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SIMULATION OF WATER HAMMER FLOWS WITH UNSTEADY FRICTION FACTOR

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ABSTRACT

The problems of unsteady flow are frequently encountered in hydraulic power plant having a long conduit without the provision of surge tank due to sudden closing of the turbine valve. The velocity in water hammer situation fluctuates within the pipe from significantly high to an extremely low value with change in its direction after some interval of time. As a result there will be tremendous fluctuation in the wall friction along with the discharge and the pressure head. Transient analysis of the pipe flow is often more important than the analysis of the steady state operating conditions that engineers normally use to withstand these additional loads resulting from rapid valve closures water hammer situations. A numerical model "UNSTD_FRIC_ WH" using MOC and Barr's explicit friction factor has been presented here for solution of the transient flow situations of water hammer where there is a provision of computation of the transient frictions along with the pressure and discharges at a particular pipe section. Assessment of friction factors at any section in this unsteady transient flow condition also clearly indicates the effectiveness of using variable friction factor in contrast to the steady state friction as in the available numerical models. Damping of pressure and discharge with time after valve closure due to friction of the pipe are clearly illustrated by "UNSTD_FRIC_WH" model. It is well applicable for all flow conditions ranging from laminar to turbulent.

Keywords: water, hammer, unsteady, resistance, numerical, model.

INTRODUCTION

The basic unsteady flow equations along pipe due to closing of the valve near the turbine are non-linear and hence analytical solutions are not possible. Allevi [1, 2] developed classical solutions by both analytical and graphical methods neglecting the. Bergeron [3, 4] also developed graphical solution. Graphical solutions mentioned above had some practical application in pipe design before the advent of computer. Streeter [5] developed a numerical model by using a constant value of turbulent friction factor. Wiggert and Sundquist [6] solved the pipeline transients using fixed grids projecting the characteristics from outside the fundamental grid size. Their analysis shows the effects of interpolation, spacing, and grid size on numerical attenuation and dispersion. Watt et al [7] have solved for rise of pressure by MoC for only 1.2 seconds and the transient friction values have not been considered. Goldberg and Wylie [8] used the interpolations in time, rather than the more widely used spatial interpolations, demonstrates several benefits in the application of the method of characteristics to wave problems in hydraulics .Shimada and Okushima[9] solved the second order equation of water hammer by a series solution method and a Newton Raphson method. They calculated only maximum water hammer pressure with constant friction factor. The solution was not carried out for sufficiently long time to demonstrate damping of pressure head with increase of time. Chudhury and Hussaini [10] solved the water hammer equations by MacCormack, Lambda, and Gabutti explicit FD schemes. I. A. Sibetheros *et al*. [11] investigated the method of characteristics (MOC) with spline polynomials for interpolations required in numerical water hammer analysis for a frictionless horizontal pipe Silva-Arya and Choudhury [12] solved the hyperbolic part of the governing equation by MoC in one dimensional form and the parabolic part of the equation by FD in quasi-two-dimensional form. Pezzinga [13] presented both quasi 2-D and 1-D unsteady flow analysis in pipe and pipe networks using finite difference implicit scheme. Head oscillations were solved only for 4 seconds with constant friction. Pezzinga [14] also worked to evaluate the unsteady flow resistance by MoC. He used Darcy-Weisback formula for friction and solved for head oscillations up to 4 seconds only. Damping with constant friction factor is presented but not much pronounced, as the solution time is very small. Ghidaoi and Kolyshkin [15] performed linear stability analysis of base flow velocity profiles for laminar and turbulent water-hammer flows. They found that the main parameters that govern the stability behavior of the transient flows are the Reynolds numbers and the dimensionless timescale and the results found were plotted in Reynolds number / timescale space. Ghidaoui etal [16] implemented and analyzed the two layer and the five layer eddy viscosity models of water hammer. A dimensionless parameter i.e., the ratio of the time scale of the radial diffusion of shear to the time scale of wave propagation has been developed for assessing the accuracy of the assumption of flow axisymmetry in both the models of water hammer. Zhao and Ghidaoui [17] have solved a quasi-two dimensional model for turbulent flow in water hammer. They have considered turbulent shear stress as resistance instead of friction factor. Bergant [18] et al incorporated two unsteady friction models proposed by Zielke [19] and Brunone [20] et al. into MOC water hammer analysis. The numerical results obtained for pressure heads, at valve section and in the mid section up to 1 sec, from the quasi-steady friction model, Zielke model and Brunone model have been compared with the results of measurements of fast valve closure in a laboratory apparatus with laminar flow and low Reynolds numbers turbulent flow condition. Zhao and Ghidaoui [21] formulated, applied and analyzed first and second –order explicit finite volume (FV) Godunov-type schemes for water hammer problems. They have compared both the FV schemes with MoC considering space line interpolation for three test cases with and without friction for Courant numbers 1, 0.5.0.1.They modeled the wall friction using the formula of Brunone [20] et al. It has been found that the First order FV Gadunov scheme produces identical results with MoC considering space line interpolation. Hence the study of the previous works denotes that there are various numerical models, which include Method of characteristics (MOC), Finite Difference (FD) and Finite Volume (FV), presented by different investigators to obtain the transient pressure and discharges in water hammer situations. Among these methods MOC proved to be the most popular one as out of 14 commercially available water hammer software found on the World Wide Web, 11 are based on MOC and 2 are based on FD. Again the fixed grid MOC is most widely accepted being simple to code, accurate, efficient. Zhao and Ghidaoui[21] advocated that although different approaches such as FV, MOC, FD and finite element (FE) provide an entirely different framework for conceptualizing and representing the physics of the flow, the schemes that result from different approaches can be similar and even identical. A numerical model "UNSTD_FRIC_ BRR_WH" using MOC and Barr's explicit friction factor has been presented here for solution of the transient flow situations of water hammer where there is a provision of computation of the transient frictions along with the pressure and discharges at a particular pipe section. Assessment of friction at any section in this unsteady transient flow condition also clearly indicates the effectiveness of using variable friction factor in contrast to the steady state friction as in the available numerical models. Damping of pressure and discharge with time after valve closure due to friction of the pipe are clearly illustrated by "UNSTD_FRIC_WH" model. It is well applicable for all flow conditions ranging from laminar to turbulent.

GOVERNING EQUATIONS

The basic equations of continuity and momentum in unsteady flow along pipe due to closing of the valve near the turbine may be written as:

Continuity: $\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0$ ……………. (1)

Momentum: $\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{f}{2gDA^2} Q|Q| = 0$ …… (2)

Where, H= pressure head, A = area of pipe or conduit, a=velocity of pressure wave, Q= discharge, g= acceleration due to gravity, t = time, f =friction factor, D= diameter of pipe or conduit x = distance along the pipe

PROPOSED TRANSIENT FRICTION MODEL

The governing equations are solved here with the assessment of friction factor at every time step by Barr's [22] explicit friction factor which is a suitable combination of the Poiseuille equation and the Colebrook-White [23, 24] function to cover the full range of flow conditions, from laminar to rough turbulent. The numerical method adopted here is the Method of Characteristics which is compared with another numerical (explicit Finite Difference) method i.e., the Lax diffusive method.

Method of Characteristics

In the MoC the partial differential equations transforms into ordinary differential equations along characteristics line. Equations1 and 2 are presented as the following finite difference equations for pressure head H and discharge Q,

$$H_k^{j+1} = \frac{1}{2}(H_{k-1}^j + H_{k+1}^j) + \frac{a}{2gA}(Q_{k-1}^j - Q_{k+1}^j) \quad (3)$$
$$+ \frac{af\Delta t}{4gDA^2}(Q_{k+1}^j|Q_{k=1}^j| - Q_{k-1}^j|Q_{k-1}^j|$$

$$Q_k^{j+1} = \frac{1}{2}(Q_{k-1}^j + Q_{k+1}^j) - \frac{gA}{2a}(H_{k+1}^j - H_{k-1}^j) \quad (4)$$
$$- \frac{af\Delta t}{4gDA^2}(Q_{k-1}^j|Q_{k-1}^j| - Q_{k+1}^j|Q_{k+1}^j|)$$

Lax finite difference explicit method

Chaudhury[25] claims that Lax explicit method yields satisfactory results in nonlinear partial difference equation with smaller time step provided initial and boundary conditions are correctly imposed. Although smaller time step apparently would increase the volume of computation time, much iteration needed in implicit method is saved leading to a net decrease in time. Hence, Lax Diffusive method has been considered for comparison.

In Lax finite difference explicit method the equation 1 and 2 have been converted to:

$$H_k^{j+1} = \frac{1}{2}(H_{k-1}^j + H_{k+1}^j) - \frac{a^2\Delta t}{gA^2}\frac{1}{2\Delta x}(Q_{k+1}^j - Q_{k-1}^j) \quad (5)$$

$$Q_k^{j+1} = \frac{1}{2}(Q_{k-1}^j + Q_{k=1}^j) + \frac{gA\Delta t}{2\Delta x}(H_{k+1}^j - H_{k-1}^j) \quad (6)$$
$$- \frac{f\Delta t}{8gA}(Q_{k-1}^j + Q_{k=1}^j)|Q_{k+1}^j + Q_{k-1}^j|$$

The friction factor f in the above equations 3 to 6 is replaced by the following Barr's[22] explicit approximations which covers full range of flow conditions, from laminar to turbulent.

$$\frac{1}{\sqrt{f}} = -2\log_{10}[\frac{5.02\log_{10}(R_e/4.518\log_{10}(R_e/7))}{R_e(1+R_e^{0.52}/29(D/k)^{0.7})} \quad (7)$$
$$+ \frac{1}{3.7(D/k)}]$$

Boundary conditions

At d/s the principle of orifice is taken to evaluate the values up to complete closure and Finite difference equation along positive characteristics is evaluated respectively as $Q_k^{j+1} = (c_d A_v)\sqrt{H_k^{j+1}}$ (8)

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Where, A_v is area of valve at j+1.

$$H_k^{j+1} = H_{L-\Delta x}^{j} + \frac{a}{gA}Q_{L-\Delta x}^{j} - \frac{a}{gA}Q_L^{j+1} - \frac{af\Delta t}{2gDA^2}Q_{L-\Delta x}^{j}\left|Q_{L-\Delta x}^{j}\right| \quad (9)$$

When the valve is completely closed,

$$Q_L^{j+1} = 0 \;\&\; A_v = 0 \quad (10)$$

At upstream i.e., reservoir is assumed to be infinite hence pressure remains constant i.e., and discharge evaluated along the negative characteristics C^- respectively as:

$$H_{x=0}^{j+1} = H_0 \quad (11)$$

$$Q_{x=0}^{j+1} = \frac{gA}{a}(H_0 - H_{0+\Delta x}^{j}) + Q_{0+\Delta x}^{j} - \frac{f\Delta t}{2DA}Q_{0+\Delta x}^{j}\left|Q_{0+\Delta x}^{j}\right| \quad (12)$$

where, Av = area of valve, C_d = coefficient of discharge, H_0 = steady pressure head, H_j = unsteady pressure head of water at j^{th} time step, k = Nikuradse's equivalent sand roughness size, *k* = Subscript of distance, L = length of pipe and conduit, p = pressure intensity, Q_0 = steady discharge, Q^j = unsteady discharge at j^{th} time step, R_e = Reynolds number flow in the pipe, Δt = increase of time, w = unit weight of water, $\forall$ = volume of water, υ = kinematic viscosity, Δx = length of section along the pipe direction.

NUMERICAL APPLICATION

The proposed model was examined for rapid valve closing in downstream of a long conduit with a infinite reservoir upstream The stability and accuracy of the solutions are tested by comparing them with the solutions obtained by an another numerical method "Explicit Finite Difference (FD) Lax Diffusive method" and solutions already available in literature for that pipe system by different investigator.

The numerical values taken to start the solution of the physical problem are: An infinite reservoir with steady head of H_0 = 600 ft, pipe length L = 12000 ft, diameter of the pipe D = 2ft, steady discharge to the turbine = 12 ft^3/sec, steady state friction factor f = 0.02, steady pressure heads at different pipe section H_1 = H_0 = 600ft, H_2 = 587.5ft, H_3 = 565ft, H_4 = 547.5ft, H_5 = 530ft, Δx = 3000ft, i.e., the pipe is divided into 4 sections. Nikuradse's sand roughness size of the pipe k = 0.007093ft, valve closure time = 4 seconds, velocity of pressure wave after valve closure a = 3000 ft/sec, kinematic viscosity of water υ = 0.000001ft^2/sec.

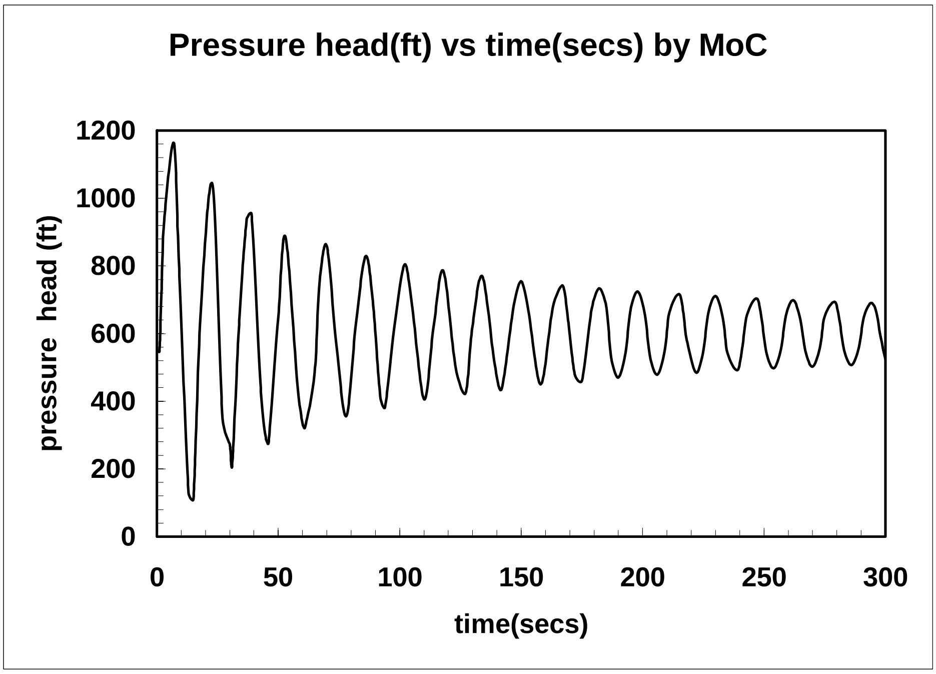

Figure-1. Pressure Head Vs time up to 300 seconds by MoC.

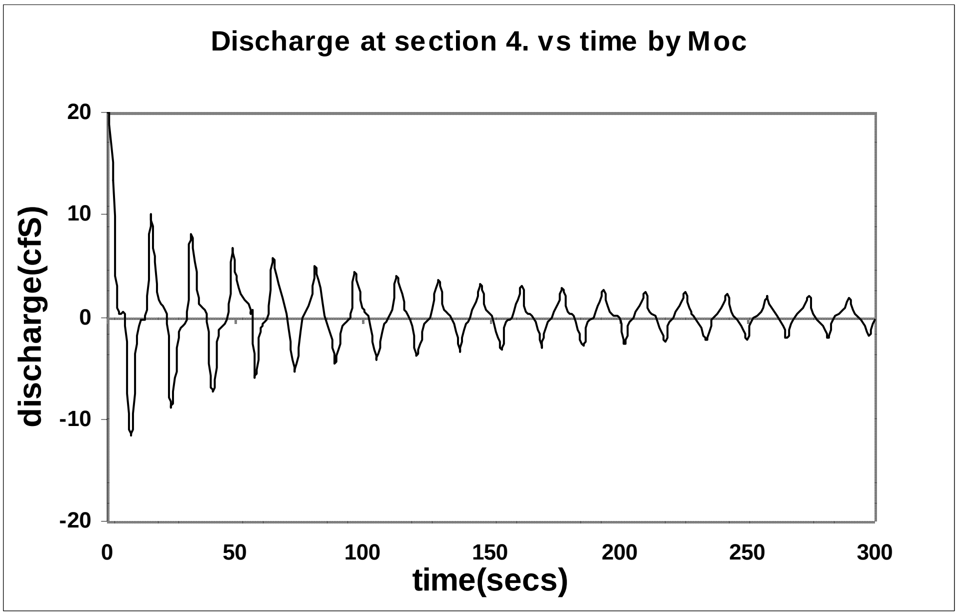

Figure-2. Discharge Vs time by MoC up to 300 seconds.

The results obtained for unsteady pressure head and discharge by the proposed model are shown in Figures1 and 2. Variation in values of friction factor obtained by the proposed model, at section 4 is also plotted in Figure-3. In Figure-4, the comparisons of solution for pressure head by both the methods are presented. Comparison with Streeter[5] solution, up to 50 seconds has been made in Figure-5.

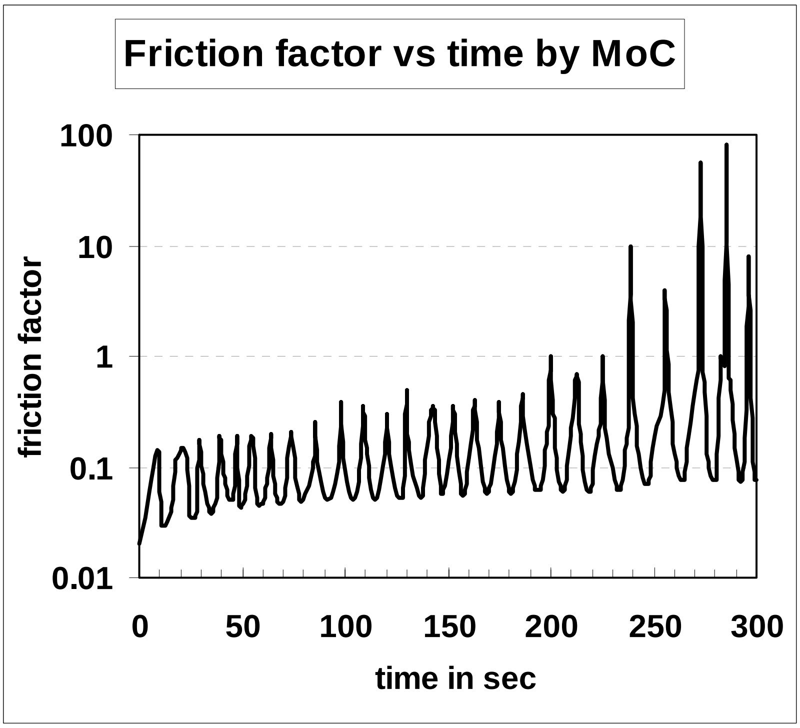

Figure-3. Transient friction factor by MoC.

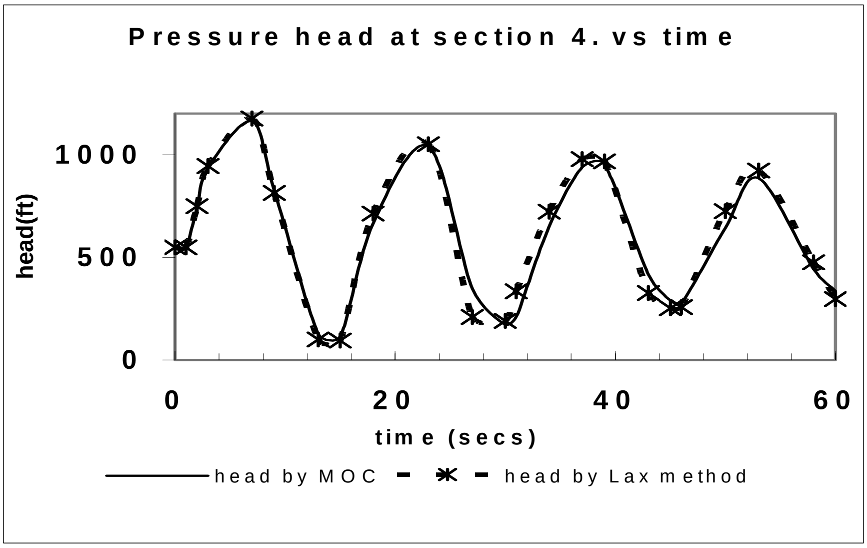

Figure-4. Comparison of pressure heads MoC and LAX FD method.

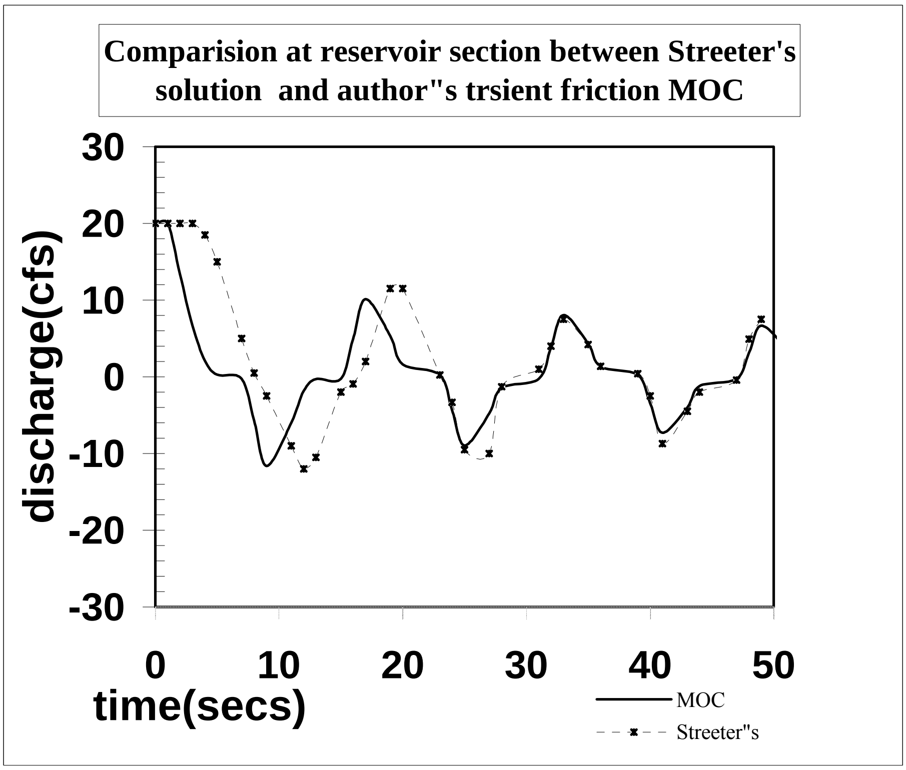

Figure-5. Comparison of discharges at section 4 by MoC and Streeter's.

CONCLUSION

Figure-1 has shown the plot of unsteady pressure at section 4 of pipe up to 300 seconds. Damping of pressure is clear in the calculated time up to 300 seconds. Similarly Figure-2 has shown the plot of unsteady discharge at section 4 of the pipe. It has also shown the damping of discharge with time due to friction up to 300 seconds. Figure-4 has shown the comparison of the solution of the two methods i.e., MOC and Lax F.D. Figure-4 is the comparison of pressure heads H. MOC gives a bit less pressure head. Plot of transient friction factor f up to time 300 seconds by MoC at section 4 has been shown in Figure-3. Undulating values of friction factor at every time step are observed .As time increases average values shoot up to very high value .It clearly indicates that use of constant friction is not recommended. In Figure-5, solution by MoC is compared with Streeter solution up to 50 seconds. Comparison appears to be quite compromising.

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