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OPTICAL BISTABILITY IN A TRIMODALE LASER CONTAINING A SATURABLE ABSORBER

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ABSTRACT

A simple mathematical model to describe the action of a saturable absorber in a laser cavity (LSA) was developed. The approach considered here takes in a phenomenological way into account the essential physical processes which makes it possible to determine the principal parameters in a LSA and their influences on the optical bistability. We will study theoretically optical bistability in a trimodale laser containing a saturable absorber in the case that the active medium and absorber show a homogeneous broadening. We determine the densities of photons as function of the pumping of the active medium and analyze the linear stability of the solutions obtained.

Keywords: optical bistability, laser, saturable absorber, homogeneous broadening, Fabry-Perot cavity.

1. INTRODUCTION

Optical bistability experienced high interest these last years. Optical bistability belongs to the most important nonlinear optical effects. An essential element of most devices that exhibit optical bistability is an optical resonator, in which a nonlinear material is placed.

The majority of optical flip-flops developed until now utilize two states, which give rise to levels of "high" and "low" energy transmission by the device. In general, optical bistability presenting the phenomenon of hysteresis; is obtained as a solution of nonlinear equations. The structure of a laser containing a saturable absorber is shown in Figure-1.

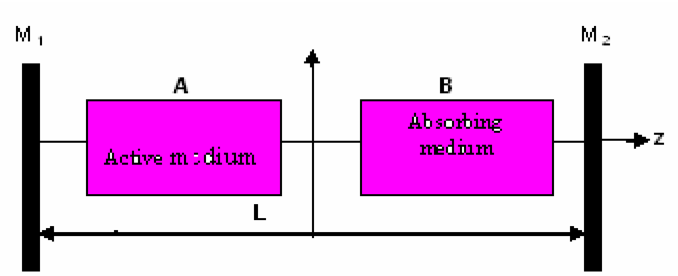

Figure-1. Structure of a laser containing a saturable absorber. L is the cavity length; Z is the direction of propagation. M1 and M2 are plane mirrors.

2. MATHEMATICAL MODEL

The system of equations of a trimode laser containing a saturable absorber consists of the following equations: The 1st equation gives the density of photons n0 in the cavity resonator of the central mode. The 2nd equation determines the density of photons n± in the cavity resonator of the modes of orders ±1. Two further rate equations determine the variation of the difference of population of the active medium Na and that of the absorbing medium Nb. These equations are written as under [1,2]:

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$$\left.\begin{aligned}
\frac{dn_0}{dt} &= -x_0 n_0 + BLg_0\left(n_0+1\right)\left[\int_{z-\frac{l}{2}}^{z+\frac{l}{2}} N_a \sin^2\left(\frac{\pi}{L}qz\right)dz - \int_{z-\frac{l}{2}}^{z+\frac{l}{2}} N_b \sin^2\left(\frac{\pi}{L}qz\right)dz\right]\\
\frac{dn_{\pm1}}{dt} &= -x_{\pm1} n_{\pm1} + BLg_{\pm1}\left(n_{\pm1}+1\right)\left[\int_{z-\frac{l}{2}}^{z+\frac{l}{2}} N_a \sin^2\left(\frac{\pi}{L}qz\right)dz - \int_{z-\frac{l}{2}}^{z+\frac{l}{2}} N_b \sin^2\left(\frac{\pi}{L}qz\right)dz\right]\\
\frac{dN_a}{dt} &= \frac{R_a}{L} - N_a\left(Bg_0 n_0 + 2Bg_{\pm1} n_{\pm1} + \gamma_a\right)\\
\frac{dN_b}{dt} &= \frac{R_b}{L} - N_b\left(Bg_0 n_0 + 2Bg_{\pm1} n_{\pm1} + \gamma_b\right)
\end{aligned}\right\} \tag{1}$$

Where:

$x_{\pm1}, x_0$ represent the coefficients of the losses of the resonator for the mode $j=0$ and $j=\pm1$, respectively. B is the coefficient of Einstein;

L the cavity length, and q is the mode index 0 or ±1.

γ_a, γ_b are the damping coefficients of the active medium and absorber, respectively.

We assume $\gamma_b = \xi\gamma_a = \xi\gamma$ with ξ a saturation coefficient: $(0 \le \xi \le 1)$

Alsorption and emission lineshapes are supposed top have a Lorentzian shape g with: $g = \dfrac{\Gamma^2}{\Gamma^2 + \Delta_j^2}$

Where Γ is the homogeneous width of line and $\Delta_j = \left|\omega_j - \omega_0\right|$ the detuning from the central frequency ω_0.

R_a, R_b are the pumping rates of the active and the absorbing medium, respectively,

The term $\left[\int_{z-\frac{l}{2}}^{z+\frac{l}{2}} N_a \sin^2\left(\frac{\pi}{L}qz\right)dz - \int_{z-\frac{l}{2}}^{z+\frac{l}{2}} N_b \sin^2\left(\frac{\pi}{L}qz\right)dz\right]$: describes the effect of interference produced in the Fabry-Pérot resonator. It can take various values which depend on the position of the two nonlinear media in the optical resonator. In general, if $l \gg \lambda$, (where λ is the wavelength of the laser modes) the populations N_a and N_b are independent of position z inside the cavity and consequently this term can be written as:

$$\left(N_a - N_b\right)\left[\int_{z-\frac{l}{2}}^{z+\frac{l}{2}} \sin^2\left(\frac{\pi}{L}qz\right)dz\right] = \left(N_a - N_b\right)\frac{A_0}{2}$$

For the case where the nonlinear media are placed in the middle of the cavity, A_0 takes the value

$$\left[1 - \frac{L}{\pi q} \sin\frac{\pi q l}{L}\right]$$

3. OPTICAL BISTABILITY

We study here optical bistability and so we are interested only in the case of stationary laser emission. We therefore consider: $\dfrac{dn_{\pm1}}{dt} = \dfrac{dn_0}{dt} = \dfrac{dNa}{dt} = \dfrac{dNb}{dt} = 0$

This leads us to the system of equations according to:

$$\left.\begin{array}{l} -x_0 n_0 + \dfrac{Bl}{2} g_0 (n_0+1)(Na-Nb)A_0 = 0 \\ -x_{\pm1} n_{\pm1} + \dfrac{Bl}{2} g_{\pm1}(n_{\pm1}+1)(Na-Nb)A_0 = 0 \\ \dfrac{Ra}{l} - Na(Bg_0 n_0 + 2Bg_{\pm1} n_{\pm1} + \gamma) = 0 \\ \dfrac{Rb}{l} - Nb(Bg_0 n_0 + 2bg_{\pm1} n_{\pm1} + \xi\gamma) = 0 \end{array}\right\} \quad (1.a)$$

We obtain from the first equations of the system (1):

$$\frac{x_0 n_0}{x_{\pm1} n_{\pm1}} = \frac{g_0(n_0+1)}{g_{\pm1}(n_{\pm1}+1)} \quad (2)$$

In order to simplify the calculations, we introduce the following changes of variables:

$Q_j = \dfrac{B}{\gamma} n_j$, $\sigma_{aj} = \dfrac{B}{\gamma}\dfrac{R_a}{x_j}$ and $\sigma_{bj} = \dfrac{B}{\gamma}\dfrac{R_b}{x_j}$ with: $j = 0, \pm1$

According to equation (2), we distinguish two cases:

3.1: Case where $x_0 g_{\pm1} - x_{\pm1} g_0 = 0$

In this case, the first terms of the system of equations (1) vanish, corresponding to vanishing spontaneous emission. Let us study the case when the coefficient of saturation is equal to unity (ξ=1) and compare it to the case $\xi \neq 1$.

a) : Saturation coefficient equal to 1 (ξ =1)

After several transformations, we obtain from equation (1) an algebraic equation of second order of the form:

$$Q_0^2 + Q_0 A_1 + B_2 = 0 \quad (3)$$

Where: $A_2 = \dfrac{1}{g_0 + 2g_{\pm1}} - \dfrac{A_0 g_0 (\sigma_{a_0} - \sigma_{b_0})}{2(g_0 + 2g_{\pm1})}$ and $B_2 = -\dfrac{A_0 g_0 (\sigma_{a0} - \sigma_{b0})\left(\dfrac{B}{\gamma}\right)}{2(g_0 + 2g_{\pm1})}$.

The necessary condition for optical bistability (OB) (at least 3 different values for one value of Q_0) cannot be fulfilled.

b) : Saturation coefficient not equal to 1 $(\xi \neq 1)$

While proceeding in the same way as above, we find an equation of third order in Q_0 of the form:

$$Q_0^3 + A_3 Q_0^2 + B_3 Q_0 + C_3 = 0 . \quad (4)$$

Where:

$$A_3 = \frac{(1+\xi)x_{\pm1}}{g_0 x_{\pm1} + 4x_{\pm1} g_{\pm1} + 4\dfrac{g_{\pm1}^3}{g_0^2} x_0} - \frac{A_0(\sigma_{a0} - \sigma_{b0})}{1 + 4\dfrac{x_{\pm1}}{x_0} + 4\dfrac{g_{\pm1}^2}{g_0^2}} + \frac{2(1+\xi)x_{\pm1}\dfrac{g_{\pm1}}{g_0}}{g_0 x_{\pm1} + 4x_{\pm1} g_{\pm1} + 4\dfrac{g_{\pm1}^3}{g_0^2} x_0} - \frac{A_0(\sigma_{a0} - \sigma_{b0})x_{\pm1} g_{\pm1}}{g_0 x_{\pm1} + 4x_{\pm1} g_{\pm1} + 4\dfrac{g_{\pm1}^3}{g_0^2} x_0} .$$

$$B_3 = \frac{\dfrac{\xi}{g_0^2}}{1 + 4\dfrac{x_{\pm1}}{x_0} + 4\dfrac{g_{\pm1}^2}{g_0^2}} - \frac{\dfrac{A_0}{2}(\sigma_{a0} - \sigma_{b0})\left(\dfrac{B}{\gamma}\right)}{1 + 4\dfrac{x_{\pm1}}{x_0} + 4\dfrac{g_{\pm1}^2}{g_0^2}} - \frac{\dfrac{A_0}{2}(\sigma_{a0}\xi - \sigma_{b0})x_{\pm1}}{g_0 x_{\pm1} + 4x_{\pm1} g_{\pm1} + 4\dfrac{g_{\pm1}^3}{g_0^2} x_0} - \frac{A_0(\sigma_{a0} - \sigma_{b0})\left(\dfrac{B}{\gamma}\right)x_{\pm1} g_{\pm1}}{g_0 x_{\pm1} + 4x_{\pm1} g_{\pm1} + 4\dfrac{g_{\pm1}^3}{g_0^2} x_0} .$$

$$C_3 = -\frac{\dfrac{A_0}{2}(\sigma_{a0}\xi - \sigma_{b0})x_{\pm1}\left(\dfrac{B}{\gamma}\right)}{g_0 x_{\pm1} + 4x_{\pm1} g_{\pm1} + 4\dfrac{g_{\pm1}^3}{g_0^2} x_0} .$$

Since the last term of the equation (5) is very small compared to unity ($B/\gamma = 10^{-11}$), the variationnel can be applied. Then, equation (5) takes the form:

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$$\left(Q_0+\frac{c_3}{B_3}\right)\left(Q_0^2+A_3Q_0+B_3\right)=0 \tag{5}$$

The solutions of this equation are easily given and one can find optical bistability.

3.2. Case where : $x_0 g_{\pm1} - x_{\pm1} g_0 \neq 0$

In this case spontaneous emission is taken into account. As above, we distinguish the cases where the coefficient of saturation is equal to the unity or differs from unity.

3.2.1. : Case where $\xi = 1$

We obtain after several transformations a cubic equation in following form:

$$Q_0^3+AQ_0^2+BQ_0+C=0 \tag{6}$$

where:

$$A=\frac{\left(\frac{B}{\gamma}\right)g_0 x_{\pm1}}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}-\frac{2\left(\frac{B}{\gamma}\right)x_0 g_{\pm1}^2}{g_0\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}+\frac{1}{g_0}-\frac{A_0}{2}\left(\sigma_{a0}-\sigma_{b0}\right).$$

$$B=\frac{\left(\frac{B}{\gamma}\right)x_{\pm1}}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}-\frac{A_0g_0x_{\pm1}\left(\sigma_{a0}-\sigma_{b0}\right)\left(\frac{B}{\gamma}\right)}{2\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}-\left(\frac{A_0}{2}\right)\left(\frac{B}{\gamma}\right)\left(\sigma_{a0}-\sigma_{b0}\right).$$

$$C=-\left(\frac{A_0}{2}\right)\left(\frac{B}{\gamma}\right)^2 g_0\frac{\left(\sigma_{a0}-\sigma_{b0}\right)}{\left(x_{\pm1}g_0-x_{\pm1}g\right)}.$$

To find at least two solutions positive i.e. the effect of the optical bistability, it is necessary that the pumping of the active medium must satisfy the following conditions

$$\begin{cases}\sigma_{a0}\prec\sigma_{a01}=\dfrac{2x_{\pm1}}{A_0\left(2x_{\pm1}g_0-x_0g_{\pm1}\right)}+\sigma_{b0}\\[2ex] \sigma_{a0}\succ\sigma_{a02}=\dfrac{2x_{\pm1}\left(g_0\left(\frac{B}{\gamma}\right)+1\right)+2x_0\left(2\left(\frac{B}{\gamma}\right)\frac{g_{\pm1}^2}{g_0}-\frac{g_{\pm1}}{g_0}\right)}{A_0\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}+\sigma_{b0}\\[2ex] \sigma_{a0}\prec\sigma_{a03}=\dfrac{2x_{\pm1}}{A_0\left(2x_{\pm1}g_0-x_0g_{\pm1}\right)}+\sigma_{b0}\end{cases} \tag{7}$$

We note that the conditions are not fulfilled at the same time and consequently there is no optical bistability if the coefficient of saturation is equal to unity

3.2.2. Case if the coefficient of saturation is different from unity $(\xi\neq1)$

By using the expressions of $Q_0, Q_{\pm1}, \sigma_{a0}\, et\, \sigma_{b0}$, we obtain after several transformations an algebraic equation of fifth order of the following form:

$$Q_0^5+A_1Q_0^4+B_1Q_0^3+C_1Q_0^2+D_1Q_0+E_1=0 \tag{8}$$

where :

$$A_1=\frac{2g_{\pm1}\left(\frac{B}{\gamma}\right)x_{\pm1}}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}+\frac{4g_{\pm1}^2x_0\left(\frac{B}{\gamma}\right)}{g_0\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}+\frac{(1+\xi)}{g_0}-\frac{A_0}{2}\left(\sigma_{a0}-\sigma_{b0}\right)$$

$$B_1=\frac{g_0^2x_{\pm1}^2\left(\frac{B}{\gamma}\right)^2}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)^2}+\frac{4g_{\pm1}^2x_{\pm1}x_0\left(\frac{B}{\gamma}\right)^2}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)^2}+\frac{2x_{\pm1}\left(\frac{B}{\gamma}\right)(1+\xi)}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}-\frac{A_0g_0x_{\pm1}^2\left(\frac{B}{\gamma}\right)\left(\sigma_{a0}-\sigma_{b0}\right)}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}+\frac{4g_{\pm1}^4x_0^2\left(\frac{B}{\gamma}\right)^2}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)^2}+\frac{\xi}{g_0^2}$$

$$+\frac{2g_{\pm1}^2(1+\xi)x_0\left(\frac{B}{\gamma}\right)}{\left(x_{\pm1}g_0-x_0g_{\pm1}\right)g_0^2}-\frac{A_0g_{\pm1}^2x_0\left(\frac{B}{\gamma}\right)\left(\sigma_{a0}-\sigma_{b0}\right)}{g_0\left(x_{\pm1}g_0-x_0g_{\pm1}\right)}-\frac{A_0\left(\sigma_{a0}\xi-\sigma_{b0}\right)}{2g_0}-\frac{A_0}{2}\left(\frac{B}{\gamma}\right)\left(\sigma_{a0}-\sigma_{b0}\right)$$

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$$C_1 = \frac{(1+\xi) g_0 x_{\pm1}^2 \left(\frac{B}{\gamma}\right)^2}{\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2} - \frac{A_0 g_0^2 x_{\pm1}^2 \left(\frac{B}{\gamma}\right)^2 (\sigma_{a0} - \sigma_{b0})}{2\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2} + \frac{2(1+\xi) g_{\pm1}^2 x_0 x_{\pm1} \left(\frac{B}{\gamma}\right)^2}{\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right) g_0} + \frac{2\xi x_{\pm1} \left(\frac{B}{\gamma}\right)}{g_0\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)}$$

$$- \frac{A_0 g_{\pm1}^2 x_0 x_{\pm1} \left(\frac{B}{\gamma}\right)^2 (\sigma_{a0} - \sigma_{b0})}{\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2} - \frac{A_0 x_{\pm1} \left(\frac{B}{\gamma}\right) (\sigma_{a0}\xi - \sigma_{b0})}{\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)} - \frac{A_0 g_0 x_{\pm1} \left(\frac{B}{\gamma}\right)^2 (\sigma_{a0} - \sigma_{b0})}{2\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2} - \frac{A_0 g_{\pm1}^2 x_0 \left(\frac{B}{\gamma}\right)^2 (\sigma_{a0} - \sigma_{b0})}{g_0\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2}$$

$$- \frac{A_0 \left(\frac{B}{\gamma}\right) (\sigma_{a0}\xi - \sigma_{b0})}{2 g_0 \left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2}.$$

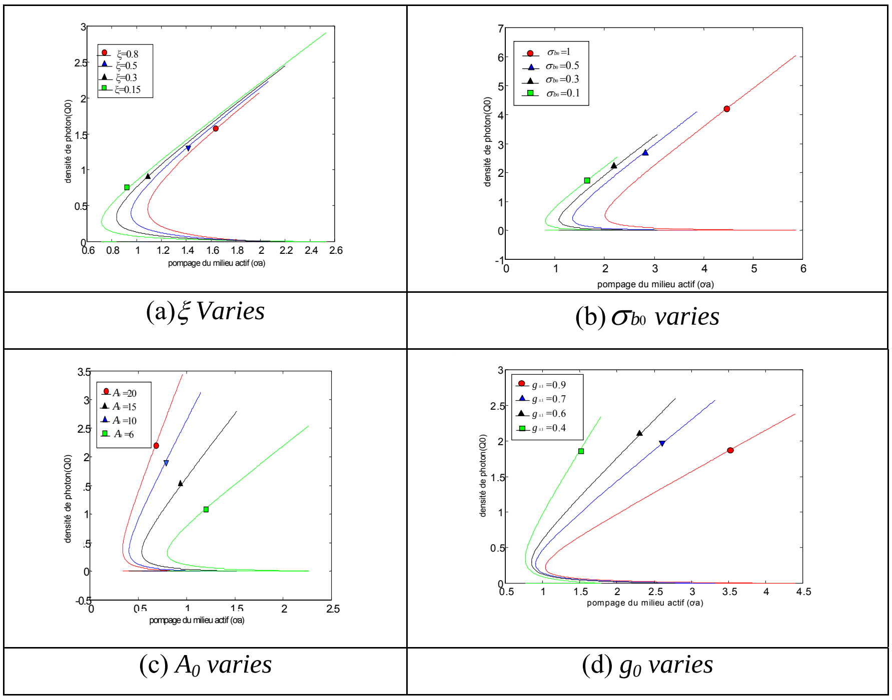
$$D_1 = \frac{\xi \left(\frac{B}{\gamma}\right)^2 x_{\pm1}^2}{\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2} - \frac{g_0 A_0 (\sigma_{a0}\xi - \sigma_{b0}) \left(\frac{B}{\gamma}\right)^2 x_{\pm1}^2}{2\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2} - \frac{g_0^2 A_0 (\sigma_{a0} - \sigma_{b0}) \left(\frac{B}{\gamma}\right)^3 x_{\pm1}^2}{2\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2} - \frac{g_{\pm1}^2 A_0 (\sigma_{a0} - \sigma_{b0}) \left(\frac{B}{\gamma}\right)^3 x_{\pm1} x_0}{\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2}$$

$$- \frac{A_0 (\sigma_{a0}\xi - \sigma_{b0}) \left(\frac{B}{\gamma}\right)^2 x_{\pm1}}{\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)}$$

and $$E_1 = \frac{-A_0 \left(\frac{B}{\gamma}\right)^3 g_0 x_{\pm1}^2 (\sigma_{a0}\xi - \sigma_{b0})}{2\left(x_{\pm1} g_0 - x_0 g_{\pm1}\right)^2}$$

3.3. Evolution of the density of photons with pumping rate of the active medium

To specify the conditions of appearance of OB, it is convenient to establish a program, which determines the intervals of OB. It is based on the condition that at least two positive densities of photons inside the cavity exist for a given pumping rate. The programme determines the values of (Q_{01} Q_{02} Q_{03}) for various values of the parameters σ_{b0} A_0, $g_{\pm1}$, g_0, ξ, while taking (σ_{a0}) as variable. Then, we plot the densities of photons as function of the pumping rate of the active medium (σ_{a0}) for the parameters σ_{b0}, A_0, $g_{\pm1}$, g_0 and ξ (Figures 2 and 3).



(a) ξ *Varies* (b) σ_{b0} *varies*

(c) A_0 *varies* (d) g_0 *varies*

Figure-2. Density of photons Q_0 as function of the pumping rate of the active medium (σ_{a0}) (case where the spontaneous emission is neglected $\xi = 0$).

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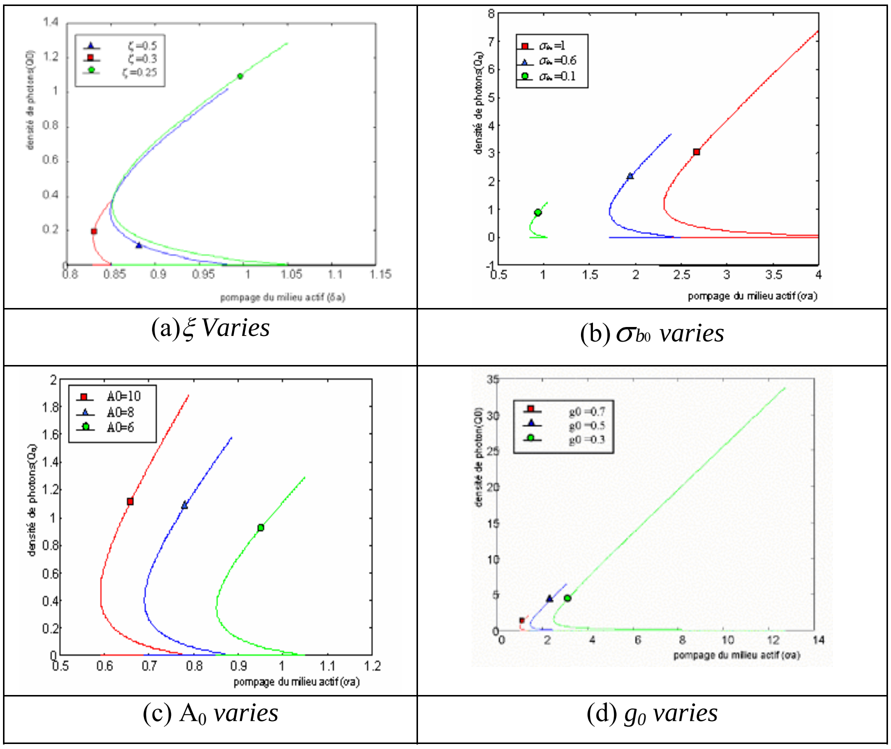

(a) ξ *Varies* (b) σ_{b0} *varies*

(c) A_0 *varies* (d) g_0 *varies*

Figure-3. Density of photons for various parameters (case where the spontaneous emission is included).

4. DISCUSSION

By examining the influence of the parameters of the L.S.A on the appearance of the optical bistability, we notice that a reduction in the coefficient of saturation, leads to an increase in the interval of the density of photons in which the system shows optical bistabitity. This can be explained in the following way: the difference in energy of the absorbing medium must correspond to the frequency of the laser transition in the active medium. In the initial state, the saturable absorber has its maximum of opacity; i.e. the state is not saturated. The luminous irradiation of the saturable absorber at the frequency of oscillation gives rise to a resonant absorption and of spontaneous emission. Then, when the pumping of the absorbing medium (σ_{b0}) increases, the saturation is increases, too. After the end of the irradiation, energy relaxation ensures the return to the fundamental state and the absorber becomes again unsaturated. On the other hand, if the pumping of the absorbing medium increases, it causes the saturation of the medium thus and generates an increase in the interval of the effect of the optical bistability and consequently the increase in extended in the density of photons.

Concerning the parameter A_0, which depends on the length of the cavity as well as on the lengths of the two nonlinear media, we notice an increase in the interval of optical bistability as well as of the extent of the density of cavity photons. That is probably due to the growth of the interfering wave amplitudes which produce an increase in the intensity in the cavity. Concerning the profiles of the lines $(g_0, g_{\pm 1})$, their increase (that is the increase in Γ or by the reduction of $\Delta_j = \omega_j - \omega_0$) lead to an increases in the interval of the effect of the optical bistability and of the density of photons. This can be explained by the following argument: if Γ increases, more modes can oscillate in the cavity. On the other hand, if Δ_j decreases, the laser tends to become monomode since ω_j approaches ω_0, giving rise to the same features.

Linearization of the system of equations in the vicinity of the stationary solution

The linear analysis of stability consists in determining the evolution of small variations of balances. If the balance is stable, their variations diminish in time. On the other hand, they diverge in the case of an unstable balance. One linearizes the system of differential equations in the vicinity of the stationary solutions and considers for this purpose:

$$
\begin{aligned}
n_0 &= n_{0s} + \Delta n_0(t) \\
n_{\pm 1} &= n_{\pm 1s} + \Delta n_{\pm 1}(t) \\
N_a &= N_{as} + \Delta N_a(t) \\
N_b &= N_{bs} + \Delta N_b(t)
\end{aligned}
$$

Where: $n_{0s}, n_{\pm 1s}, N_{as}, N_{bs}$ are the stationary values.

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$\Delta n_0, \Delta n_{\pm 1}, \Delta N_a, \Delta N_b$ are small corresponding variations of quantities with respect to their stationary values. When replacing $n_0, n_{\pm 1}, N_a$ *et* N_b in the equations of the system (1) and neglecting all quantities which are of second order in smallness Δ, one obtains finally a system of linear differential equations for the variations $\Delta n_0(t), \Delta n_{\pm 1}(t), \Delta N_a(t), \Delta N_b(t)$ which is given by:

$$\begin{bmatrix} \frac{d\Delta n_0}{dt} \\ \frac{d\Delta n_{\pm 1}}{dt} \\ \frac{d\Delta N_a}{dt} \\ \frac{d\Delta N_b}{dt} \end{bmatrix} = A \begin{bmatrix} n_0 \\ n_{\pm 1} \\ N_a \\ N_b \end{bmatrix} \qquad où \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{9}$$

A is the matrix representing the characteristic equation of the system of linear differential equations.
Where:

$$a_{11} = -x_0 + \frac{B\lg_0}{2} A_0 (N_{as} - N_{bs}) \ , \ a_{13} = (n_{0s} + 1)\frac{Bl}{2} g_0 A_0 = -a_{14} \ , \ a_{22} = -x_{\pm 1} + \frac{B\lg_{\pm 1}}{2} A_0 (N_{as} - N_{bs})$$

$$a_{12} = a_{21} = a_{34} = a_{43} = 0.$$

$$a_{23} = (n_{\pm 1s} + 1)\frac{Bl}{2} g_{\pm 1} A_0 = -a_{24} \ , \ a_{23} = (n_{\pm 1s} + 1)\frac{Bl}{2} g_{\pm 1} A_0 = -a_{24} \ , \ a_{23} = (n_{\pm 1s} + 1)\frac{Bl}{2} g_{\pm 1} A_0 = -a_{24}.$$

$$a_{31} = -N_{as} Bg_0 \ , \ a_{32} = -2Bg_{\pm 1} N_{as} \ , \ a_{33} = -(Bg_0 n_{0s} + 2Bg_0 n_{\pm 1s} + \gamma_a)$$

$$a_{41} = -N_{bs} Bg_0, \ a_{42} = -2Bg_{\pm 1} N_{bs} \ , \ a_{44} = (Bg_0 n_{0s} + 2Bg_0 n_{\pm 1s} + \gamma_b),$$

The calculation of the determinant of $(A + I\lambda)$ gives the characteristic equation of fourth order in λ :

$$\lambda^4 + k_1 \lambda^3 + k_2 \lambda^2 + k_3 \lambda + k_4 = 0 \tag{10}$$

λ is the eigenvalue of the matrix and I is the unit matrix of order four.
The k_i have t-he values:

$$k_1 = a_{11} + a_{22} + a_{33} + a_{44}.$$

$$k_2 = a_{11}(a_{33} + a_{22} + a_{44}) + a_{22}a_{44} + a_{33}a_{44} + a_{22}a_{33} - a_{23}a_{32} - a_{24}a_{42} - a_{13}a_{31} - a_{14}a_{41}.$$

$$k_3 = a_{11}a_{22}a_{44} + a_{11}a_{33}a_{44} + a_{22}a_{33}a_{11} - a_{23}a_{32}a_{11} - a_{24}a_{42}a_{11} + a_{22}a_{33}a_{44} - a_{23}a_{32}a_{44} - a_{24}a_{42}a_{33} - a_{31}(a_{22} + a_{44})a_{13} - a_{41}a_{41}(a_{22} + a_{33}).$$

$$k_4 = a_{22}a_{33}a_{44}a_{11} - a_{23}a_{32}a_{44}a_{11} - a_{24}a_{42}a_{33}a_{11} - a_{31}a_{22}a_{44}a_{13} + a_{24}a_{31}a_{42}a_{13} - a_{32}a_{41}a_{24}a_{13} - a_{22}a_{33}a_{41}a_{14} - a_{14}a_{23}a_{31}a_{42} + a_{32}a_{41}a_{23}a_{14}$$

To know if the state is stable or unstable, one has to determine the roots of eq. 10. Indeed if the real parts of the four roots of the characteristic equation (10) are positive, the state is stable. If, on the contrary, at least one of these roots has a negative real part, this state will be unstable. The question of stability or instability of the initial stationary state covers the fundamental importance in order to determine whether an automatic damping of perturbations occurs in a saturable absorber laser. If the initial state is stable, fluctuations of the amplitude of the field and the density of inversion of populations cannot make oscillate the laser. In other words, the stability of the stationary state means that pumping of the active element alone is unable to give rise to laser action without the support of a secondary source

5. CONCLUSION

The analysis of the linear stability of the stationary solutions showed us the existence of two stable solutions. Consequently the system shows optical bistability if the spontaneous emission is neglected in the system of rate equations. According to the results obtained in the analysis of optical bistability, we note that one branch is unstable because it has two positive roots and a negative root of the characteristic equation, independent which parameters of the L.S.A are taken within the interval of the optical bistability. On the other hand, the two other branches and are stable.

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