22 – 24) as follows:
Equation (17), employing Eq.(18), becomes

$$\nabla^2\Psi = K\Psi \tag{25}$$

Plane wave solution of the Helmholtz Eq. (25) exists in the form

$$\Psi(x,y) = N(\xi) \tag{26}$$

Where

$$\xi = \left(x\cos\theta + y\sin\theta\right), \qquad -\pi \le \theta < \pi \tag{27}$$

Equation (25), on using Eq. (26), yields

$$N''(\xi) - K\,N(\xi) = 0 \tag{28}$$

Equation (28) possesses solutions for the following cases:

Case-I : $K = -n^2$, $n > 0$

Case-II: $K = m^2$, $m > 0$

We now consider these cases separately and also determine the solutions of Eqs. (22-24) for these cases as follows:
Our strategy is that first we find the function $\Psi(x, y)$ from Eqs. (26) and (28), and use this Ψ to determine μ, H and T from system of Eqs.(22-24).

CASE-I:

For this case the solution of Eq. (28) is given by

$$N(\xi) = A_1(\theta)\cos\left(n\,\xi + A_2(\theta)\right) \tag{29}$$

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Where

$$\xi = \left(x\cos\theta + y\sin\theta \right) \quad , \qquad -\pi \le \theta < \pi$$

and $A_1(\theta)$ and $A_2(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
The function $\Psi(x, y)$, therefore is given by

$$\Psi = A_1(\theta)\cos\left(n(x\cos\theta + y\sin\theta) + A_2(\theta)\right) \tag{30}$$

Equation (22), utilizing Eq. (30), becomes

$$Z_{\eta\eta} = A_3(\theta)\sin\eta \tag{31}$$

Where

$$Z = \mu\cos\eta \tag{32}$$

$$\eta = n\,\xi + B(\theta) \tag{33}$$

$$A_3(\theta) = \frac{\mathrm{Re}\,K\,U\left(\cos\theta - \sin\theta \right)}{n^3}$$

The solution of Eq. (31) is

$$Z = -A_3(\theta)\sin\eta + A_4(\theta)\,\eta + A_5(\theta) \tag{34}$$

Where $A_4(\theta)$ and $A_5(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
Equations (32) and (34) give

$$\mu = -A_3(\theta)\tan\eta + A_4(\theta)\,\eta\sec\eta + A_5(\theta)\sec\eta \tag{35}$$

Equation (23), utilizing Eq. (30), becomes

$$H_{\eta\eta} - A_6(\theta)\,H_\eta = 0 \tag{36}$$

Where

$$A_6(\theta) = \frac{R_\sigma\,U\left(\cos\theta - \sin\theta \right)}{n}$$

$$\eta = n\,\xi + A_2(\theta)$$

The solution of Eq. (36) is

$$H = A_7(\theta)\,e^{A6(\theta)\eta} + A_8(\theta) \tag{37}$$

Where $A_7(\theta)$ and $A_8(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
Equation (25), using Eqs. (30), (33), (35) and (37), becomes

$$T_{\eta\eta} + A_9(\theta)\,T_\eta = -2A_{10}(\theta)\sin 2\eta + A_{11}(\theta)\,\eta\cos\eta + A_{12}(\theta)\cos\eta + A_{13}(\theta)e^{2A6(\theta)\eta} \tag{38}$$

Where

$$A_9(\theta) = -\frac{U\,\mathrm{Re}\,\Pr(\cos\theta - \sin\theta)}{n}$$

$$A_{10}(\theta) = -\frac{K\,U\,\mathrm{Re}\,\Pr\,\mathrm{Ec}\,A_1^2(\theta)\,(\cos\theta - \sin\theta)}{4n}$$

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$$A_{11}(\theta) = -n^2 \Pr Ec\, A_1(\theta) A_4(\theta)$$

$$A_{12}(\theta) = -n^2 \Pr Ec\, A_1(\theta)\, A_5(\theta)$$

$$A_{13}(\theta) = \frac{Re \Pr Ec\, R_H\, A_6^2(\theta)\, A_7^2(\theta)}{R_\sigma}$$

The solution of Eq. (38) is

$$T = A_{14}(\theta)(2\sin 2\eta + A_9(\theta)\cos 2\eta) + A_{15}(\theta)(\sin\eta + A_9(\theta)\cos\eta) + A_{16}\,(A_9(\theta)\sin\eta - \cos\eta)$$
$$+ A_{17}(\theta)\,\eta\,(A_9(\theta)\sin\eta - \cos\eta) + A_{18}(\theta)\, e^{2A6(\theta)\eta} + A_{19}(\theta)\, e^{-A9(\theta)\eta} + A_{20}(\theta) \qquad (39)$$

Where

$$\eta = n\,\xi + A_2(\theta)$$

$$A_{14}(\theta) = \frac{A_{10}(\theta)}{4 + A_9^2(\theta)}$$

$$A_{15}(\theta) = \frac{A_{11}(\vartheta)}{1 + A_9^2(\theta)} + \frac{A_{11}(\theta)}{\left(1 + A_9^2(\theta)\right)^2}$$

$$A_{16}(\theta) = \frac{A_{12}(\theta)}{1 + A_9^2(\theta)} - \frac{A_9(\theta) A_{11}(\theta)}{\left(1 + A_9^2(\theta)\right)^2}$$

$$A_{17}(\theta) = \frac{A_{11}(\theta)}{1 + A_9^2(\theta)}$$

$$A_{18}(\theta) = \frac{A_{13}(\theta)}{2A_6(\theta)\,(2\,A_6(\theta) + A_9(\theta))}$$

Where $A_{19}(\theta)$ and $A_{20}(\theta)$ are real constants dependent on the parameter θ , and $-\pi \le \theta < \pi$.
The exact solutions of system of Eqs. (22- 24), in this case is given by

$$\mu = -A_3(\theta)\tan\eta + A_4(\theta)\eta\,\sec\eta + A_5(\theta)\,\sec\eta$$

$$H = A_7(\theta)e^{A6(\theta)\eta} + A_8(\theta)$$

$$T = A_{14}(\theta)(2\sin 2\eta + A_9(\theta)\cos 2\eta) + A_{15}(\theta)(\sin\eta + A_9(\theta)\cos\eta) + A_{16}\,(A_9(\theta)\sin\eta - \cos\eta)$$
$$+ A_{17}(\theta)\,\eta\,(A_9(\theta)\sin\eta - \cos\eta) + A_{18}(\theta)\, e^{2A6(\theta)\eta} + A_{19}(\theta)\, e^{-A9(\theta)\eta} + A_{20}(\theta)$$

The stream function ψ for this case is given by

$$\psi = U(x + y) + A_1(\theta)\cos\left(n(x\cos\theta + y\sin\theta) + A_2(\theta)\right) \qquad (40)$$

It represent a uniform stream U in the positive x direction plus a perturbation that is periodic in x and y. Some typical streamline patterns are given in Figures (1 - 4).

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CASE-II:

For this case

$$N(\xi) = B_1(\theta)\, e^{m\xi} + B_2(\theta)(\theta)\, e^{-m\xi}$$

Where $B_1(\theta)$ and $B_2(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
The function $\Psi(x, y)$, therefore is given by

$$\Psi(\xi) = B_1(\theta)\, e^{m\xi} + B_2(\theta)(\theta)\, e^{-m\xi} \tag{41}$$

Equations (22), utilizing Eqs. (41), becomes

$$G_{\xi\xi} = m^2 B_3(\theta)\, e^{m\xi} + m^2 B_4(\theta)\, e^{-m\xi} \tag{42}$$

Where

$$G = \mu \Psi \tag{43}$$

$$B_3(\theta) = \frac{K\ U\ Re\ B_1(\theta)\,(\cos\theta - \sin\theta)}{m^3}$$

$$B_4(\theta) = -\frac{K\ U\ Re\ B_2(\theta)\,(\cos\theta - \sin\theta)}{m^3}$$

The solution of Eq. (42) is

$$G = B_3(\theta)\, e^{m\xi} + B_4(\theta)\, e^{-m\xi} + B_5(\theta)\, \xi + B_6(\theta) \tag{44}$$

Where $B_5(\theta)$ and $B_6(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
Equation (44), on using Eqs. (41) and (43) give

$$\mu = \frac{B_3(\theta)\, e^{m\xi} + B_4(\theta)\, e^{-m\xi} + B_5(\theta)\, \xi + B_6(\theta)}{B_1(\theta)\, e^{m\xi} + B_2(\theta)\, e^{-m\xi}} \tag{45}$$

Equation (23), utilizing Eqs. (41) ,becomes

$$H_{\xi\xi} - B_7(\theta)\, H_{\xi} = 0 \tag{46}$$

Where

$$\xi = (x\cos\theta + y\sin\theta)\ , \qquad -\pi \le \theta < \pi$$

$$B_7(\theta) = R_\sigma U\ (\cos\theta - \sin\theta)$$

The solution of Eq. (46) is

$$H = B_8(\theta)\, e^{B7(\theta)\,\xi} + B_9(\theta) \tag{47}$$

Where $B_8(\theta)$ and $B_9(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.

Equation (25), using Eqs.(27), (41),(45) and (47), becomes

$$T_{\xi\xi} + B_{10}(\theta)\ T_{\xi} = -\Pr Ec\ m^4\ G\Psi + B_{11}(\theta)\, e^{2B7(\theta)\xi} \tag{48}$$

Where

$$B_{10}(\theta) = -\mathrm{Re}\,\mathrm{Pr}\ U\ (\cos\theta - \sin\theta)$$

$$B_{11}(\theta) = \frac{\mathrm{Re}\,\mathrm{Pr}\,\mathrm{Ec}\ R_H\ B_7^{\,2}(\theta)\ B_8^{\,2}(\theta)}{R_\sigma}$$

The solution of Eqs. (48) is

$$T = -\mathrm{Pr}\,\mathrm{Ec}\,m^4 \Big[B_{12}(\theta)\, e^{2m\xi} + B_{13}(\theta)\, e^{-2m\xi} + B_{14}(\theta)\, \xi\, e^{m\xi} + B_{15}(\theta)\, \xi\, e^{-m\xi} + B_{16}(\theta)\, e^{m\xi} + B_{17}(\theta) e^{-m\xi} + B_{18}(\theta)\, (\xi - 1) \Big] + B_{19}(\theta)\, e^{2B7(\theta)\xi} + B_{20}(\theta)\, e^{-B11(\theta)\xi} + B_{21}(\theta) \tag{49}$$

Where

$$\xi = (x\cos\theta + y\sin\theta), \qquad -\pi \le \theta < \pi$$

$$B_{12}(\theta) = \frac{B_1(\theta)\, B_3(\theta)}{2m\,(2m + B_{10}(\theta))}$$

$$B_{13}(\theta) = \frac{B_2(\theta)\, B_4(\theta)}{2m\,(2m - B_{10}(\theta))}$$

$$B_{14}(\theta) = \frac{B_1(\theta)\, B_5(\theta)}{m\,(m + B_{10}(\theta))}$$

$$B_{15}(\theta) = \frac{B_2(\theta)\, B_5(\theta)}{m\,(m - B_{10}(\theta))}$$

$$B_{16}(\theta) = \frac{B_1(\theta)\, B_6(\theta)}{m\,(m + B_{10}(\theta))} - \frac{B_1(\theta)\, B_5(\theta)}{m^2\,(m + B_{10}(\theta))} - \frac{B_1(\theta)\, B_5(\theta)}{m\,(m + B_{10}(\theta))^2}$$

$$B_{17}(\theta) = \frac{B_2(\theta)\, B_6(\theta)}{m\,(m - B_{10}(\theta))} + \frac{B_2(\theta)\, B_5(\theta)}{m^2\,(m - B_{10}(\theta))} + \frac{B_2(\theta)\, B_5(\theta)}{m\,(m + B_{10}(\theta))^2}$$

$$B_{18}(\theta) = \frac{B_1(\theta)\, B_4(\theta) + B_2(\theta)\, B_3(\theta)}{B_{10}(\theta)}$$

$$B_{19}(\theta) = \frac{B_{11}(\theta)}{2\, B_7(\theta)(2\, B_7(\theta) + B_{10}(\theta))}$$

Where $B_{20}(\theta)$ and $B_{21}(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
The exact solutions of system of Eqs.(5-10), in this case is given by

$$\mu = \frac{B_3(\theta)\, e^{m\xi} + B_4(\theta)\, e^{-m\xi} + B_5(\theta)\, \xi + B_6(\theta)}{B_1(\theta)\, e^{m\xi} + B_2(\theta)\, e^{-m\xi}}$$

$$H = B_8(\theta)\, e^{B7(\theta)\,\xi} + B_9(\theta)$$

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$$T = -\Pr Ec\, m^4 \Big[B_{12}(\theta)\, e^{2m\xi} + B_{13}(\theta)\, e^{-2m\xi} + B_{14}(\theta)\, \xi\, e^{m\xi} + B_{15}(\theta)\, \xi\, e^{-m\xi} + B_{16}(\theta)\, e^{m\xi}$$

$$+ B_{17}(\theta) e^{-m\xi} + B_{18}(\theta)\, (\xi - 1) \Big] + B_{19}(\theta)\, e^{2B_7(\theta)\xi} + B_{20}(\theta)\, e^{-B_{11}(\theta)\xi} + B_{21}(\theta)$$

For this case the stream function

$$\psi = U(x + y) + B_1(\theta)\, e^{m(x\cos\theta + y\sin\theta)} + B_2(\theta)\, e^{-m(x\cos\theta + y\sin\theta)} \tag{50}$$

represents a uniform stream U in the positive *x* direction plus a perturbation that is not periodic in *x* and *y*. Some typical streamline patterns are given in Figures (5-10).

If in particular we assume K = 0, then the solution of Eq. (21) is

$$N(\xi) = C_1(\theta)\, \xi + C_2(\theta), \qquad -\pi \le \theta < \pi$$

Where $C_1(\theta)$ and $C_2(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
The function $\Psi(x, y)$, therefore is given by

$$\Psi = C_1(\theta)\, \xi + C_2(\theta) \tag{51}$$

When K = 0, Eq. (22) is identically satisfied and therefore the viscosity μ is arbitrary and Eq. (23) provides the same solution as in case-II, that is

$$H = C_4(\theta)\, e^{C_3(\theta)\,\xi} + C_5(\theta) \tag{52}$$

Where $C_4(\theta)$ and $C_5(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
Employing Eqs. (51) and (52) in Eq. (24), we get

$$T_{\xi\xi} - U \,Re \Pr\, (\cos\theta - \sin\theta)\, T_\xi = \frac{Re\ \Pr R_H\, Ec\ C_3^{\,2}(\theta)\ C_4^{\,2}(\theta)}{R_\sigma} \tag{53}$$

The solution of Eq. (53) is

$$T = \frac{R_H\, Ec\ C_3^{\,2}(\theta)\, C_4^{\,2}(\theta)}{U R_\sigma\, (\cos\theta - \sin\theta)}\, \xi + C_8(\theta)\, e^{C_6(\theta)\xi} + C_9(\theta) \tag{54}$$

Where

$$\xi = (x\cos\theta + y\sin\theta), \qquad -\pi \le \theta < \pi$$

and $C_8(\theta)$ and $C_9(\theta)$ are real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
The exact solutions of system of Eqs. (22-24), in this case is given by

μ is arbitrary

$$H = C_4(\theta)\, e^{C_3(\theta)\,\xi} + C_5(\theta)$$

$$T = \frac{R_H\, Ec\ C_3^{\,2}(\theta)\, C_4^{\,2}(\theta)}{U R_\sigma\, (\cos\theta - \sin\theta)}\, \xi + C_8(\theta)\, e^{C_6(\theta)\xi} + C_9(\theta)$$

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K = 0, corresponds to an irrotational flow and it is the following uniform flow

$$\psi = \left(U + C_1(\theta)\cos\theta\right)x + \left(U + C_1(\theta)\sin\theta\right)y + C_2(\theta) \quad (55)$$

4. CONCLUSIONS

Some exact solutions of equations of motion of a finitely conducting incompressible fluid of variable viscosity in the presence of a transverse magnetic field are determined, using transformation method. These solutions consist of flows for which the vorticity distribution is proportional to the stream function perturbed by a uniform stream. Streamline patterns for some of the solutions are also presented. By assigning different values to the constants there in the solutions, we can generate more streamline patterns.

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NOMENCLATURE

u, v, w	__	non-dimensional velocity component
H_1, H_2, H_3	__	non-dimensional components of the magnetic field vector **H**
H	__	non-dimensional transverse components of the magnetic field vector **H**
p	__	non-dimensional pressure
T	__	non-dimensional temperature
μ	__	viscosity of the fluid
Re	__	Reynolds number
R_H	__	magnetic pressure number
R_σ	__	magnetic Reynolds number
Pr	__	Prandtl number
Ec	__	Eckert number
ψ	__	stream function
J	__	generalized energy function
ω	__	vorticity function
Ψ, L, M, N, Z, H, G, P	__	functions
x, *y*,ξ,η, θ	__	variables
K, U, m, n	__	real constants
$A_1(\theta),\ldots,A_{20}(\theta)$	__	real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
$B_1(\theta),\ldots,B_{21}(\theta)$	__	real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.
$C_1(\theta),\ldots,C_9(\theta)$	__	real constants dependent on the parameter θ, and $-\pi \le \theta < \pi$.

Subscripts

x, y, z, xx, yy	__	differentiation with respect to cartesian coordinates *x* and *y*.
ξ, η, ξξ, ηη	__	differentiation with respect to ξ and η.

Superscripts

″	__	differentiation with respect to the argument.

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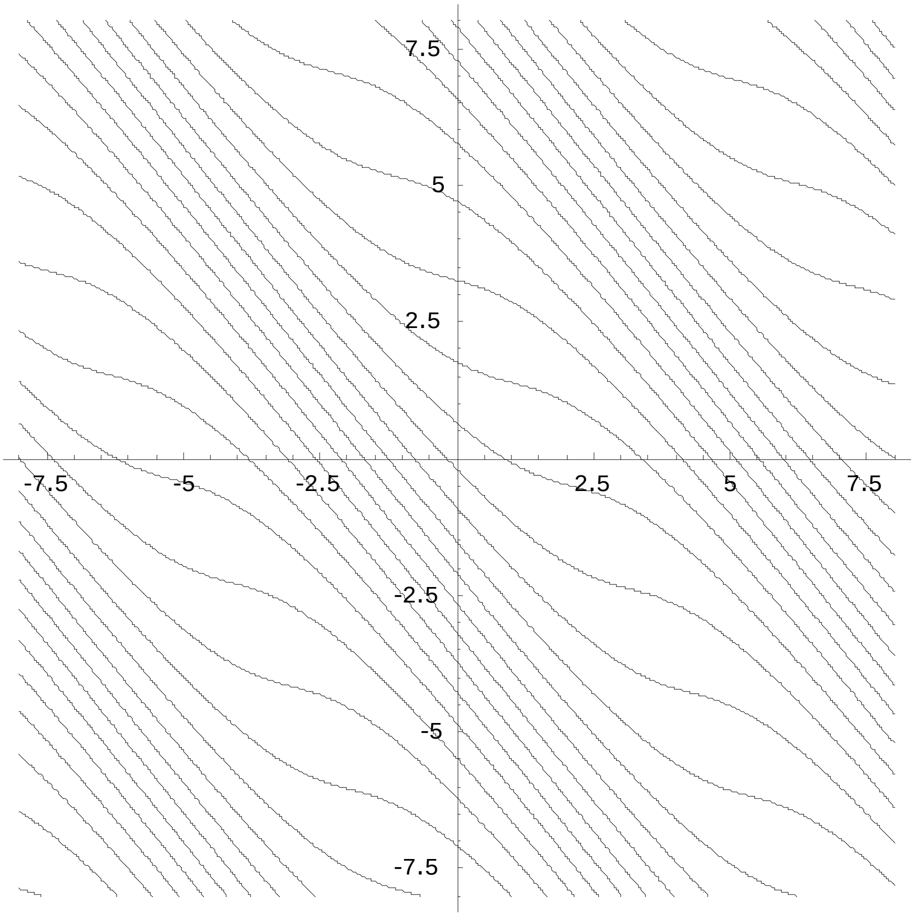

Figure-1. Streamline pattern for $\mathrm{Cos}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}y\right)+x+y=$ constant

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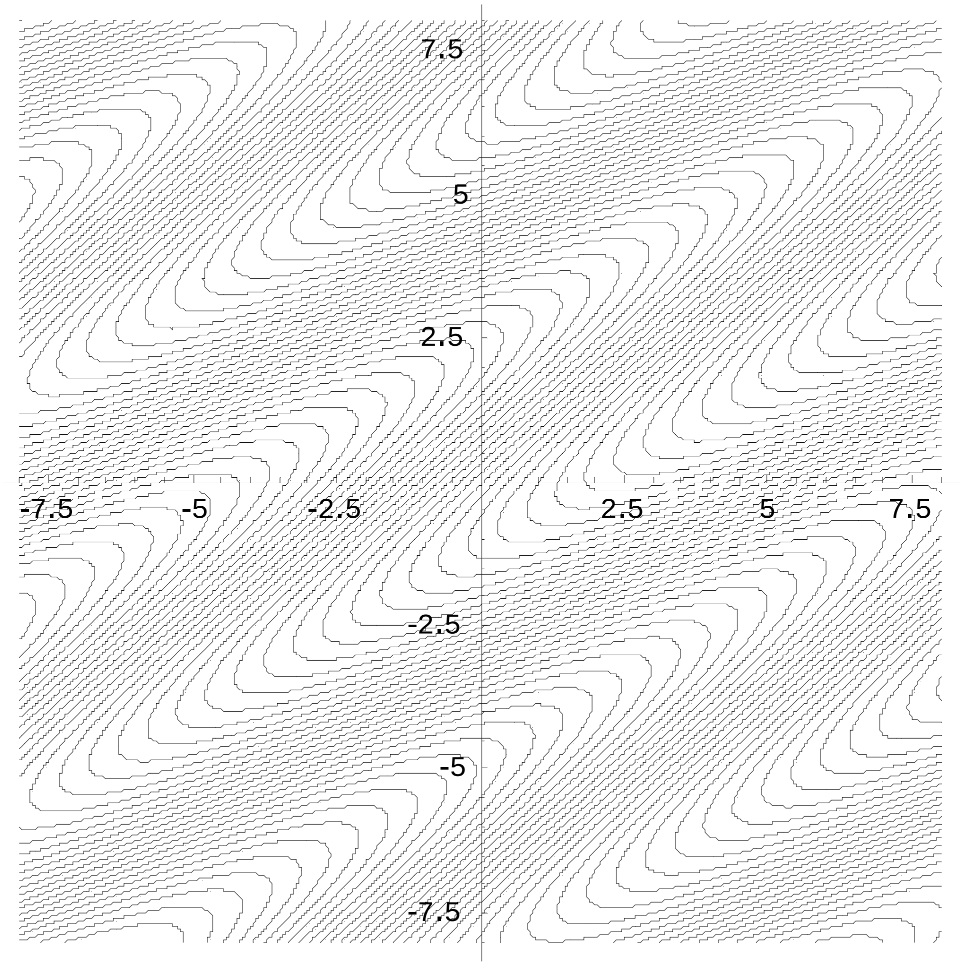

Figure-2. Streamline pattern for $7\,\mathrm{Cos}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}y - 1\right) + x + y = \text{constant}$

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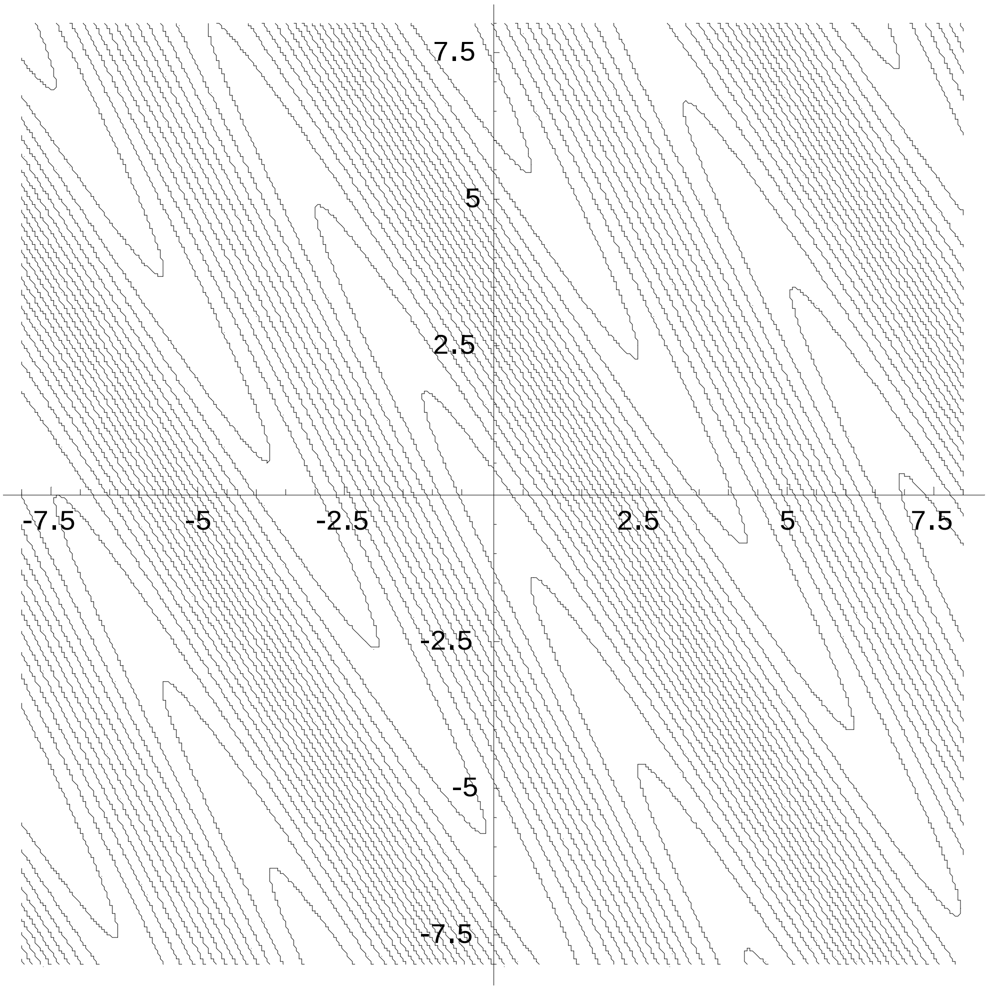

Figure-3. Streamline pattern for $-6\,\mathrm{Cos}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}y\right)+0.6\left(x+y\right)=\text{constant}$

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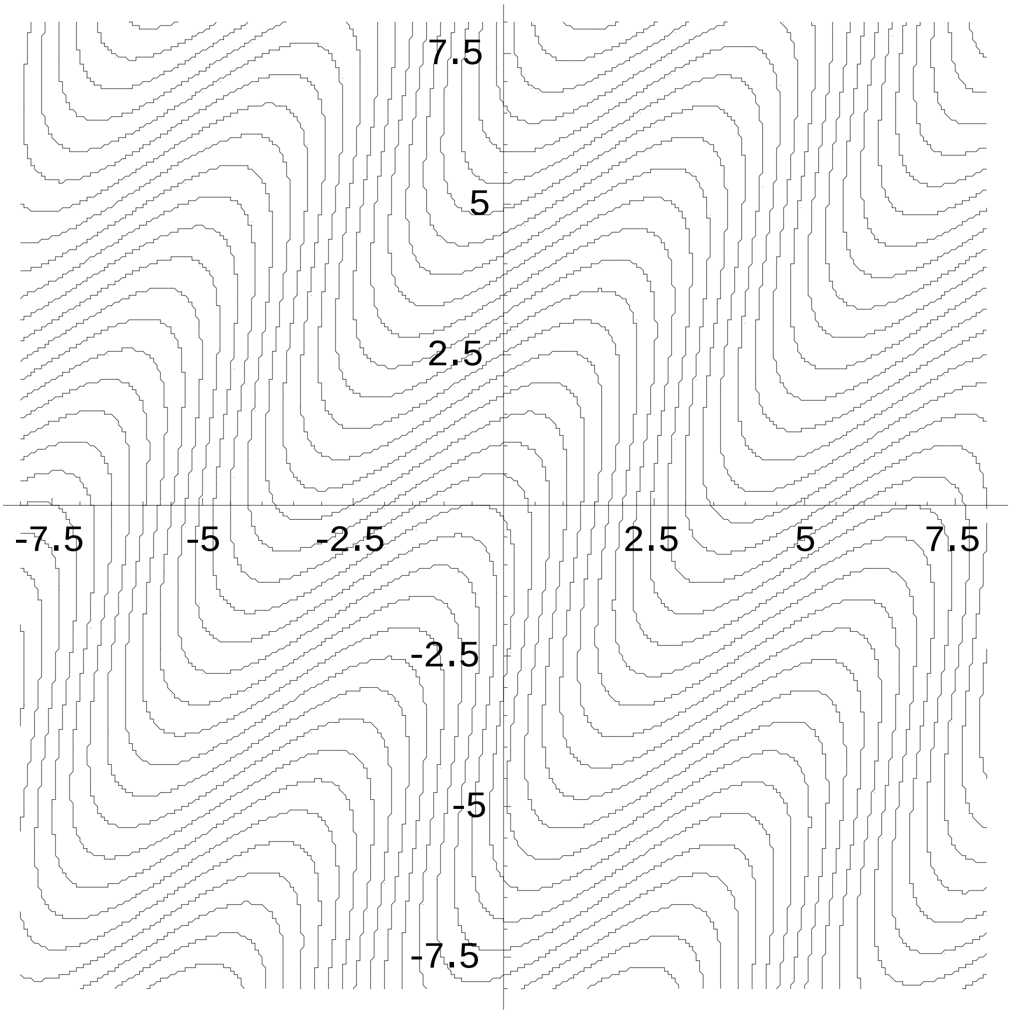

Figure-4. Streamline pattern for $-3\mathrm{Cos}\left(\frac{\sqrt{3}}{2}x-\frac{1}{2}y\right)+x+y=\text{constant}$

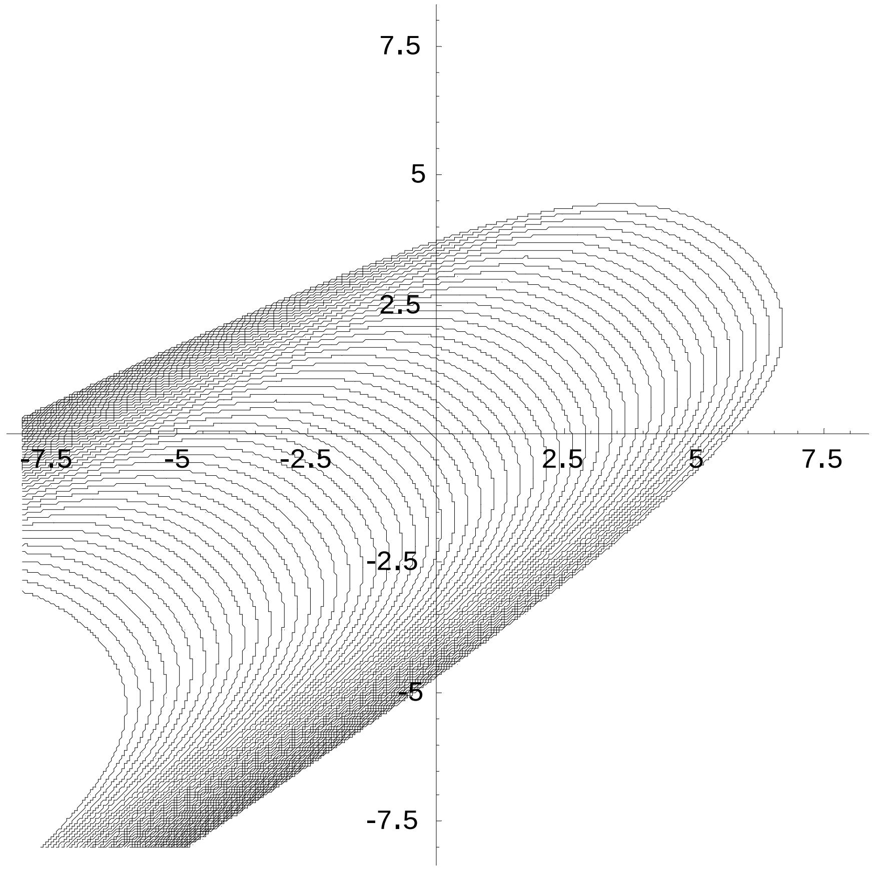

Figure-5. Streamline pattern for $\operatorname{Cosh}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}y\right) + 2\ (x\ +\ y)\ = \text{constant}$

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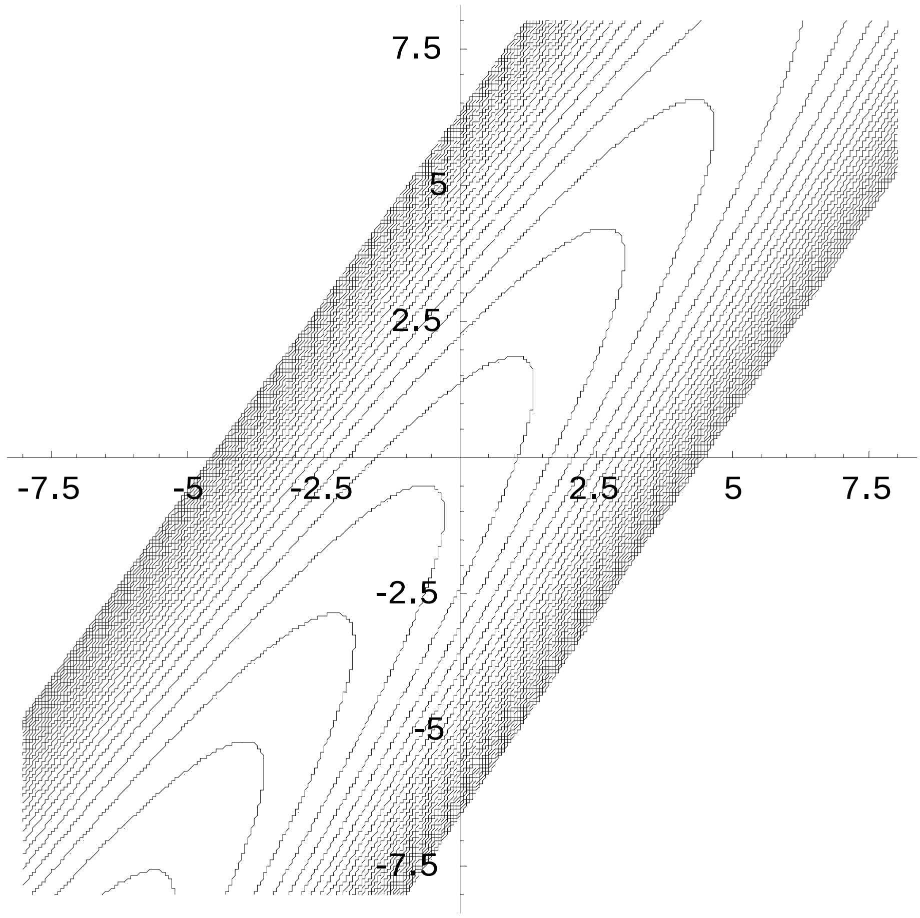

Figure-6. Streamline pattern for $\mathrm{Cosh}\left(0.819x - 0.574y\right) + 0.2\,(x + y) = \text{constant}$

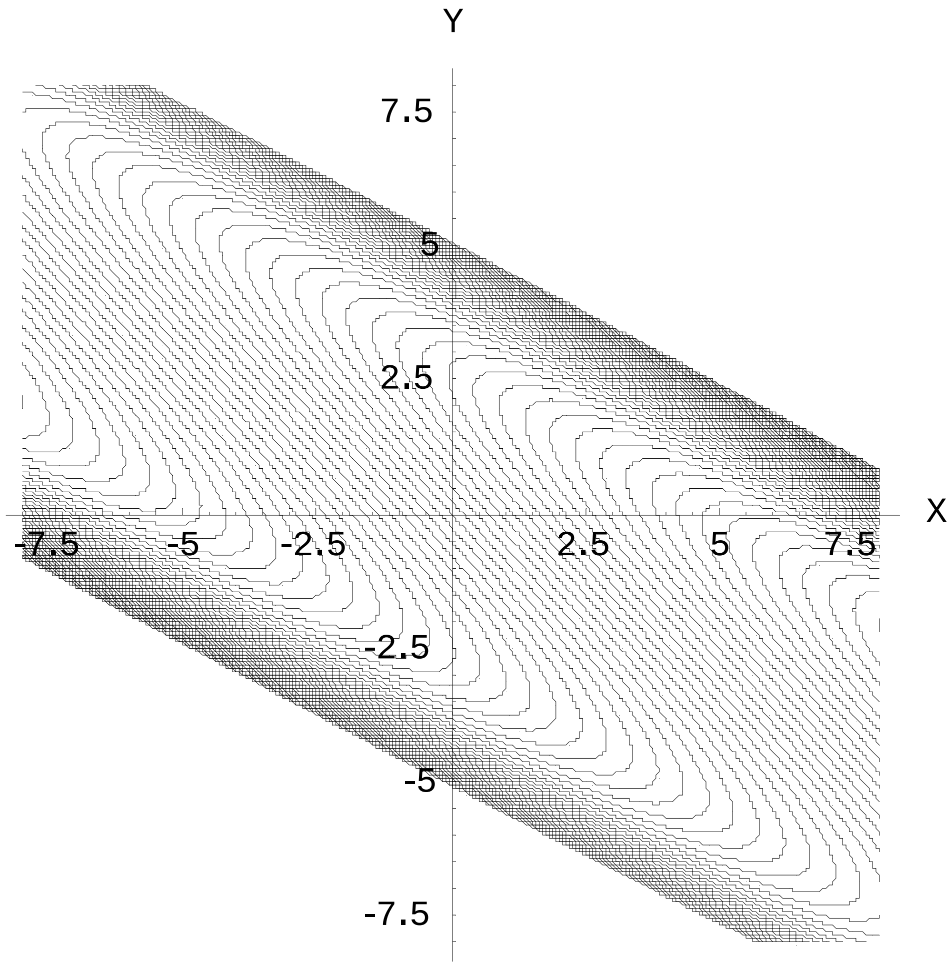

Figure-7. Streamline pattern for $\text{Sinh}\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right) - 4(x + y) = \text{constant}$

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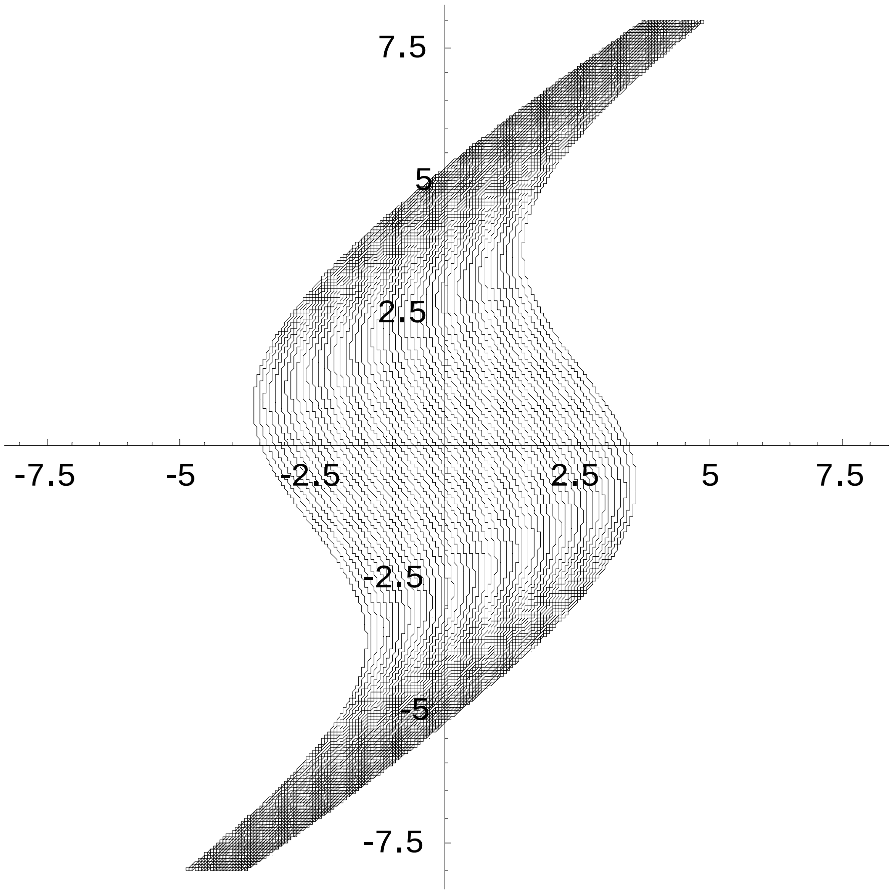

Figure-8. Streamline pattern for $\text{Sinh}\left(\frac{1}{2}x - \frac{\sqrt{3}}{2}y\right) + 5(x + y) = \text{constant}$

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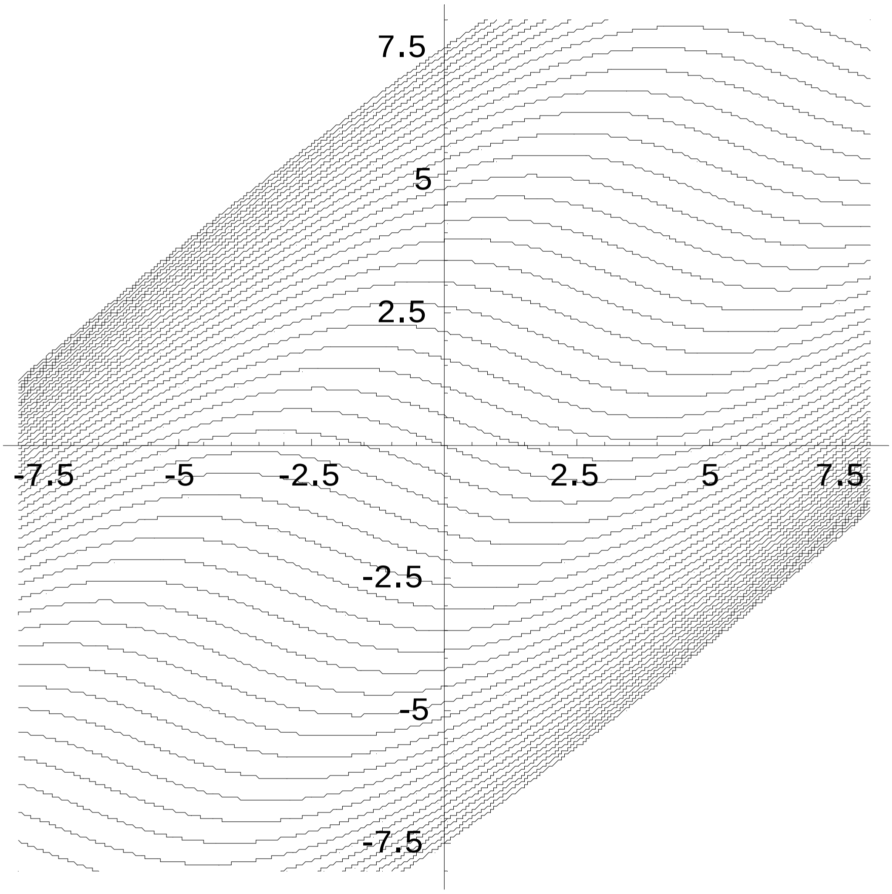

Figure-9. Streamline pattern for $\mathrm{Sinh}\left(0.61\left(\frac{x-y}{\sqrt{2}}\right)\right) - (x + y) = \text{constant}$

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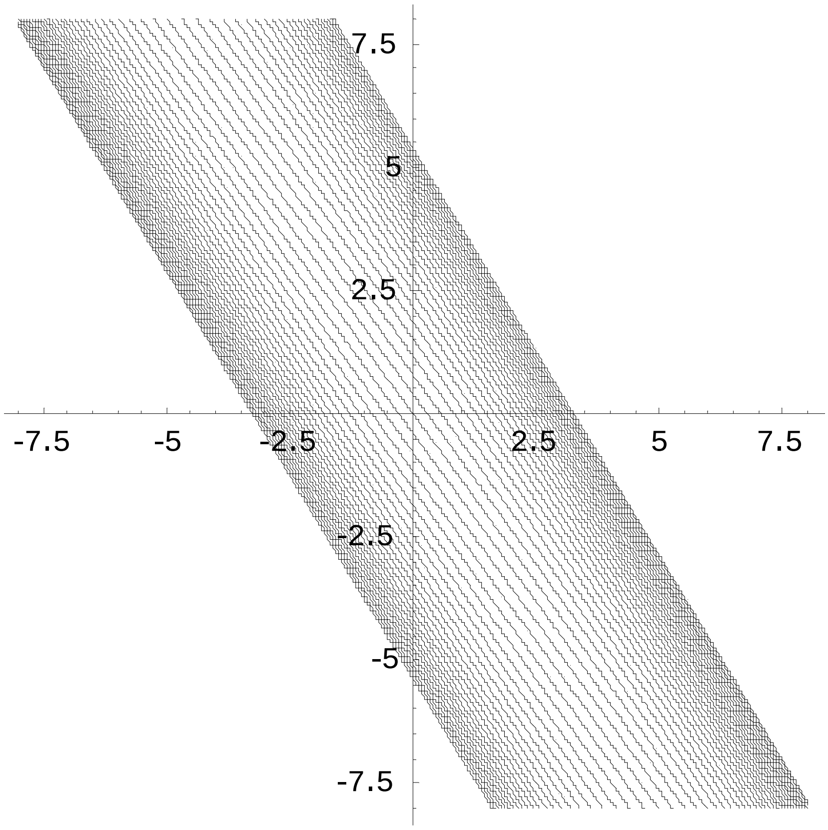

Figure-10. Streamline pattern for $\mathrm{Exp}\left(-\frac{\sqrt{3}}{2}x+\frac{1}{2}y\right)-\mathrm{Exp}\left(-\left(-\frac{\sqrt{3}}{2}x+\frac{1}{2}y\right)\right)-(x+y)=\text{constant}$