ARPN Journal of Engineering and Applied Sciences

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DESIGN OF NON-UNIFORM FILTER BANKS- QUADRATIC OPTIMIZATION WITH LINEAR CONSTRAINTS

G.A.E. Satish Kumar[1], P. Chandrasekhar Reddy[2] and B. Stephen Charles[3]
[1]ECE Department, St. Johns College of Engineering and Technology, Yemmiganur, India
[2]ECE Department, JNTU College of Engineering, Hyderabad, India
[3]St. Johns College of Engineering and Technology, Yemmiganur, India
E-mail: gae_sathi@rediffmail.com

ABSTRACT

This paper presents methods for the design of non-uniform filter banks. The filter bank structure is obtained from a uniformly modulated filter bank by using an allpass transform which has a lossless frequency function and a nonlinear phase function. The proposed design method includes quadratic optimization with linear constraints. Considered applications are subband adaptive filtering and subband coding. Analysis filter banks and synthesis filter banks are designed in two subsequent stages, and design objectives include minimization of subband aliasing as well as reconstruction output residual aliasing components on an individual basis. This way to formulate design objectives is appropriate for filter banks used in subband adaptive filtering. Other design objectives are to optimize the overall filter bank response for low amplitude and phase distortion. Designs with phase compensation for linear phase overall response are included. Examples are included of filter banks with increasing bandwidth.

KEYWORDS: filter bank, analysis, synthesis, quadratic optimization, linear constraints.

1. INTRODUCTION

Filter banks with the aliasing cancellation property have been of great interest in numerous applications, and design methods taking aliasing into account have been considered in an early stage [1, 2]. An overview is presented in [3].

Non-uniform filter banks have been of interest in speech enhancement, since by appropriate design it is possible to get a model, corresponding to the human auditory system [7]. They are also successfully applied to, speech recognition and speech coding. Non-uniform filter banks have also been proposed for subband adaptive filtering, e.g. in spectral subtraction for speech enhancement, [8], and beamforming for subband microphone arrays [9]. The filter banks addressed in this paper are non-uniform filter banks with polyphase structure. They utilize a lossless frequency transformation similar to a bilinear transform to obtain the non-uniformity [10]. These frequency transformed filter banks have previously been presented [11, 12], and are known to approximate the Bark frequency scale, or critical band scale, very accurately [7]. However, these filter banks are also known to cause phase distortion, which is inappropriate for coding or communications applications. The phase distortion can be compensated for by phase compensation filters [13].

This paper proposes novel methods for the design of filter banks in two stages. First a non-uniform analysis bank is designed and then a matching synthesis filter bank is designed, given the analysis filter bank. Quadratic criteria with linear constraints is used and evaluated. A common aim is to design the analysis and synthesis banks with pre-specified parameters, such as number of subbands, filter lengths, delays and decimation factors.

In the first stage the analysis filter bank is designed in such way that aliasing terms in the subbands are minimized. In the second stage the synthesis filter bank is designed, based on the analysis filter bank, such that the overall response is optimized and the reconstruction aliasing terms are minimized.

Generally, filter bank design methods are reduced to the design of a single prototype for the analysis and synthesis filter banks in order to obtain nearly perfect reconstruction properties. In the two stage design methods the amplitude distortion, phase distortion (delay) and aliasing distortion can be minimized or controlled for the analysis and synthesis filter banks separately.

2. QUADRATIC OPTIMIZATION WITH LINEAR CONSTRAINTS

The minimization of aliasing energy on an individual basis with respect to response constraints can be done by combining the quadratic cost functions 3 J^{II}_{A} (**a**) and J^{II}_{S} (**b**) with the linear constraints, for the analysis and synthesis filter bank respectively.

2.1. Analysis of filter bank design criterion

The minimization of subband aliasing energy with respect to constraints on the passband ripple can be formulated as

$$\begin{cases} \min_{\mathrm{a}} J_A^{II}(\mathrm{a}) \\ \left| H_m(e^{j\omega}) - H_m^D(e^{j\omega}) \right| \le \sigma, \omega \in \Omega_m^p, \forall m \end{cases} \tag{1}$$

Using the matrix notations, the formulation in Eq. (1) can be approximated by the finite dimensional quadratic program

$$\begin{cases} \min_{\mathrm{a}} \mathrm{a}^T \left\{ \sum_{m=0}^{M-1} \Phi_m^{S^H} \Phi_m^S \right\} \mathrm{a} \\ \mathrm{Re}\{e^{j2\pi c/C} \Phi_m^p\}\, \mathrm{a} \le \mathrm{Re}\{e^{j2\pi c/C} \mathrm{H}_m^D\} + \sigma, \forall c, \forall m \end{cases} \tag{2}$$

2.2. Synthesis of filter bank design criterion

The minimization of reconstruction aliasing energy with respect to constraints on the overall response ripple can be formulated as

$$\begin{cases}\min\limits_{b} J_S^{II}(a)\\ \left|T(e^{j\omega})-T^D(e^{j\omega})\right|\le\sigma,\omega\in\Omega\end{cases} \qquad (3)$$

Using previous notations, the formulation in Eq. (3) can be approximated by the finite dimensional quadratic program

$$\begin{cases}\min\limits_{b} b^T\Lambda b\\ \mathrm{Re}\{e^{j2\pi c/C}\Psi\}\,b\le \mathrm{Re}\{e^{j2\pi c/C}T^D\}+\sigma,\forall c\end{cases} \qquad (4)$$

3. DESIGN EXAMPLES

In both the analysis and synthesis filter bank design, the maximum response ripple is set to $\sigma = 0.01$ and parameter C is set to $C = 8$.

3.1 Example without phase compensation - QP[1]

Filter banks are designed without phase compensation in the synthesis filter bank. The resulting analysis filters $Hm(z)$ and synthesis filters $Gm(z)$ are shown in Figure-1. The stopbands of the analysis and synthesis filters clearly exhibit minimum energy characteristics. The magnitude and the group delay of the overall response, $T(e^{j\omega})$ and the average magnitude of the aliasing terms Savg(ω) are shown in Figure-2.

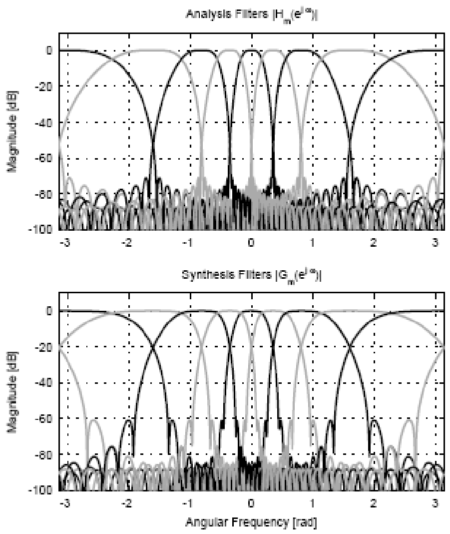

Figure-1. **Design Example QP**[1]. *Magnitude responses of the analysis filters* $|H_m(e^{j\omega})|$ *and synthesis filters* $|G_m(e^{j\omega})|$, *for* $m = 0, \ldots, M-1$. *The analysis and synthesis filter banks are designed using quadratic optimization with linear constraints. There is no phase compensation in the synthesis filter bank. The cost function value for the analysis filter bank design is* $J^{II}_A(\mathbf{a}) = -81.6$ *dB.*

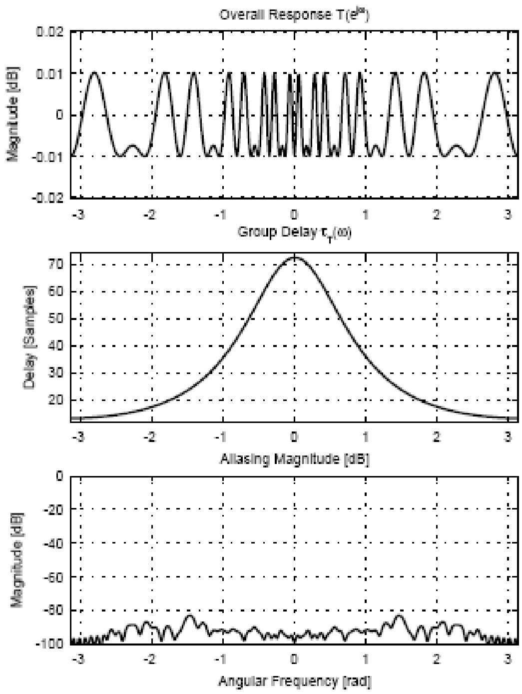

Figure-2. **Design Example QP**[1]. *Overall magnitude response* $|T(e^{j\omega})|$, *group delay and maximum output signal aliasing magnitude* $S_{avg}(\omega)$. *The cost function value for the synthesis filter bank design is* $J_{IIS}(\mathbf{b}) = -91.7$ *dB.*

3.2. Example with phase compensation - QP[2].

Filter banks are designed without phase compensation in the synthesis filter bank. The delay parameter for the phase compensation filters is set to $p = 6$. The resulting analysis filters $Hm(z)$ and synthesis filters $Gm(z)$ are shown in Figure-3. The magnitude and the group delay of the overall response, $T(e^{j\omega})$ and the average magnitude of the aliasing terms Savg(ω) are shown in Figure-4.

VOL. 2, NO. 3, JUNE 2007 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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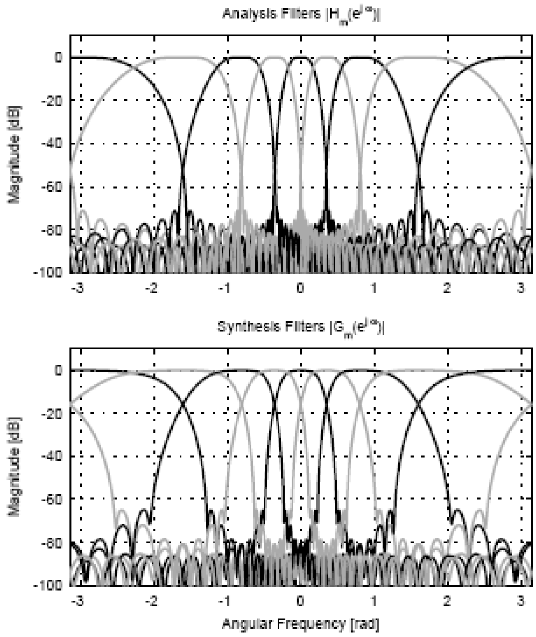

Figure-3. **Design Example QP**^{2}. *Magnitude responses of the analysis filters* $|H_m(e^{j\omega})|$ *and synthesis filters* $|G_m(e^{j\omega})|$, *for* $m = 0, \ldots, M-1$. *The analysis and synthesis filter banks are designed using quadratic optimization with linear constraints. Phase compensation is used in the synthesis filter bank. The cost function value for the analysis filter bank design is* $J^{II}_A(\mathbf{a}) = -81.6$ *dB.*

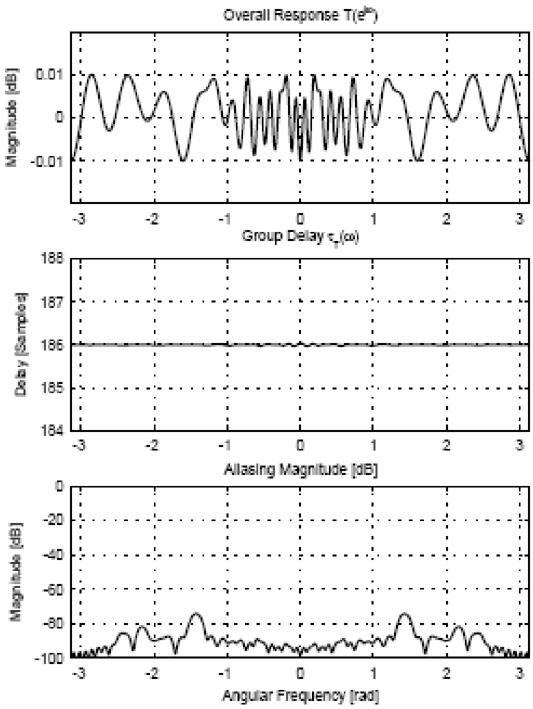

Figure-4. **Design Example QP**^{2}. *Overall magnitude response* $|T(e^{j\omega})|$, *group delay and maximum output signal aliasing magnitude* $S_{avg}(\omega)$. *The cost function value for the synthesis filter bank design is* $J^{II}_S(\mathbf{b}) = -86.6$ *dB.*

4. Conclusions

In this paper, two stage design methods are presented for the design of non-uniform filter banks using Quadratic optimization with linear constraints. The polyphase structure with allpass functions is used with or without the compensation of nonlinear phase in the synthesis filter bank. One of the design objectives of the proposed method is to minimize the magnitude of all aliasing components individually, such that aliasing distortion is minimized although phase alterations occur in the subbands as in subband adaptive filtering. Design examples are given of filter banks with increasing bandwidth with a comparison of the results obtained with different design criteria.

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