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FREQUENCY ANALYSIS OF CONSECUTIVE DAYS MAXIMUM RAINFALL AT BANSWARA, RAJASTHAN, INDIA

S. R. Bhakar, Anil Kumar Bansal, Neeraj Chhajed and R. C. Purohit
Department of Soil and Water Engineering, CTAE, Udaipur, Rajasthan, India
E-mail: srbhakar@rediffmail.com

ABSTRACT

Annual one day maximum rainfall and two to five days consecutive days maximum rainfall corresponding to return period varying from 2 to 100 years are used by design engineers and hydrologists for the economic planning, design of small and medium hydrologic structures and determination of drainage coefficient for agricultural fields. A maximum of 154.31mm in 1 day, 250.88mm in 2 days, 270.15mm in 3 days, 284.18mm in 4 days and 295.54mm in 5 days is expected to occur at Udaipur, Rajasthan every 2 years. For a recurrence interval of 100 years, the maximum rainfall expected in 1 day, 2, 3, 4 and 5 days is 773.6mm, 849.34mm, 874.19mm, 931.78mm and 957.89mm, respectively. The magnitudes of 1 day as well as 2 to 5 consecutive days annual maximum rainfall corresponding to 2 to 100 years return period were estimated using Gamma function. Various probability distributions and Transformations can be applied to estimate one day and two to five consecutive days annual maximum rainfall of various return periods. Three commonly used probability distributions (viz: Normal, Log Normal and Gamma distribution) were tested by comparing the Chi-square value. Gamma distribution was found to be best fit for the region.

Keywords: frequency, analysis, rainfall, gamma, distribution, probability.

INTRODUCTION

Most of the hydrological events occurring as natural phenomena are observed only once. One of the important problem in hydrology deals with interpreting past records of hydrologic events in terms of future probabilities of occurrence. The procedure for estimating frequency of occurrence of a hydrological event is known as frequency analysis.

Analysis of consecutive days maximum rainfall of different return periods is a basic tool for safe and economical planning and design of small dams, bridges, culverts, irrigation and drainage work etc. Though the nature of rainfall is erratic and varies with time and space, yet it is possible to predict design rainfall fairly accurately for certain return periods using various probability distributions (Upadhaya and Singh, 1998). Frequency analysis of rainfall data has been attempted for different places in India (Jeevrathnam and Jaykumar, 1979; Sharda and Bhushan, 1985; Prakash and Rao, 1986; Aggarwal *et al.*, 1988; Bhatt *et al.*, 1996; Mohanty *et al.*, 1999; Rizvi *et al.*, 2001; Singh, 2001).

At present a few studies have been done in India and these studies were mainly carried out to validate the statistical procedure of different types of probability distribution function, viz., Normal, Log Normal and Gamma, keeping in view the importance of watershed development programme. In the present paper frequency analysis of annual 1 day and 2 to 5 consecutive days maximum rainfall data of Banswara has been carried out using frequency for watershed management planning in South Eastern Rajasthan.

MATERIALS AND METHODS

The daily rainfall data recorded at Agricultural Research Station, Banswara (23° 33'N latitude, 74° 27'E longitude and 220 m above MSL) for a period of 21 years (1971-1991) were used in this analysis. The daily data in a particular year is converted to 2 to 5 days consecutive days rainfall by summing up the rainfall of corresponding previous days. The maximum amount of 1 day to 2 to 5 days consecutive days rainfall for each year was taken for analysis. The statistical parameters of annual 1 day as well as consecutive days maximum rainfall are shown in Table-1. One day to five day maximum rainfall data were fitted to various probability distribution functions (Table-2).

Testing the goodness of fit

Comparing the theoretical and sample values of the relative frequency of the cumulative frequency function can test the goodness of fit of a probability distribution. In case of the relative frequency function, the Chi-square test is used. The sample value of the relative frequency of interval i is calculated by the following equation:

$$f_s(x_i) = \frac{n_i}{n} \qquad \text{..... (1)}$$

The theoretical value of the relative probability function is

$$p(x_i) = F(x_i) - F(x_{i-1})$$

The Chi-square test statistic χ_c^2 is given by the equation (2)

$$\chi_c^2 = \sum_{i=1}^{m} n \left[\frac{(f_s(x_i) - p(x_i))^2}{p(x_i)} \right] \qquad \text{..... (2)}$$

$n f_s(x_i) = n_i$ is the observed number of occurrences in interval i

$n\,p(x_i)$ is the corresponding expected number of occurrences in interval i

The χ^2 distribution functions are tabulated in many statistics texts. In the χ^2 test,

ν = m-p-1

A confidence level is chosen for the test, it is often express as 1-α, where 'α' is termed as the significant level. A typical value for the confidence level is 95 per cent. The null hypothesis for the test is that the proposed probability fits the data adequately. This hypothesis is rejected if the value of $\chi_c^{\,2}$ is larger than a limiting value, $\chi^2_{\nu,\,1-\alpha}$ (which is determined from the χ^2 distribution with ν degree of freedom at 5 % level of significance. Otherwise it was rejected.

Frequency analysis using frequency factors

Chow (1988) has shown that many frequency analyses can be reduced to the form

$$X_T = \overline{X}(1 + C_V\, K_T) \qquad \text{..... (3)}$$

For Normal and Log Normal distribution, the frequency factor can be expressed by the following equation (Chow, 1988)

$$K_T = \frac{(x_T - \mu)}{\sigma} \qquad \text{..... (4)}$$

This is the same as the standard normal variable z. The value of z corresponding to an exceedence of p (p = 1/T) can be calculated by finding the value of an intermediate variable w:

Where,

$$w = \left[\ln\left(\frac{1}{p^2}\right)\right]^{1/2} \qquad (0 < p \le 0.5) \qquad \text{..... (5)}$$

Then calculating z using the equation

$$z = w - \left[\frac{(2.515517 + 0.802853w + 0.010328w^2)}{(1 + 1.432788w + 0.189269w^2 + 0.001308w^3)}\right] \qquad \text{..... (6)}$$

When p > 0.5, 1-p is substituted for p in equation (5) and the value of z is computed by equation (6) is given a negative sign. The frequency factor K_T for the normal distribution is equal to z, as mentioned above.

In case of Gamma distribution, frequency analysis was done by the method as described by Hann (1994) as given in Table-2.

Table-1. Statistical parameters of annual 1 day as well as consecutive days maximum rainfall.

S. No.	Parameters	1 day	2 day	3 day	4 day	5 day
1.	**Minimum** (mm)	2.0	2.0	2.0	2.0	2.0
2.	**Maximum** (mm)	622.3	685.3	685.3	685.3	689.5
3.	**Mean** (mm)	181.6	229.9	248.3	261.9	272.5
4.	**Standard deviation** (mm)	138.42	157.32	162.12	166.82	169.26
5.	**Coefficient of variation**	76.22	68.42	65.29	63.71	62.11
6.	**Coefficient of skewness**	1.71	1.28	1.10	0.98	0.99
7.	**Kurtosis**	4.08	2.22	1.37	0.79	0.77

Table-2. Description of various probability distribution functions.

Distribution	Probability density function	Range	Equation for the parameters in terms of the sample moment
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$	-α ≤ x ≤ α	$\mu = \overline{x}$, $\sigma = S_x$
Log Normal	$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{(y-\mu_y)^2}{2\sigma_y^{\,2}}\right\}$	x > 0	$\mu_y = y$, $\sigma_y = S_y$
Gamma	$f(x) = \frac{\lambda^\beta x^{\beta-1} e^{-\lambda x}}{\Gamma(\beta)}$	x ≥ 0	$\lambda = \frac{\overline{x}}{s^2 x}$, $\beta = \left(\frac{\overline{x}}{s_x}\right)^2$

RESULTS AND DISCUSSION

The data presented in Table-3 reveal that 1 day, 2, 3, 4 and 5 consecutive days maximum rainfall followed Normal and Gamma distribution. As per the Chi-square values (Table-3), Normal distribution function was found to be best fit function for one, two and three consecutive days and Gamma distribution function for four and five consecutive days. Table-4 gives the 1 day and consecutive days maximum rainfall for different return periods as determined by selected distribution. A maximum of 154.31mm in 1 day, 250.88mm in 2 days, 270.15mm in 3 days, 284.18mm in 4 days and 295.54mm

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in 5 days is expected to occur at Udaipur, Rajasthan every 2 years. For a recurrence interval of 100 years, the maximum rainfall expected in 1 day, 2, 3, 4 and 5 days is 773.6mm, 849.34mm, 874.19mm, 931.78mm and 957.89mm, respectively. The magnitudes of 1 day as well as 2 to 5 consecutive days annual maximum rainfall corresponding to 2 to 100 years return period were estimated using Gamma function. It is generally recommended that 2 to 100 years is the sufficient return period for Soil and Water Conservation measures, construction of dams, irrigation and drainage works.

The data presented in Table-3 revealed that the computed Chi-square values for three probability distribution i.e. Normal, Log Normal and Gamma were found to be less than the critical value of Chi-square at 95% confidence level for 1 day as well as consecutive days maximum rainfall series. Gamma distribution gave minimum value of χ^2 for annual 1 day and 2 to 5 days consecutive maximum rainfall series. The statistical comparison by Chi-square test for goodness of fit clearly shows that Gamma distribution was best fitting representative function for rainfall frequency analysis in this region.

Table-3. Chi-square value for different distribution.

Consecutive days	Normal	Log Normal	Gamma
One day	2.428571	3.190476	2.428571
Two day	1.285714	6.619048	2.809524
Three day	2.047619	7	3.952381
Four day	5.095238	9.285714	2.809524
Five day	4.714286	9.285714	2.809524

Table-4. 1 day as well as consecutive days maximum rainfall for various return periods.

S. No.	Return period	**Maximum rainfall** (mm)				
		1 day	**2 day**	**3 day**	**4 day**	**5 day**
1.	2	154.31	250.88	270.15	284.18	295.54
2.	5	294.66	357.69	379.66	396.66	409.2
3.	10	366.41	438.49	462.5	481.75	495.18
4.	20	458.04	544.46	571.44	598.23	610.66
5.	50	624.59	709.69	736.27	778.05	796.59
6.	100	773.6	849.34	874.19	931.78	957.89

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List of symbols

n_i	:	number of observation in interval i
n	:	total number of observation
m	:	number of interval
p	:	number of parameters used in fitting the proposed distribution
C_V	:	coefficient of variation
K_T	:	frequency factor
$\overline{X}$	:	mean value of X
X_T	:	magnitude of the event having a return period t
μ	:	mean of the sample
σ	:	standard deviation of the sample
y	:	ln x
x	:	a variable
μ_y	:	mean of y
σ_y	:	standard deviation of y
z	:	standard normal variable
ν	:	degree of freedom
α	:	significance level
$\Gamma(\beta)$	:	gamma function