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PRIMITIVE VARIABLE APPROACH TO CALCULATE SEPARATION POINT IN LAMINAR BOUNDARY LAYER ALONG A FLAT PLATE

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ABSTRACT

Primitive variable approach has been used to model the Navier Stokes Equation. This is the method by which one can see what is going on in boundary layer. In the present work that primitive variable approach is considered to calculate incompressible, steady state laminar boundary layer along a flat plate with different flow conditions. This approach uses implicit finite difference method which consists of replacing the partial derivatives with respect to flow direction by finite differences. As a result the partial differential equations are approximated by ordinary differential equations. This method is capable of solving any flow problem for which the boundary layer equations remain valid. This implicit finite difference scheme is advantageous comparing to explicit finite difference scheme because the results are unconditionally stable regardless the step size. In the present work, the method is used to compute accurately the separation points of flow.

Keywords: Navier, stokes, equation, model, laminar, boundary, layer, implicit, finite, difference.

INTRODUCTION

The determination of the separation point in boundary layer flows has been the subject of many investigations over the past few decades. The solution of boundary layer calculation for a given pressure distribution has received a great deal of attention in the past century as well as present. With modern computers, it is now possible to obtain a very accurate solution for two dimensional laminar flow up to separation point. *Transition* and *turbulence* still require some empiricism, but various developments have come to the forefront to make general formulations more attractive.

For a fixed pressure distribution, the boundary layer equations become singular at separation point with back flow near the wall past separation, the equation become unstable in the down stream direction. Further more, when separation is involved, there is a strong interaction between the free stream and the boundary layer and the two can not be determined separately. To determine the separation point, the usual procedure is to apply numerical methods to the governing partial differential equations, compute the full- field solution, and thereby obtain the streamwise station at which the wall shear stress approaches zero. This solution procedure is not free from its difficulties; it is well known that the wall shear stress approaches zero in a singular fashion at the separation point, a fact that invariably gives rise to problems of numerical convergence there.

Primitive variable approach has been used to model the Navier Stokes equation. By this method one can see what is going on within the boundary layer. For the reason of efficient computation, the most popular codes use transformed variables- stream function coordinates, Falkner Skan stretching.

Finite difference method is used to solve either the Euler equation or the “thin –layer” Navier Stokes equation subjected to the arbitrary boundary conditions. It consists of replacing the partial derivative with respect to the flow direction by finite differences, so that the partial differential equation becomes approximated by an ordinary differential equation. Implicit finite difference scheme is used which is having more advantages compared to the explicit finite difference method, like the results are unconditionally stable, regardless of the step sizes.

The initial profiles are calculated using similarity variables to remove the singularity at the starting point to start the calculations. In transformed plane, boundary layer is nearly of uniform thickness for many flow situations. The thrust of the current work is towards combining general transformations, and implicit finite difference algorithms into available and versatile flow program. Comparisons are made with other exact solutions with the present method. It appears capable of solving any flow problem for which the boundary layer equations themselves remain valid.

REVIEW OF LITERATURE

Boundary layer theory is one of those inventions that allow a giant step to be taken to understand. The concept of a boundary layer was first introduced by Ludwig Prandtl, a German aerodynamicist, in 1904. Prandtl showed that many viscous flows can be analyzed by dividing the flow into two regions, one close to solid boundaries, the other covering the rest of the flow. Only in the thin region adjacent to a solid boundary, which is actually known as boundary layer, is the effect of viscosity important.

The boundary layer concept provided the link that had been missing between theory and practice. Furthermore, the boundary layer concept permitted the solution of viscous flow problems that would have been impossible through application of the Navier-Stokes equations to the complete flow field (Today, computer solutions of the Navier-Stokes equations are common). Thus the introduction of the boundary layer concept marked the beginning of the modern era of fluid mechanics. The idea of boundary layer completed the theory of attached flows at high Reynolds numbers and placed potential flow in its proper perspective. The step-by- step solution of the first order boundary layer equations was considered by Prandtl in 1938.

Kosson (1963) presented an approximate solution for two dimensional, incompressible, laminar boundary layer flow with arbitrary pressure gradient. Paskonov (1963) studied the solution of the boundary layer equations in the physical coordinates; using the Implicit finite difference scheme. The governing equations are replaced with finite differences such that the coupling between equations is initially neglected. Then an iteration process is employed to obtain desired accuracy of the dependent variables. In his work a procedure is described which allow the step size across the boundary layer to vary.

At about the same time, a similar implicit technique was developed independently for the boundary layer equations in physical coordinates by Blottner (1970). In this work coupling between the equations is allowed which was absent in previous work. This results in tri diagonal matrix with elements, which is some what more complicated to solve than the uncoupled equations. The transformed boundary layer equations were replaced by an implicit finite difference scheme and coupling between equations was included. Krause (1967) showed same approach of solution technique in his paper

One of the problems with all previous methods is the starting of the solution of the equations. Initial profiles of the dependent variables are required across the boundary layer at some point and then the solution proceeds downstream. For sharp bodies one would want to start the solution at the tip, where as for blunt body the solution should start at the stagnation point. At the tip of the sharp body, the boundary thickness goes to zero and the finite difference scheme is inappropriate in the physical coordinates.

So the boundary layer equations are transformed into similarity variables, then in the transformed plane boundary layer thickness is nearly of uniform thickness for many flow situations. The partial differential equations reduce to ordinary differential equations at the tip of the body or at stagnation point. The solution of this ordinary differential equations provides initial conditions for a finite difference solution, which can start at the beginning of the body.

In a paper by Fussell and Hellums (1965) an implicit finite difference procedure is applied to similarity form of a boundary layer equation. The method of treatment of boundary conditions involving normal derivatives has an important influence on accuracy. Moses (1985) described the method of solution using implicit finite difference approach for the viscous layer based on the strip integral equation. A line relaxation procedure was used for inviscid flow, solved simultaneously with the boundary layer in each step. Steger (1978) in his work showed solution technique of two dimensional geometries of any shape. The arbitrary shaped body can be rearranged into a definite shape so that it can be solved using finite difference formula. He mainly showed the blending of an implicit finite difference scheme with transformations that permit the use of automatic grid generation technique. Jameson et al (1986) introduced a different approach of solution of Euler equation. They used multigrid solution using implicit finite scheme. It was shown in that paper that the schemes of the approximate factorization type can be adapted for use in conjunction with a multigrid technique to produce a rapidly convergent algorithm for calculating steady state solution of the Euler equation.

Smith *et. al.* (1963) in his one paper presented a method for solving the complete incompressible laminar boundary layer equations, both two dimensional and axisymmetric, in essentially full generality and with speed. In subsequent papers (1970, 1972), he showed application potential flow and boundary layer theory in aerodynamics. He also showed the solution technique of the laminar boundary layers by means of differential difference method. Wehrle (1986) in later time presented an analytical scheme for determination of the separation point in laminar boundary layer flows. Unlike conventional approaches the scheme does not require the full-field solution of the governing partial difference equation, but rather the solution of a first order set of boundary layer equations defined in the neighborhood of the leading age.

In the present case solution of the boundary layer equations in untransformed or primitive variables approach is considered as of Blottner. At the starting point of solution the boundary layer equations are transformed to similarity variables. *Implicit finite difference* scheme is used to solve the boundary layer equations. The scheme is demonstrated to compute accurately the separation points of several flows for which comparison with previously published results are possible.

MATHEMATICAL MODEL

The influence of viscosity at high Reynolds number makes the Navier-Stokes equation simple and yield approximate solution. For the sake of simplicity, consider two dimensional flow of a fluid with very small viscosity, flowing over a surface. With the exception of the immediate neighborhood of the surface, the velocity, U_∞, and the pattern of the stream lines and the velocity distribution deviate only slightly from those in frictionless (potential) flow.

The two dimensional (2-D), incompressible equations of motion with constant transport properties are,

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

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x-momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (2.1)$$

y- momentum:

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 v}{\partial y^2} \qquad (2.2)$$

According to Prandtl the 2-D, steady, incompressible boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (3)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho}\frac{\partial P}{\partial x}=\nu\frac{\partial^2 u}{\partial y^2} \qquad (4)$$

With boundary conditions:

$$\begin{array}{ll} y=0 & u=0,\ v=0 \\ y=\infty & u=U(x) \end{array} \qquad (5)$$

The equations (3) and (4) are the simplified Navier Stokes Equation. It can be referred as the system of two simultaneous equations for the two unknown *u* and *v*. The pressure ceased to be an unknown function and can be evaluated from the potential flow solution for the body with the aid of the Bernoulli equation.

At the outer edge of the boundary layer, for steady flow, the parallel component u becomes equal to that in the outer flow U(x). Since there is no large velocity gradient at this point the equation (4) is further simplified by putting

$$U\frac{dU}{dx}=-\frac{1}{\rho}\frac{dP}{dx} \qquad (6)$$

Finally we get the simplified 2-D steady state boundary layer equations in the following form

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (7)$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\frac{\partial^2 u}{\partial y^2} \qquad (8)$$

METHOD OF SOLUTION

A simple and direct method, known as primitive variable approach, is used in this work. In this method the primitive variables (u , v) and (x , y) are used without transformation for various laminar boundary layers.

The finite difference model of each term of the equation (8) represent the level 'n' of the finite difference mesh. To keep the model at level 'n', combination of forward difference and central difference formula are used for different terms of the equation (8). All terms are centered at level 'n' for the numerical accuracy.

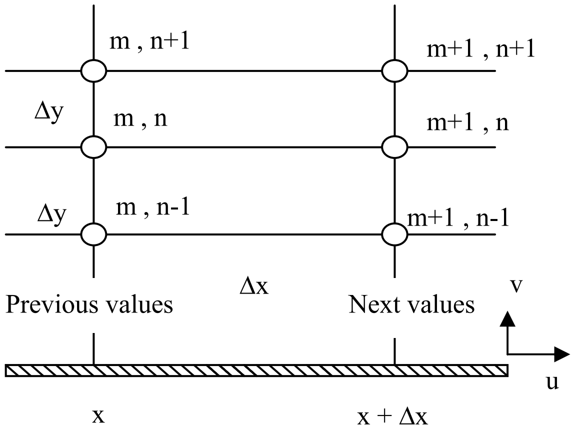

Figure-1. Finite difference mesh for a two dimensional boundary layer.

$$u_{m,n}\frac{u_{m+1,n}-u_{m,n}}{\Delta x}+v_{m,n}\frac{u_{m,n+1}-u_{m,n-1}}{2\Delta y}=\frac{U_{m+1}^2-U_m^2}{2\Delta x}+\frac{u_{m+1,n+1}-2u_{m+1,n}+u_{m+1,n-1}}{\Delta y^2} \qquad (9)$$

Multiplying the equation (9) by $\frac{\Delta x}{u_{m,n}}$ yields,

$$\left[u_{m+1,n}-u_{m,n}\right]+\frac{v_{m,n}\Delta x}{2u_{m,n}\Delta y}\left[u_{m,n+1}-u_{m,n-1}\right]=\frac{U_{m+1}^2-U_m^2}{2u_{m,n}}+\frac{u_{m+1,n+1}-2u_{m+1,n}+u_{m+1,n-1}}{\Delta y^2}\frac{\Delta x}{u_{m,n}} \qquad (10)$$

$$-\alpha u_{m+1,n+1}-\alpha u_{m+1,n-1}+(1+2\alpha)u_{m+1,n}= \Rightarrow u_{m,n}+\frac{U_{m+1}^2-U_m^2}{2u_{m,n}}-\beta\left[u_{m,n+1}-u_{m,n-1}\right] \qquad (11)$$

$$\text{Where, } \alpha=\frac{\Delta x}{\Delta y^2 u_{m,n}} \qquad \beta=\frac{v_{m,n}\Delta x}{2u_{m,n}\Delta y} \qquad (12)$$

Equation (11) has to be simultaneously solved for $u_{m+1,n}$.

To find the $v_{m+1,n}$ equation (7) is modeled using forward difference

$$\frac{u_{m+1,n}-u_{m,n}}{\Delta x}+\frac{v_{m+1,n}-v_{m+1,n-1}}{\Delta y}=0 \qquad (13)$$

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In this case the first term is at level 'n' and second term is at level $'n-\frac{1}{2}'$ leading to a poor numerical accuracy. Bringing $\partial u/\partial x$ down to $'n-\frac{1}{2}'$ by using an average value.

$$\left.\frac{\partial u}{\partial x}\right|_{avg} = \frac{1}{2}\left[\frac{u_{m+1,n}-u_{m,n}}{\Delta x}+\frac{u_{m+1,n-1}-u_{m,n-1}}{\Delta x}\right] \quad (14)$$

To solve for $v_{m+1,n}$, the required form of the equation is

$$v_{m+1,n} = v_{m+1,n-1} - \frac{\Delta y}{2\Delta x}\left[u_{m+1,n}-u_{m,n}+u_{m+1,n-1}-u_{m,n-1}\right] \quad (15)$$

Boundary conditions

No slip $u_{m,1}= v_{m,1} = 0$

Known initial profiles $u_{1,n}$, $v_{1,n}$

Inversion of Tri-Diagonal Matrix

Assuming that n=1 is the wall and n=N is the free stream , equation (11) represents (N-2) numbers of equations, each with three unknown namely $u_{m+1,n-1}$; $u_{m+1,n}$; and $u_{m+1,n+1}$. The set of algebraic equations are then written in tri-diagonal matrix form. Gauss elimination method is used to invert the matrix. The equation (11) can be rewritten in following form

$$A_n u_{m+1,n-1} + B_n u_{m+1,n} + C_n u_{m+1,n+1} = D_n \quad (16)$$

where,

$$A_n = -\alpha\left(m+1,n\right)$$
$$B_n = 1+2\alpha\left(m+1,n\right)$$
$$C_n = -\left(m+1,n\right)$$
$$D_n = \text{right hand side of the equation (11)}$$

There are only two unknowns at the bottom, n=2, where u_{n+1}=0 (no slip condition) and only two unknowns at the top, n=N, where u_n= U(x). Writing the equation for grid points across the boundary layer

$$\begin{aligned} A_2u_1+B_2u_2+C_2u_3 &= D_2\\ A_3u_2+B_3u_3+C_3u_4 &= D_3\\ A_4u_3+B_4u_4+C_4u_5 &= D_4 \end{aligned} \quad (17)$$

$$\begin{aligned} A_{n-1}u_{n-2}+B_{n-1}u_{n-1}+C_{n-1}u_n &= D_{n-1}\\ A_nu_{n-1}+B_nu_n+C_nu_{n+1} &= D_n \end{aligned}$$

Applying the boundary conditions u_1=0 and u_{n+1}=1, the first and last equations become

$$\begin{aligned} B_2u_2+C_2u_3 &= D_2\\ A_nu_{n-1}+B_nu_n &= D_n-C_n \end{aligned} \quad (18)$$

Beginning from the bottom and eliminating one variable at a time until reaching the top, where u_{n-1} is found. Following the back substitution, picking up in terms of u_{n+1} until reaching the value of u_2. After solving the tri diagonal matrix, the values $u_{m+1,n}$ are to be substituted in the equation (15) to get the normal velocity at all positions across the boundary layer. Therefore the solution process marches towards downstream by knowing the values of $u_{m+1,n}$ and $v_{m+1,n}$ at the next station.

Initial profiles

In the stagnation point (Hiemenz flow) or streaming flow over a flat plate (Blasius flow), two independent variables are combines into one dependent variable η.

$$\eta = \frac{y}{\delta(x)} \quad (19)$$

Where, $\delta(x)$ – scaling function for 'y'

Assuming a scaling function $u_e(x)$ to make the velocity profile similar.

$$\frac{u\,(x,y)}{u_e\,(x)} = f(\eta) \quad (20)$$

At this stage $u_e(x)$ and δ (x) are undetermined functions. Analysis of boundary layer equations will determine $u_e(x)$ and x. At any 'x' location the stream function is

$$\psi = \int_0^y \frac{\partial\psi}{\partial y}dy = u_e\delta\int_0^\eta f'\,d\eta \quad (21)$$

Since ψ = 0 at y = 0 for all x, f(0) = 0

$$\psi=u_e\,\delta\, f(\eta) \quad (22)$$

Vertical velocity can be found by applying the chain rules to the above equation.

$$v = -\frac{\partial\psi}{\partial x} = -f\left[u_e\delta\right]' + \eta\, u_e\, f\delta' \quad (23)$$

The momentum equation for a steady boundary layer flow is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = u_eu_e{}' + \frac{\partial^2 u}{\partial y^2} \quad (24)$$

Combining equations (19, 20) and equations (23, 24) yield

$$u_ef'\left[u_e{}'f'-\frac{\eta}{\delta}u_e\delta'f''\right]+\left[\eta\, u_e\delta'f'-f\left(\delta\, u_e\right)'\right]\frac{u_e}{\delta}f'' = u_eu_e{}' - \frac{\nu\, u_e}{\delta^2}f''' \quad (25)$$

Then the equation is written as

$$f'''+\alpha ff''+\beta(1-f'^2)=0 \quad (26)$$

Where the coefficients α and β are defined as

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$$\alpha = \frac{\delta}{\nu}\frac{d}{dx}\left(u_e\delta\right) \quad (27.1)$$

$$\beta = \frac{\delta^2}{\nu}\frac{du_e}{dx} \quad (27.2)$$

Equation (26) is an ordinary differential equation of f(η) only if α and β do not depend on 'x' and are constant.

The solution of the coupled equation can further be simplified by putting

$$2\alpha - \beta = \frac{1}{\nu}\frac{d}{dx}\left(\delta^2 u_e\right) \quad (28.1)$$

$$\Rightarrow (2\alpha - \beta)\,(x - x_0) = \tfrac{1}{\nu}\delta^2 u_e \quad (28.2)$$

Solving the equation (28.2) yield

$$\delta = \sqrt{\frac{(2\alpha - \beta)\,\nu x}{u_e}} \quad (28.3)$$

Generally u_e and x both will be positive; in certain situation x and u_e may have opposite signs.

$$\delta = \sqrt{\frac{\nu x}{u_e}} \quad (29)$$

The external velocity $u_e(x)$ is found by using the equations (27.2) and (28.3)

$$\beta = \pm\frac{x}{u_e}\frac{du_e}{dx} \quad (30)$$

After integration we get $u_e = u_0\,(x/L)^m$ (31)
where the u_0 and L are arbitrary constants and

$$m = \begin{cases} \beta \\ -\beta \end{cases} \text{ if } u_e \text{ and x have } \begin{cases} \text{Same sign} \\ \text{Opposite sign} \end{cases}$$

The equation that governs the stream function is equation (26) without α and β

$$f''' + \tfrac{1}{2}\,(m+1)ff'' + m(1-f'^2) = 0 \quad (32)$$

Where $$\eta = \frac{y}{\delta} = \frac{y}{\sqrt{\frac{\nu L}{u_0}\left(\frac{x}{L}\right)^{m+1}}} \quad (33)$$

The above equation is known as *Falkner-Skan* equation, Substituting m = β as considering u_e and x have the same sign.

$$f''' + \tfrac{1}{2}(\beta+1)ff'' + \beta\left(1 - f'^2\right) = 0 \quad (34)$$

The *Falkner-Skan* profiles supply most of the initial conditions like

β = 1 plane stagnation point
1<β<0 wedge half angle
β = 0 flat plate with sharp leading edge.

RESULTS AND DISCUSSION

The numerical calculation is applied to analyze different types of flow. The flows considered here are flowing over flat plate with sharp leading edge. The family of potential flows $u(x) = u_0 - ax^n$ (n = 1 , 2 , 3…) causes separation of the boundary layer (laminar) in a relatively short distance. This case provides a good test of the scheme for strong adverse pressure gradients since reliable results have been obtained by analytical and as well as numerical solutions. Howrath first gave the analytical solution of the problem.

In the simplest case with n = 1, which was treated by L. Howrath , is another example of a boundary layer for which the velocity profiles are not similar. L. Howrath introduced in this case a new independent variable

$$\eta = \frac{1}{2}\,y\sqrt{\frac{u_0}{\nu x}}$$

Another assumption is $x/L = x^* = \frac{ax}{u_0}$

A simple decelerating non similar velocity distribution for n = 1 as given by Howrath

$u(x) = u_0\,(1 - x/L)$ (35)

Where,

u_0= free stream velocity distribution
L = Reference length

As mentioned earlier, the implicit finite difference model is used to predict the separation point for the above non similar flow. Arbitrary values can be taken for u_0 and L, since the results are non-dimensional.

For sharp bodies the solution starts at the tip of the body where the boundary layer thickness is zero and the finite difference scheme is inappropriate. So the boundary layer equations are transformed into similarity variables in order to make the boundary layer thickness uniform.

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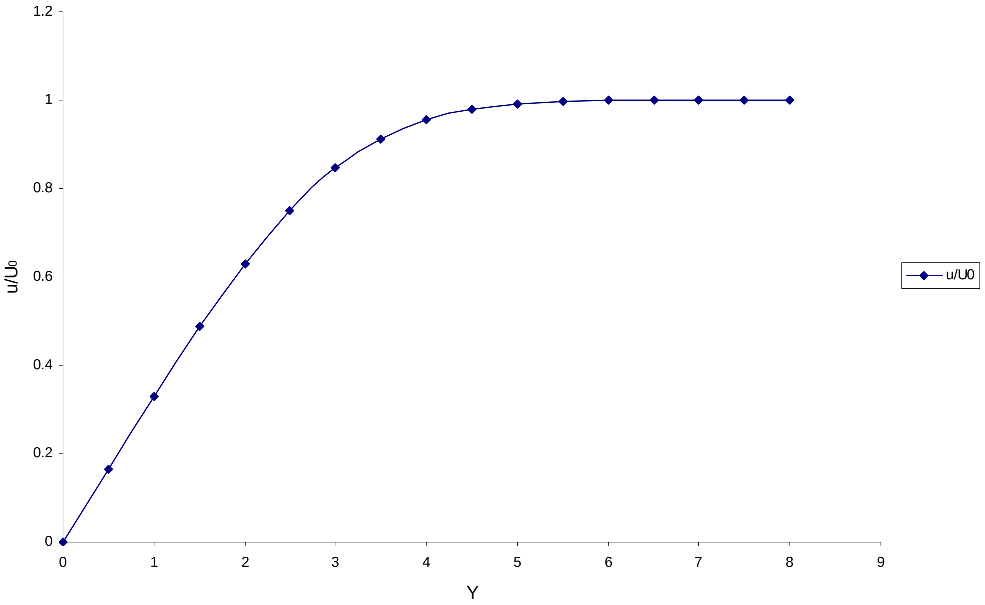

Figure-2. Velocity distribution in the boundary Layer along a flat plate, after Balsius.

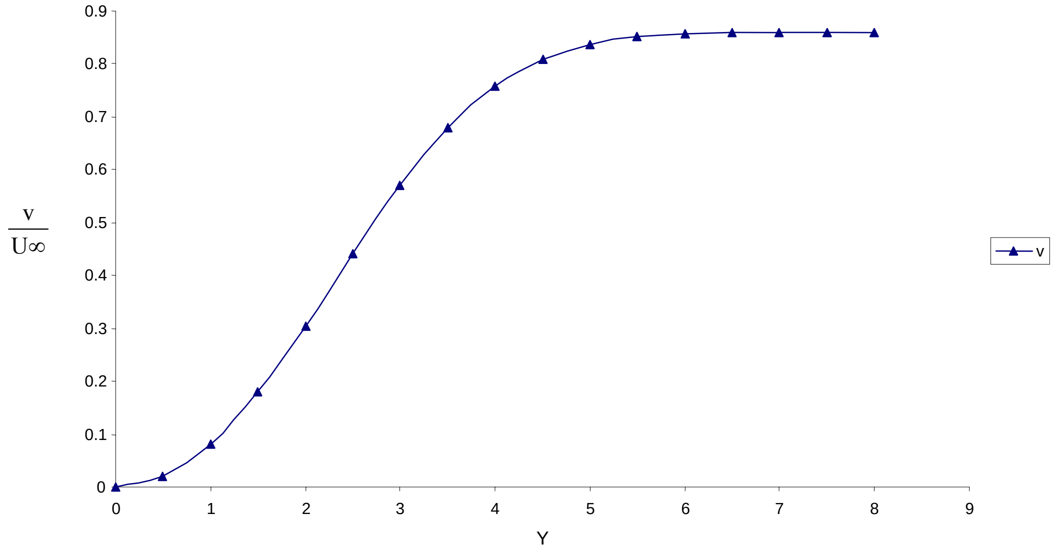

Figure-3. Transverse velocity component in boundary layer along a flat plate.

The non linear third order differential equation after simplification is solved numerically with this current method. After solution we get the velocity profiles of well know Blasius equation. Figure-3 depicts the velocity distribution in the boundary layer along a flat plate. It is clearly visible form the figure that the longitudinal velocity component beyond the boundary layer becomes same as free stream velocity. Within the boundary layer the obvious variation of velocity is observed. The velocity profile here posses a very small curvature at the wall and turns abruptly further in order to reach the asymptotic value.

The transverse velocity component is observed in the Figure-3. This velocity component exhibits same asymptotic nature beyond the boundary layer thickness. As flow moves from the leading edge the component differ form zero value. That means at the outer edge there is a flow outward which is due to the fact that increasing boundary layer thickness causes the fluid to be displaced form the wall as it flows along it. There is no boundary layer separation in these cases as the pressure gradient is equal to zero.

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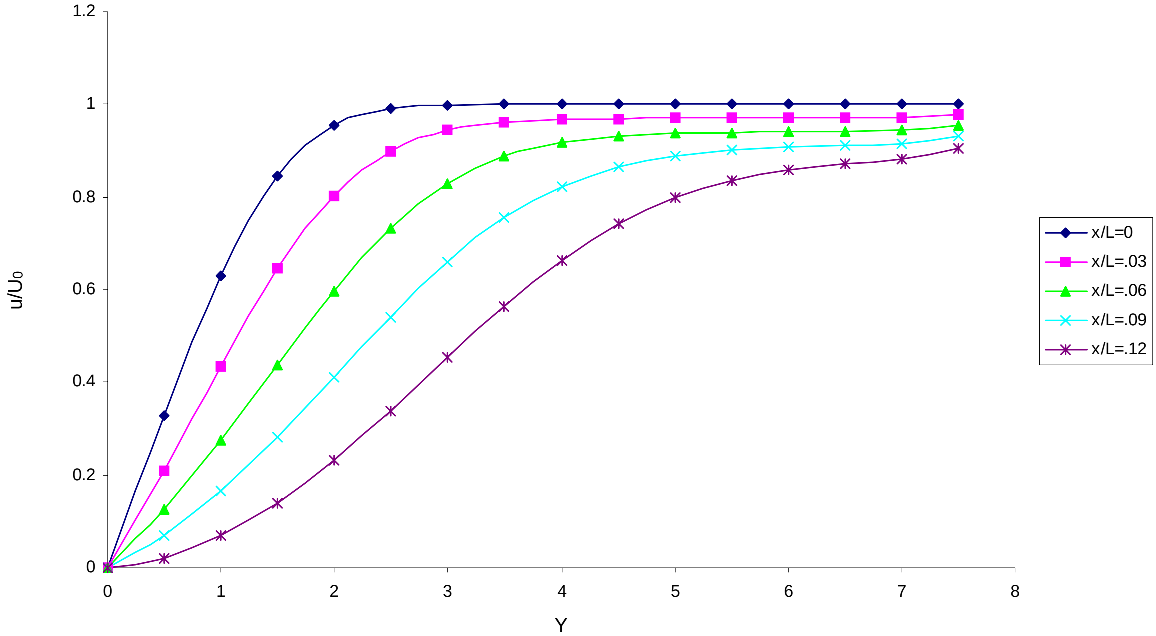

Figure-4. Velocity profiles for Howrath flow.

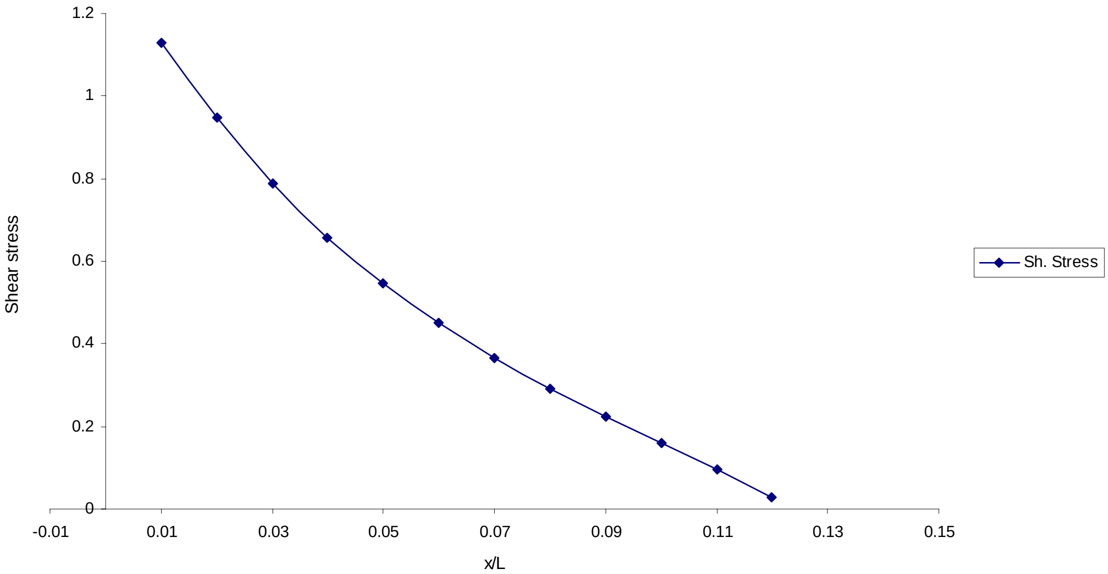

Figure-5. Shear stress distribution for Howrath flow.

The initial profiles are calculated for $\beta = 0$. The step sizes for the method of calculating the initial profile are $\Delta x = 0.01$ & $\Delta y = 0.1$. Calculation of the initial profile has to be specified accurately to have the agreeable results further downstream. Figure 4 shows the velocity profiles and figure 5 shows the computed values of shear stress for Howrath flow. The profiles become increasingly S-shaped in the downstream and finally the separation occurs at x/L ~0.120.

The exact separation point, i.e. the point where $f'' = 0$, can not be calculated for the linearly retarded flow because the boundary layer becomes singular there. That too the pressure distribution around the separation point can not be taken arbitrarily but must satisfy certain conditions connected with the existence of back flow downstream of separation. In the region of separation the solution becomes very sensitive to the value of f'', which makes it difficult to find the exact value that satisfies the outer boundary conditions.

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COMPARISON OF RESULTS OBTAINED BY PRESENT METHOD WITH CLASSICAL ANALYTICAL SOLUTIONS

Other classical solutions of the boundary layer equations for special u(x) are,

Tani Flow: $u = u_0(1 - x^{*n})$ (36)

where, $n = 2, 4, 8$ and $x^* = x/L$

$n = 2$; Quadratically retarded flow

$n = 4$; Quartically retarded flow

$n = 5$; Octally retarded flow

Gortler Flow: $u = u_0(1 - x)^n$ (37)

where, $n = \frac{1}{2}, 2$

$u = u_0(1 + x)^n$ (38)

$n = -1, -2$

For the Howrath – Tani type of retarded flows, a graph is drawn between the separation point and the value of 'n' as shown in figure 6. In the same manner graphs are drawn for Gortler flows for different values of 'n' as shown in Figures 7 and 8.

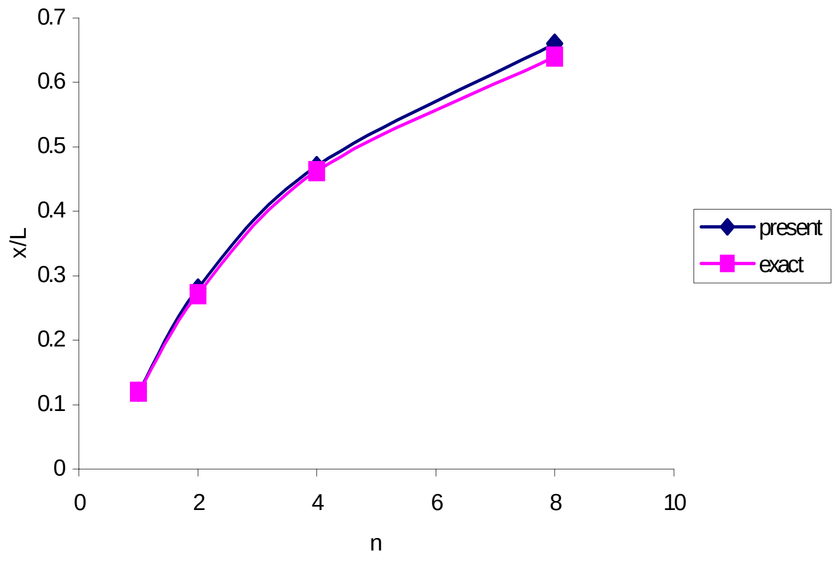

Figure-6. Comparison of separation of Howrath-Tani flows between present method and exact method for different values of exponent n.

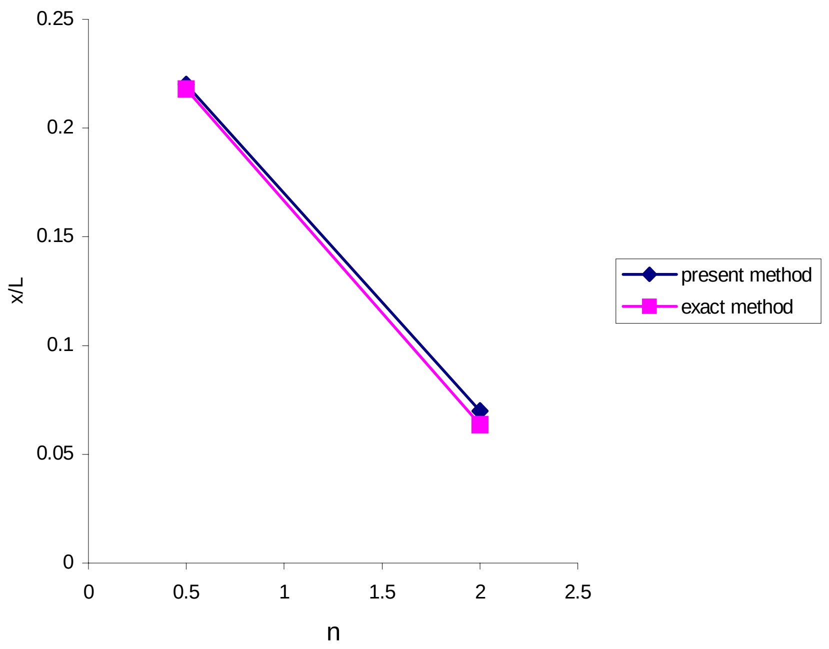

Figure-7. Comparison of separation of Gortler flow between present method and exact method for different values of exponent n; $[(1\text{-}x)^n$; n = 1/2 , 2].

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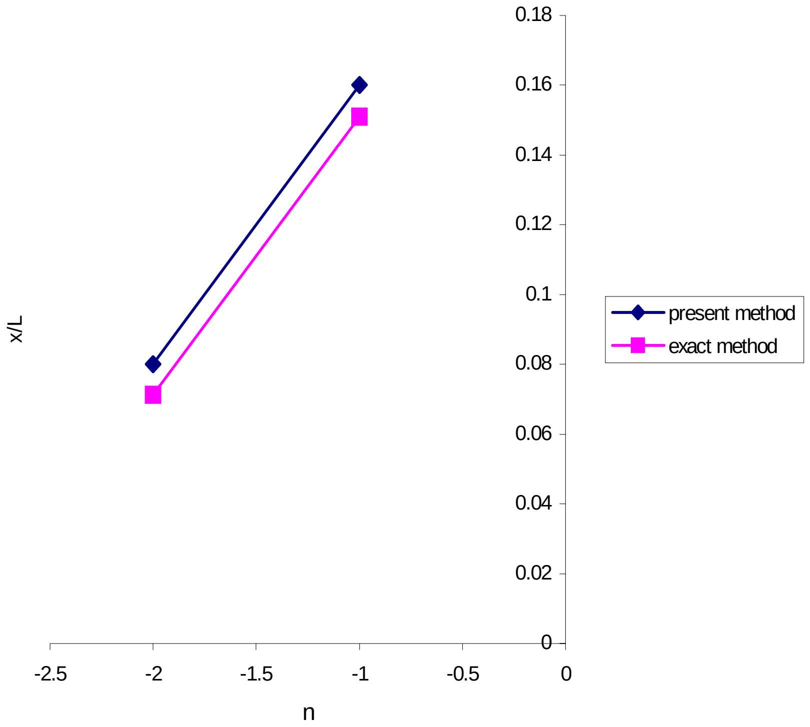

Figure-8. Comparison of separation of Gortler flow between present method and exact method for different values of exponent n; $[(1+x)^n$; n = -1 , -2].

Computational effort of the present method

Considering the above figures it is clearly visible that the present method to calculate the separation point of laminar boundary layer convincingly agrees with previously published results. This Implicit method is algebraic model of continuity and momentum equation which points on the downstream iteration. Computation time per step Δx is larger than for explicit scheme but the point to mention in favor of the method is that of no numerical instability. The step size can be as large as possible but it is only subjected to the normal truncation errors, which do not oscillate.

This Implicit finite difference scheme reveals that only those boundary flows can be calculated for which the normal velocity component at the initial station is specified in addition to the tangential velocity component. The specification of the 'v' becomes necessary as the implicit scheme is formulated by discarding the continuity equation and using the momentum equation to determine the 'x' derivatives of 'u'. The continuity equation is then employed to evaluate 'v' at the next 'x' station.

In this current method, the step sizes Δx and Δy need not to be equal. The results are unconditionally stable regardless of step sizes Δx and Δy. Δy is selected in such a way that 20 or more points exist within the boundary layer, and Δx should be small enough that changes in $u_{m,n}$ from station to station are less than 5 percent.

CONCLUSION

This new approach deals with the solution of the laminar boundary layers. We have presented in this work the primitive variable approach as calculating tool of the boundary layer problem. Primitive variable approach has given the results which are matching with the previously published results quite accurately.

It has been concluded that implicit finite difference method along with the transformations to remove singularity at the starting point is very useful in analyzing the different types of flows. In this approach visualization of the things can be done which is not possible in transformation plane like Falkner-Skan stretching, stream function coordinates.

This method is highly depends on the initial profile as well as derivation of the ordinary differential equations. The overall analysis is extremely sensitive, if the separation region is relatively small, as the method does not involve iteration process.

NOMENCLATURE

u ,v	velocity components
Δx	step size in 'x' direction
Δy	step size in 'y' direction
x , y	coordinates along and normal to the surface of

	the body
$f(\eta)$	function related to stream function
L	reference length
μ	viscosity coefficient
τ	shear stress
ψ	stream function
ν	kinematic viscosity
η	transformed 'y' coordinate in the boundary layer equation
U	velocity of the main stream at the edge of the boundary layer
m	exponent in free stream velocity variation of similar flow, $U = cx^m$
α , β	finite difference mesh size parameters
δ	boundary layer thickness

Subscripts

∞	free stream conditions
e	external conditions
i	designation of mesh point in x-direction
J	designation of mesh point in y-direction

Superscript

primes (′) differentiation with respect to η

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