VOL. 2, NO. 1, FEBRUARY 2007 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

BUCKLING LOAD OF A BEAM-COLUMN FOR DIFFERENT END CONDITIONS USING MULTI-SEGMENT INTEGRATION TECHNIQUE

Goutam Saha[1] and Sajeda Banu[1]
[1]Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh
Email: rana_math06@yahoo.com

ABSTRACT

A method of identifying the buckling load of a beam-column is presented based on a technique named 'Multi-segment Integration technique'. This method has been applied to a number of problems to ascertain its soundness and accuracy. We consider a boundary-value problem for the beam-column equation, in which the boundary conditions mean that i) it is hinged at both ends; ii) it is fixed at both ends; and iii) it is fixed at one end and hinged at the other end. The results obtained by Finite Difference method are compared in order to determine the efficiency of this method.

Keywords: beam, column, boundary, finite method, multi-segment integration technique.

INTRODUCTION

The beam-column problem is generally approached from the standpoint of the strength of materials, which drastically simplify the more precise methods of the theory of elasticity and plasticity. The term "beam-column" is used here to specify a structural member which is subjected simultaneously to axial compressive force and bending moment. The first investigation of the buckling of columns under axial compression go back about two centuries to Euler and his study of the elastica, while the initial investigations of necking in bars are already more than a century old. In the early years, columns were designed empirically and their ultimate strength was determined entirely by the crushing strength of the material similar to that of the fracture strength in tension members.

It was vaguely understood that column strength is somehow related to the column length. Van Musschenbroek (1729) first recognized this and presented an empirical formula for column strength in terms of column length *l*. Euler (1759) was the first to derive the Euler column formula and proved theoretically that there is another criterion for column strength which is independent of crushing or yielding of material. In this early development, the column behavior is analyzed by using the linear theory based on linear elastic material property and small deflection approximation of the column. Amba Rao (1967) is one of many authors who have shown that in the presence of a compressive axial load, the natural frequency of a beam reduces and finally becomes zero when the critical Euler buckling load is reduced. Euler developments of columns and beam-columns have been reviewed by Bleich (1952) and Timoshenko (1953). Elastic beam-columns were solved by Timoshenko and Gere (1961), Thompson and Hunt (1973) and many others, for various end conditions. Plastic studies of beam-columns were started by Von Karman (1908, 1910) and Chwalla (1928).

In this paper, we first introduce the basic equation of beam-column theory and we present the analytical expression of the deflection equation for different boundary conditions. Then we present the Euler buckling load for $q(x) = 0$. Next we present the formulation of the fourth order non-homogeneous beam-column equation using both Finite Difference method and Multi-segment Integration technique. Finally we establish some results on critical load and graphical presentation of buckling load obtained from Multi-segment Integration technique.

MATHEMATICAL MODEL

The basic equation of beam-column theory is a differential equation, linking the displacement of the center line $w(x)$ to the axial compressive load P and the lateral load $q(x)$. That is,

$$EI\frac{d^4w}{dx^4} + P\frac{d^2w}{dx^2} = q \qquad (1.1)$$

together with the boundary condition

i) $w(0) = w''(0) = w(l) = w''(l) = 0$ (1.2a)

ii) $w(0) = w'(0) = w(l) = w'(l) = 0$ (1.2b)

ii) $w(0) = w'(0) = w(l) = w''(l) = 0$ (1.2c)

where E is the Young's modulus of the beam, I is the area moment of inertia of the beam's cross section.

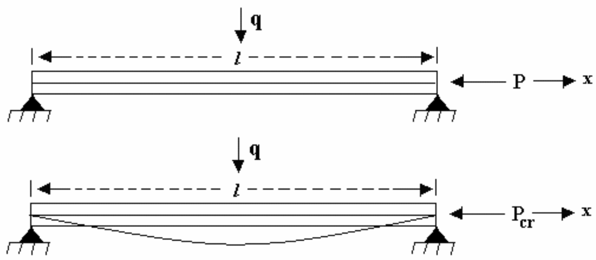

Figure-1(a). Beam-column with both ends hinged.

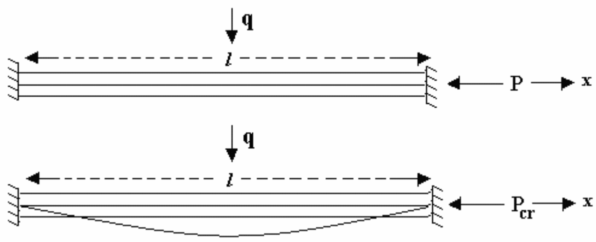

Figure-1(b). Beam-column with both ends fixed.

VOL. 2, NO. 1, FEBRUARY 2007 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

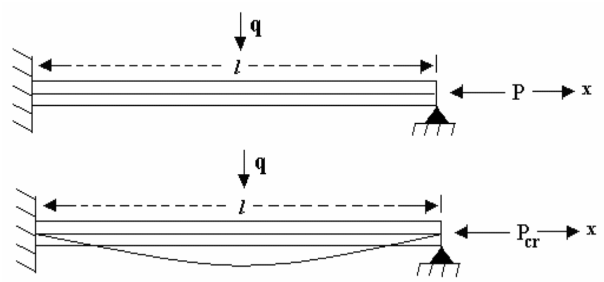

Figure-1(c). Beam-column with one end fixed and other end hinged.

Equation (1.1) and the boundary conditions (1.2) arise from the study of elastic stability and have definite physical meanings. Equation (1.1) describes the deflection or deformation of a beam-column under a certain force. The boundary condition (1.2a), (1.2b), and (1.2c) means that the beam-column is hinged at the end $x = 0$ and $x = l$, is fixed at the end $x = 0$ and $x = l$, is fixed at the end $x = 0$ and is hinged at the end $x = l$ respectively. Now the general solution of Equation (1.1) is

$$w(x) = A \cos k x + B \sin k x + C x + D + f(x) \qquad (1.3)$$

in which

$$f(x) = \frac{q}{2P} x^2 .$$

CALCULATION OF DEFLECTION FUNCTION

When we consider the beam-column with both ends hinged, then using the boundary conditions given in Equation (1.2a) and applying those in Equation (1.3), we get the following deflection equation

$$w(x) = -\frac{2q}{k^2 P}\left(\frac{k\ell - kx}{2}\,\frac{kx}{2} - \frac{\sin\frac{kx}{2}}{\cos\frac{kx}{2}} \sin\frac{k\ell - kx}{2} \right)$$

The maximum deflection occurs at the mid-span and then deflection equation becomes,

$$w\left(\frac{\ell}{2}\right) = -\frac{2q}{k^2 P}\left[\frac{k\ell}{4}\,\frac{k\ell}{4} - \frac{1 - \cos\frac{k\ell}{2}}{2\cos\frac{k\ell}{2}} \right]$$

When we consider the beam-column with both ends fixed, we use the boundary conditions given in Equation (1.2b) and applying those in Equation (1.3), we get the following deflection equation

$$w(x) = \frac{q}{2kP}\left(kx(x-\ell) + \ell(-1 + \cos kx)\cot\left(\frac{k\ell}{2}\right) + \ell \sin kx \right)$$

The maximum deflection occurs at the mid-span and then deflection equation becomes,

$$w\left(\frac{\ell}{2}\right) = \frac{q}{2kP}\left(-\frac{k\ell^2}{4} + \ell\left(-1 + \cos\frac{k\ell}{2}\right)\cot\left(\frac{k\ell}{2}\right) + \ell \sin\frac{k\ell}{2} \right)$$

When we consider the beam-column with one end fixed and the other end hinged, we use the boundary conditions given in Equation (1.2c) and applying those in Equation (1.3), we get the following deflection equation

$$w(x) = \frac{-q\begin{pmatrix} 2k\ell - 2kx - kx\left(-2 + k^2\ell(-\ell + x)\right)\cos k\ell \\ -2k\ell\cos kx - k^2\ell^2 \sin k\ell + k^2x^2 \sin k\ell \\ +2\sin\left(k(\ell - x)\right) + k^2\ell^2 \mathrm{Sin}\left(k(\ell - x)\right) + 2\sin kx \end{pmatrix}}{2k^2 P\left(k\ell\cos k\ell - \sin k\ell\right)}$$

CALCULATION OF BUCKLING LOAD

We consider here a beam subject to an axial compressive load P. The buckling load P_{cr} then satisfies the equation

$$EI\frac{d^4w}{dx^4} + P_{cr}\frac{d^2w}{dx^2} = 0 \qquad (1.4)$$

where E is the Young's modulus of the beam, I is the area moment of inertia of the beam's cross section, P_{cr} is the buckling load, and w is the transverse displacement.

The general solution of Equation (1.4) is

$$w(x) = A \cos k x + B \sin k x + C x + D \qquad (1.5)$$

where

$$k = \sqrt{\frac{P}{EI}} .$$

Applying the boundary conditions given in Equation (1.2a) in Equation (1.5), we get

$A + D = 0,$

$B \sin k\ell + A \cos k\ell + C\ell + D = 0,$

$A = 0,$

$B \sin k\ell + A \cos k\ell = 0.$

For non-trivial solution, the characteristic equation is

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ \cos k\ell & \sin k\ell & l & 1 \\ -k^2 & 0 & 0 & 0 \\ -k^2\cos k\ell & -k^2 \sin k\ell & 0 & 0 \end{vmatrix} = 0,$$

which yields $A = C = D = 0$ and $B \sin k\ell = 0$.

The solution of this equation is $k\ell = n\pi$

$$\Rightarrow k^2 = \frac{n^2\pi^2}{\ell^2},$$

$$\Rightarrow \frac{P}{EI} = \frac{n^2\pi^2}{\ell^2},$$

$$\Rightarrow P = EI\,\frac{n^2\pi^2}{\ell^2},\ n = 1, 2, \ldots$$

Therefore the buckling load occurs when $n = 1$ and we get

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} .$$

Again applying the boundary conditions given in Equation (1.2b) in Equation (1.5), we get

$A + D = 0,$

$B \sin k\ell + A \cos k\ell + C\ell + D = 0,$

$A = 0,$

$B \sin k\ell + A \cos k\ell = 0.$

For non-trivial solution, the characteristic equation is

VOL. 2, NO. 1, FEBRUARY 2007 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ \cos k\ell & \sin k\ell & l & 1 \\ 0 & k & 1 & 0 \\ k\cos k\ell & -k\sin k\ell & 1 & 0 \end{vmatrix} = 0$$

The expansion of this determinant leads to

$$\sin\left(\frac{k\ell}{2}\right)\left[k\ell\cos\left(\frac{k\ell}{2}\right) - 2\sin\left(\frac{k\ell}{2}\right)\right] = 0.$$

One solution of this equation is

$$k = \frac{2n\pi}{\ell},$$

$$\Rightarrow \frac{P}{EI} = \frac{4n^2\pi^2}{l^2}$$

$$\Rightarrow \quad P = \frac{4\pi^2 EI}{\ell^2} n^2, \; n = 1, 2, \ldots\ldots$$

Therefore the buckling load occurs when n = 1 and we get

$$P_{cr} = \frac{4\pi^2 EI}{\ell^2}.$$

Again applying the boundary conditions given in Equation (1.2c) in Equation (1.5), we get

$A + D = 0,$

$B \sin k\ell + A\cos k\ell + C\ell + D = 0,$

$k B + C = 0,$

$-k^2 B\sin k\ell - k^2 A\cos k\ell = 0.$

For non-trivial solution, the characteristic equation is

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ \cos k\ell & \sin k\ell & l & 1 \\ 0 & k & 1 & 0 \\ -k^2\cos k\ell & -k^2\sin k\ell & 0 & 0 \end{vmatrix} = 0.$$

The expansion of this determinant leads to

$$k\ell\cos k\ell - \sin k\ell = 0$$

$$\Rightarrow \tan k\ell = k\ell.$$

The solution of this equation is $k\ell$ = 4.493. So we can write

$$k^2 = \frac{(4.493)^2}{\ell^2}$$

$$\Rightarrow \frac{P}{EI} = \frac{(4.493)^2}{\ell^2}$$

$$\Rightarrow P = \frac{(4.493)^2}{\ell^2} EI.$$

This equation determines the value of the buckling load.

But in case of a beam-column problem, it is impossible to find the buckling load manually by traditional methods for the problem (1.1)-(1.2). For this reason we use Finite Difference method and Multi-segment Integration technique to calculate the buckling load and the answers obtained are not the exact but approximate solutions.

FINITE DIFFERENCE METHOD

The problem (1.1)-(1.2) is formulated from a mathematical point of view.

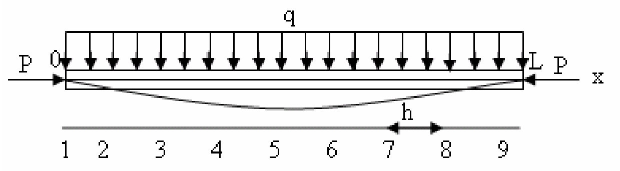

Figure-2. Discretization of a domain.

Finite Difference Formulae for Governing Differential Equation (1.1):

1. Forward Difference Formula:

$$-EIf_{i+7} + 12EIf_{i+6} - 59EIf_{i+5} + 154EIf_{i+4} - (231EI + 4Ph^2)f_{i+3} +$$
$$(200EI + 16Ph^2)f_{i+2} - (93EI + 20Ph^2)f_{i+1} + (18EI + 4Ph^2)f_i = 4h^4 q$$

2. Backward Difference Formula:

$$-EIf_{i-7} + 12EIf_{i-6} - 59EIf_{i-5} + 154EIf_{i-4} - (231EI + 4Ph^2)f_{i-3} +$$
$$(200EI + 16Ph^2)f_{i-2} - (93EI + 20Ph^2)f_{i-1} + (18EI + 4Ph^2)f_i = 4h^4 q$$

3. Central Difference Formula:

$$EIf_{i+3} - 2EIf_{i+2} + (4Ph^2 - EI)f_{i+1} + (4EI - 8Ph^2)f_i +$$
$$(4Ph^2 - EI)f_{i-1} - 2EIf_{i-2} + EIf_{i-3} = 4h^4 q$$

We use all three formulae to determine the deflection of the beam-column. We use forward difference formula for node 2, backward difference formula for node 8 and central difference formula for node 4, 5 and 6.

Finite Difference Formulae for boundary conditions (1.2a):

$f_1 = 0,$ at node 1,

$-f_4 + 4f_3 - 5f_2 + 2f_1 = 0,$ at node 3,

$f_9 = 0,$ at node 9,

$-f_6 + 4f_7 - 5f_8 + 2f_9 = 0,$ at node 7.

Finite Difference Formulae for boundary conditions (1.2b):

$f_1 = 0,$ at node 1,

$-f_3 + 4f_2 - 3f_1 = 0,$ at node 3,

$f_9 = 0,$ at node 9,

$f_7 - 4f_8 + 3f_9 = 0,$ at node 7.

Finite Difference Formulae for boundary conditions (1.2c):

$f_1 = 0,$ at node 1,

$-f_3 + 4f_2 - 3f_1 = 0,$ at node 3,

$f_9 = 0,$ at node 9,

$-f_6 + 4f_7 - 5f_8 + 2f_9 = 0,$ at node 7.

VOL. 2, NO. 1, FEBRUARY 2007 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

MULTI-SEGMENT INTEGRATION TECHNIQUE

The Multi-segment Integration technique is developed by Kalnins and Lestingi (1967) and it involves much less computational work. This method has been applied to a number of problems to ascertain its soundness and accuracy. But the success of this method is limited to the very simple problems which can be managed just as well by direct integration. Finite element formulation of such problems, which ultimately resolves into solution of a large number of linear algebraic equations, has often met the problem of non-convergence. To overcome the difficulty of direct integration of the following problem, Kalnins and Lestingi (1967) developed a Multi-segment method of Integration which avoided integration over large ranges of x.

The linear ordinary differential equation of order m is given in matrix form as

$$\frac{d}{dx}\left[y(x) \right] = A(x).y(x) + B(x)$$

where $y(x) = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_m \end{bmatrix}$,

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & \dots & \dots a_{1m} \\ a_{21} & a_{22} & \dots & \dots & \dots a_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & \dots & \dots a_{mm} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_m \end{bmatrix}.$$

We can write the given boundary conditions in the matrix form

$$C\,y(a) + D\,y(b) = E \tag{1.6}$$

Let the solution be

$$y(x) = Y(x)\,.\,G + Z(x) \tag{1.7}$$

where

G is known as the constants of integration,
Y(x) is known as general solution and
Z(x) is known as particular solution.

Consider

$$\frac{d}{dx}\left[Y(x) \right] = A(x).Y(x) \text{ with } Y(a) = I \tag{1.8}$$

$$\frac{d}{dx}\left[Z(x) \right] = A(x).Z(x) + B(x) \text{ with } Z(a) = 0 \tag{1.9}$$

Evaluating Equation (1.7) at $x = a$, we get

$$y(a) = Y(a)\,G + Z(a).$$

This follows that $G = Y(a)$.

Again evaluating Equation (1.7) at $x = b$, we get

$$y(b) = Y(b)\,G + Z(b). \tag{1.10}$$

where *Y(b)* is evaluated at Equation (1.8) by Runge Kutta method of order 4 with initial condition $Y(a) = I$ and $Z(b)$ is evaluated at Equation (1.9) by Runge Kutta method of order 4 with initial condition $Z(a) = 0$.

Solving Equation (1.6) and (1.10), we get

$$C\,y(a) + D\left[Y(b).\,y(a) + Z(b) \right] = E.$$

$$\Rightarrow\ y(a)\left[C + D.Y(b) \right] = E - D.Z(b).$$

$$\Rightarrow\ y(a) = \left[C + D.Y(b) \right]^{-1}\left[E - D.Z(b) \right].$$

By this way we can obtain the numerical solution of *y(x)* at the intermediate grid points between $x = a$ and $x = b$.

FORMULATION OF THE PROBLEM (1.1-1.2)

Let

$$y = y_1,\ \frac{dy}{dx} = y_2,\ \frac{d^2y}{dx^2} = y_3,\ \frac{d^3y}{dx^3} = y_4.$$

Then we get

$$\frac{dy_1}{dx} = y_2,\ \frac{dy_2}{dx} = y_3,\ \frac{dy_3}{dx} = y_4,\ \frac{dy_4}{dx} = \frac{q}{EI} - \frac{P}{EI}y_3.$$

The governing differential equation in matrix form is

$$\frac{d}{dx}\left[y(x) \right] = A(x) \times y(x) + B(x)$$

$$\Rightarrow\ \frac{d}{dx}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{P}{EI} & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{q}{EI} \end{bmatrix}.$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{P}{EI} & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{q}{EI} \end{bmatrix}.$$

The boundary conditions in matrix form is

$$C \times y(a) + D \times y(b) = E$$

where

$$y(a) = \begin{bmatrix} y_1(a) \\ y_2(a) \\ y_3(a) \\ y_4(a) \end{bmatrix},\ y(b) = \begin{bmatrix} y_1(b) \\ y_2(b) \\ y_3(b) \\ y_4(b) \end{bmatrix}.$$

i) For boundary conditions (1.2a):

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},\ E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

ii) For boundary conditions (1.2b):

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

VOL. 2, NO. 1, FEBRUARY 2007 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

iii) For boundary conditions (1.2c):

$$C=\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix},\ D=\begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix},\ E=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}.$$

RESULTS

To illustrate these procedures consider a beam-column that is i) hinged at both ends, ii) fixed at both ends, iii) fixed at one end and hinged at other end subject to an axial force P and lateral load q as shown in Figures 1(a), 1(b), 1(c), respectively. The buckling loads P_{cr} of a beam-column with length $l = 2\,m$, $EI = 100\,KNm^2$, and q = 0.01, 0.05, 0.10, 0.15, 0.20, 0.25 *kN/m* calculated using both Finite Difference method and Multi-segment Integration technique that is used in this paper, are given in Table-1, and the buckling shape of the beam-column is also shown in Figures 1(a), 1(b) and 1(c), respectively.

The Figures 3(a), 3(b) and 3(c) show that for a lateral load q, the lateral displacement produce but when the lateral load q reaches the critical value P_{cr} then it will not return to its initial position and if q exceeds P_{cr} then the beam-column will collapse after a slight disturbance.

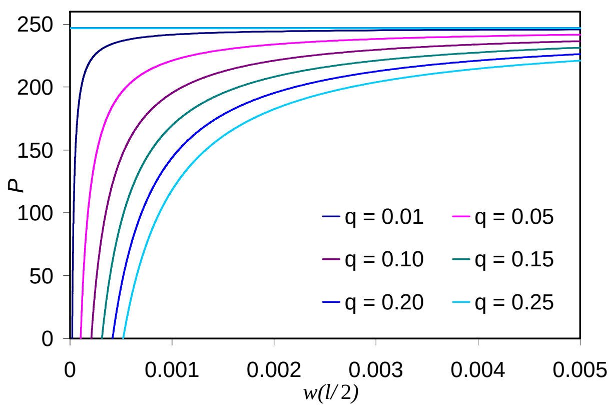

Figure-3(a). Critical load of a beam-column with both end hinged.

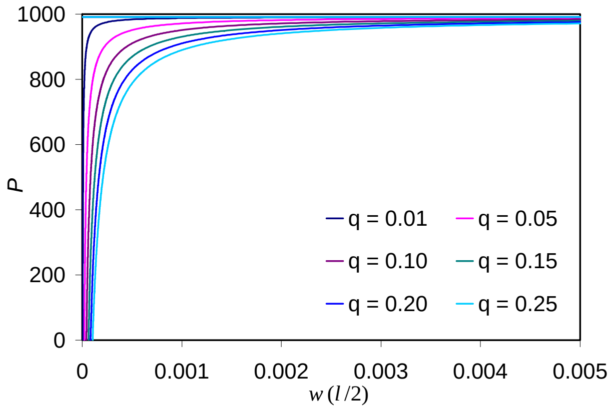

Figure-3(b). Critical load of a beam-column with both end fixed.

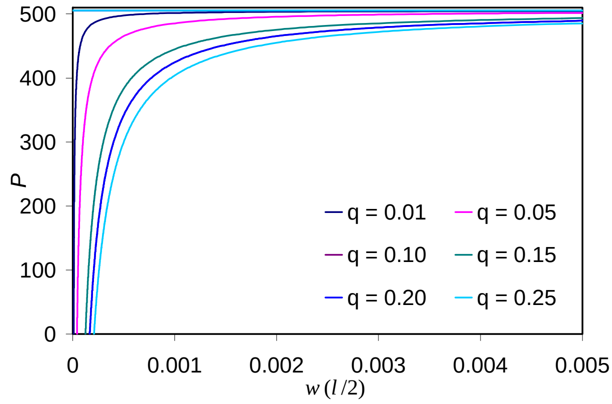

Figure-3(c). Critical load of a beam-column with one end fixed and the other end hinged.

From the following results, it is apparent that the approximate solutions yield very good estimates of the values obtained from both the methods, and that any error is simply a function of the number of iterations used. The proposed methods therefore provide a straightforward and effective numerical technique for the problem (1.1)-(1.2).

Table-1. Buckling load of a beam-column

Boundary conditions	Euler critical load	Finite difference method	Multi-segment Integration technique
Both ends hinged	246.74	247.23	246.82
Both ends fixed	986.96	994.65	991.02
One end fixed and the other end hinged	504.75	505.11	504.69

CONCLUSION

This paper makes a brief comparison between the proposed method and the familiar finite difference technique. From the results, we have found that the Multi-segment Integration technique is seen to be capable of solving this kind of problem and that the method is both effective and efficient.

REFERENCES

Amba Rao, C.L. 1967. Effect of end conditions on the lateral frequencies of uniform straight columns. Journal of the Acoustical Society of America. 42, pp. 900-901.

Bleich, F. 1952. Buckling strength of metal structures. Eng. Soc. Monograph, McGraw-Hill, New York.

Chwalla, E. 1928. Die Stabilitat zentrisch und exzentrisch gedruckter Stabe aus Baustahl. Sitzungsberichte der Akademie de Wissenschaften in Wien, Abt. IIa, p. 469.

Euler, L. 1744. De curvis elasticis. Lausanne and Geneva. The Euler formula was derived in a later paper, "Sur la force de colonnes," published 1759 in the Memoires de V Academie de Barlin.

Kalnins, A. and Lestingi, J. F. 1967. On nonlinear analysis of elastic shells of revolution. J. Appl. Mech., Ser. E. Vol. 34(1): 59-64.
Timoshenko, S. P. and Gere, J. M. 1961. Theory of elastic stability. McGraw-Hill, New York.

Thompson, J.M.T. and Hunt, G.W. 1973. A general theory of elastic stability. New York: Wiley.

Van Musschenbrock, P. 1729. Introductio ad cohaerentiam corporum firmorum. Lugduni.

Von Karman, T. 1908. Die Knickfestigkeit gerader Stabe. Physikalische Zeitschrift. Vol. 9. p. 136.

APPENDIX

Formulas for Computing Second Derivative.

Derivative		Formula	Error
f"(x$_i$)	Forward Difference	$\frac{-f_{i+3}+4f_{i+2}-5f_{i+1}+2f_i}{h^2}$	O(h^2)
	Backward Difference	$\frac{2f_i-5f_{i-1}+4f_{i-2}-f_{i-3}}{h^2}$	O(h^2)
	Central Difference	$\frac{f_{i+1}-2f_i+f_{i-1}}{h^2}$	O(h^2)

Formulas for Computing Fourth Derivative.

Derivative		Formula	Error
f""(x$_i$)	Forward Difference	$\frac{1}{4h^4}\begin{pmatrix}-f_{i+7}+12f_{i+6}-59f_{i+5}+154f_{i+4}- \\ 231f_{i+3}+200f_{i+2}-93f_{i+1}+18f_i\end{pmatrix}$	O(h^2)
	Backward Difference	$\frac{1}{4h^4}\begin{pmatrix}-f_{i-7}+12f_{i-6}-59f_{i-5}+154f_{i-4} \\ -231f_{i-3}+200f_{i-2}-93f_{i-1}+18f_i\end{pmatrix}$	O(h^2)
	Centered Difference	$\frac{1}{4h^4}\begin{pmatrix}f_{i+3}-2f_{i+2}-f_{i+1}+ \\ 4f_i-f_{i-1}-2f_{i-2}+f_{i-3}\end{pmatrix}$	O(h^2)