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# MHD UNSTEADY FREE CONVECTIVE WALTER'S MEMORY FLOW WITH CONSTANT SUCTION AND HEAT SINK

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## ABSTRACT

The study of unsteady hydromagnetic free convective memory flow of incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and heat absorbing sinks have been made. Approximate solutions have been derived for the mean velocity, mean temperature, mean skin-friction and mean rate of heat transfer using multi-parameter perturbation technique. It is observed that magnetic field strength decreases the mean velocity of the fluid. Also the mean skin-friction and mean rate of heat transfer of the conducting fluid decreases with the increase in magnetic field strength.

Keywords: memory fluid, flow, suction, heat, sink, field, magnetic, velocity.

#### **INTRODUCTION**

The effect of magnetic field on free convective flow of electrically conducting fluids past a semiinfinite flat plate has been analysed by (Gupta, 1960), (Singh and Cowling, 1963) and (Nanda and Mohanty, 1970). The unsteady free convective flow past an infinite plate with constant suction and heat sources has been studied by (Pop and Soundalgekar, 1974). (Raptis and Kofousias, 1982) had studied the magnitohydrodynamic (MHD) free convection flow and mass transfer through a porous medium bounded by an infinite vertical plate with constant heat flux. (Sacheti et al., 1994) have obtained an exact solution for the unsteady MHD problem. MHD free convective flow with Hall current in a porous medium for electrolytic solution (viz. salt water) was studied by (Sattar and Alam, 1995). But they have neither considered the effect of constant suction nor included the heat absorbing sink and viscous dissipation. The propagation of thermal energy through mercury and electrolytic solution in the presence of external magnetic field and heat absorbing sinks has wide range of application in chemical and aeronautical engineering, atomic propulsion, space science etc. Unsteady effect on MHD free convective and mass transfer flow through porous medium with constant suction and constant heat flux in rotating system was studied by (Sharma, 2004). MHD convective flow of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption was studied by (Rahman and Sattar, 2006).

The objective of this study was to extend the work of (Sahoo et al., 2003) to memory fluid. The mixture of polymethyl mehacrylate and pyridine at 25°C containing 30.5g of polymer per liter behaves very nearly as the (Walter's liquid model B, 1960, 1962).

## FORMULATION OF THE PROBLEM

Let the x-axis be taken in the vertically upward direction along the infinite vertical plate and y-axis normal to it. Neglecting the induced magnetic field and applying Boussineqs approximation, the equations of the flow is governed as: (1)

$$\partial v / \partial y = 0$$
 (1)  
i.e,  $v = -v_0$  (constant) (2)

$$\partial \mathbf{u} / \partial \mathbf{t} + \mathbf{v} \partial \mathbf{u} / \partial \mathbf{y} = \mathbf{g} \beta (\mathbf{T} - \mathbf{T}_{\infty}) + \mathbf{v} \partial^2 \mathbf{u} / \partial \mathbf{y}^2 - \mathbf{B}_1 (\partial^3 \mathbf{u} / \partial \mathbf{v}^2 + \mathbf{v} \partial^3 \mathbf{u} / \partial \mathbf{v}^3) - \sigma \mathbf{B}_0^2 \mathbf{u} / \rho$$
(3)

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} + S(T-T_{\infty}) + v/C_p$$

$$(\frac{\partial u}{\partial y})^2$$
(4)

On disregarding the Joulean heat dissipation, the boundary conditions of the problem are:

$$y = 0: u = 0, v = -v_0, T = T_w + \varepsilon (T_w - T_\infty) e^{i\omega t}$$
  
$$y \to \infty : u \to 0, T \to T_\infty.$$
(5)

Introducing the non-dimensional quantities and parameters,

$$y^* = y v_0 / v,$$
  $t^* = t v_0^2 / 4 v,$   $\omega^* = 4 v \omega / v_0^2,$   
 $u^* = u / v_0$ 

$$v = \dot{\eta}_0 / \rho,$$
  $Pr = v / K',$   $S * = 4 S v / v_0^2,$   
 $K = K_0 / \rho C_p$ 

$$T^* = (T - T_{\infty}) / (T_w - T_{\infty}), \qquad Gr = \nu g \beta (T_w - T_{\infty}) / v_0^3,$$

$$Ec = v_0^2 / C_p (T_w - T_\infty), \quad M = (\sigma B_0^2 / \rho) \nu / v_0^3, \quad R_m = B_1 v_0^2 / \nu^2$$
(6)

where g,  $\beta$ ,  $\nu$ , B<sub>0</sub>,  $\sigma$ , B<sub>1</sub>,  $\rho$ ,  $\kappa$ , C<sub>p</sub>, Pr, Gr, S, Ec, M and R<sub>m</sub> are acceleration due to gravity, kinematic visco-elasticity, kinematic viscosity, magnetic field of uniform strength, electrical conductivity, coefficient of volumetric expansion, density, thermal conductivity, specific heat at constant pressure, Prandtl number,

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Grashoff number, Sink strength, Eckert number, Hartmann number and Magnetic Reynolds number, respectively. Using equations (5) and (6), equations (2) and (3) become:

$$\begin{array}{l} 1/4\partial \ u \ / \ \partial \ t \ - \ \partial \ u \ / \ \partial \ y = Gr \ T \ + \ \partial^2 \ u \ / \ \partial \ y^2 \ - \ R_m \ ( \ 1/4(\partial^3 \ u \ / \ \partial \ t^2) \ - \ \partial^3 \ u \ / \ \partial \ y^3 \ ) \ - \ M \ u \ (7) \\ and \\ (Pr \ / \ 4) \ \partial \ T \ / \ \partial \ t \ - \ Pr \ \partial \ T \ / \ \partial \ u = \ \partial^2 \ T \ / \ \partial \ y^2 \ + \ Pr \ S \ T \ / \ 4 \ + \\ Pr \ Ec \ ( \ \partial \ u \ / \ y)^2 \ (8) \\ (after \ dropping \ the \ asterisks) \end{array}$$

The corresponding boundary conditions are:

$$\begin{array}{ccc} y = 0; & u = 0, & T = 1 + \varepsilon e^{i \omega t} \\ y \to \infty; & u \to 0, & T \to 0 \end{array}$$
 (9)

To solve equations (7) and (8), we assume  $\omega$  to be very small and the velocity and temperature in the neighbourhood of the plate as

where  $u_0$  and  $T_0$  are mean velocity and mean temperature. Substituting (10) in equations (7) and (8), equating harmonic and non-harmonic terms for mean velocity and mean temperature, after neglecting coefficient of  $\varepsilon^2$ , we get

$$R_{m} u_{0}^{111} + u_{0}^{11} + u_{0}^{1} - M u_{0} = -Gr T_{0}$$
(11)

$$T_0^{11} + Pr T_0^{1} + Pr S T_0 / 4 = -Pr Ec (u_0^{1})^2$$
 (12)

The equation (11) is third order differential equation due to presence of elasticity. Therefore  $u_0$  is expanded using (Beard and Walters rule, 1964).

$$u_0 = u_{00} + R_m u_{01} \tag{13}$$

## Zero-order of $R_m$

$$u_{00}^{11} + u_{00}^{1} - M u_{00} = - Gr T_0$$
 (14)

#### First-order of R<sub>m</sub>

$$\mathbf{u}_{01}^{11} + \mathbf{u}_{01}^{1} - \mathbf{M} \mathbf{u}_{01} = -\mathbf{u}_{00}^{111}$$
(15)

Using multiparameter perturbation technique and assuming  $Ec \ll 1$ , we write

 $\mathbf{u}_{00} = \mathbf{u}_{000} + \mathbf{E}\mathbf{c} \,\mathbf{u}_{001} \tag{16}$ 

 $\mathbf{u}_{01} = \mathbf{u}_{011} + \mathbf{E}\mathbf{c} \,\mathbf{u}_{012} \tag{17}$ 

 $T_{0} = T_{00} + Ec T_{01}$ (18)

Using equations (16), (17) and (18) in the equations (12), (14) and (15) and equating the coefficient of  $\text{Ec}^{-0}$  and  $\text{Ec}^{-1}$ ,

we get the following sets of differential equations

#### Zero-order of Ec

$$u_{000}^{11} + u_{000}^{1} - M u_{000} = - Gr T_{00}$$
(19)

$$\mathbf{u}_{011}^{11} + \mathbf{u}_{011}^{1} - \mathbf{M} \mathbf{u}_{011} = -\mathbf{u}_{000}^{111}$$
(20)

$$T_{00}^{11} + Pr T_{00}^{1} + Pr S T_{00} / 4 = 0$$
(21)

## First – order of Ec :

$$u_{001}^{11} + u_{001}^{1} - M u_{001} = - Gr T_{01}$$
(22)

$$\mathbf{u}_{012}^{11} + \mathbf{u}_{012}^{11} - \mathbf{M} \mathbf{u}_{012} = -\mathbf{u}_{001}^{111}$$
(23)

$$T_{01}^{11} + Pr T_{01}^{1} + Pr S T_{01} = -2 Pr (u_{000}^{1})^2$$
 (24)

The corresponding boundary conditions are:

$$\begin{array}{c} y = 0: u_{000} = u_{001} = u_{011} = u_{012} = 0, \\ T_{00} = 1, T_{01} = 0 \\ y \rightarrow \infty: u_{000} \rightarrow u_{001} \rightarrow u_{011} \rightarrow u_{012} \rightarrow 0, \\ T_{00} \rightarrow T_{01} \rightarrow 0 \end{array}$$

$$(25)$$

#### SOLUTION OF THE PROBLEM

Solving these differential equations from (19-24), using boundary conditions (25), and then making use of equations (16-18).

Finally with the help of equation (13), we obtain the mean velocity  $u_0$  and mean temperature T<sub>0</sub> as follows

 $\begin{array}{l} u_{0} = \{ \left[ t_{2} \left( \exp(-l_{2} y) - \exp(-l_{1} y) \right) + Ec \left( t_{22} \exp(-l_{2} y) - t_{18} \exp(-l_{1} y) + t_{19} \exp(-2l_{2} y) + t_{20} \exp(-2l_{1} y) - t_{21} \exp(-t_{3} y) \right) \right] + R_{m} \left[ \left( -t_{33} \exp(-l_{2} y) - y t_{32} \exp(-l_{2} y) + t_{33} \exp(-l_{1} y) \right) + Ec \left( \left( t_{35} + y t_{34} \right) \exp(-l_{2} y) - t_{28} \exp(-l_{1} y) + t_{29} \exp(-2l_{2} y) + t_{30} \exp(-2l_{1} y) - t_{31} \exp(-t_{3} y) \right) \right] \}$   $\begin{array}{l} \end{array}$   $\begin{array}{l} (26) \end{array}$ 

 $T_{0} = \{ \exp(-l_{1} y) + Ec(t_{14} \exp(-l_{1} y) - t_{11} \exp(-2 l_{2} y) - t_{12} \exp(-2 l_{1} y) + t_{13} \exp(-t_{3} y)) \}$ (27)

#### Mean Skin-Friction and Mean Rate of Heat Transfer

The mean skin friction at the plate in dimensionaless form is given by

$$\tau_{\omega}^{m} = (\partial u_{0} / \partial y)_{y=0} = u_{0}'(0)$$
(28)

$$\tau_{\omega}^{m} = \{ t_{2} (l_{1}-l_{2}) + Ec (l_{2}(t_{22}-2 t_{19}) + l_{1}(t_{18}-2 t_{20}) + t_{3} t_{21}) \} + R_{m} \{ (t_{33} l_{2} - t_{32} - l_{1} t_{33}) + Ec (-t_{35} l_{2} + t_{34} + t_{28} l_{1} - 2 l_{2} t_{29} - 2 l_{1} t_{30} + t_{3} t_{31}) \}$$
(29)

Similarly, the mean rate of heat transfer at the plate is given by

$$q_{\omega}^{m} = (\partial T_{0} / \partial y)_{y=0} = T_{0}'(0)$$
 (30)

 $q_{\omega}^{m} = -l_{1} + \text{Ec} \left[ -l_{1} t_{14} + 2 l_{2} t_{11} + 2 l_{1} t_{12} - t_{3} t_{13} \right]$ (31)  $l_{1} = 1/2 [\text{Pr} + \sqrt{(\text{Pr}^{2} - \text{Pr} S)}], l_{2} = 1/2 [1 + \sqrt{(1+4M)}]$  © 2006-2007 Asian Research Publishing Network (ARPN). All rights reserved.



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where  $t_1$  to  $t_{35}$  are constants and their expressions are not presented here for the sake of brevity.

#### **DISCUSSION AND CONCLUSIONS**

Table-1 showed the mean skin-friction for mercury and electrolytic solution. It was noticed that the increase in magnetic field strength decreases the mean skin-friction, for both mercury and electrolytic solution. Similar effect noted in sink strength.

<b>Table-1.</b> Values of mean skin-friction $\tau_{\omega}^{m}$ for fixed values of Gr	= 5.0,
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Ec = 0.001 and  $\omega = 5.0$ 

Pr	Μ	S	$\tau_{\omega}^{m}$
Mercury ( Pr = 0.025 )	1.0	-0.05	2.73809
	5.0	-0.05	1.56413
	5.0	-0.10	1.55651
Electrolytic solution (Pr = 1.0)	1.0	-0.05	7.21121
	5.0	-0.05	2.50342
	5.0	-0.10	2.49426

Table-2 showed the mean rate of heat transfer for mercury and electrolytic solution. It was observed that the mean rate of heat transfer decreases with the increase in magnetic field strength or sink strength for both mercury and electrolytic solution.

**Table-2.** Values of mean rate of heat transfer q  $_{\omega}$  <sup>m</sup> for fixed values of Gr = 5.0, Ec = 0.001 and  $\omega$  = 5.0

Pr	Μ	S	q ω <sup>m</sup>
Mercury ( Pr = 0.025)	1.0	-0.05	-0.0336
	5.0	-0.05	-0.0341
	5.0	-0.10	-0.0404
Electrolytic solution (Pr = 1.0)	1.0	-0.05	-0.9946
	5.0	-0.05	-1.0111
	5.0	-0.10	-1.0232

The profiles of mean velocity are shown in Figure-1. It depicts the effects of Hartmann number, sink-strength and Prandtl number on the mean velocity. It was observed that the increase in external magnetic field strength and sink-strength decreases the mean velocity. It was also clear from Figure-1 that the mean velocity was greater for mercury (Pr = 0.025) than that of electrolytic solution (Pr = 1.0).

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**Figure-1.** Effects of Pr, S and M on mean velocity for fixed values of (Gr = 5.0, Ec = 0.001 and  $\omega = 5.0$ ).

Figure-2 depicts the effects of M, Pr, Gr, Ec and S on the mean temperature. It was observed that the increase in Hartmann number M decreases the mean temperature. But it is different in decrease in sink-strength increases the mean temperature. It was also observed that increase in Grashoff number, thereby heating the plate, increases the mean temperature. Also increase in Eckert number increases the fluid temperature. It was observed that the mean temperature is more for mercury (Pr = 0.025) than for electrolytic solution (Pr = 1.0).



	Μ	Pr	Gr	Ec	S
Ι	1.0	0.025	5.0	0.001	-0.05
II	1.0	1.0	5.0	0.001	-0.05
III	5.0	1.0	5.0	0.001	-0.05
IV	1.0	1.0	10.0	0.001	-0.05
V	1.0	1.0	5.0	0.001	-0.10
VI	1.0	1.0	5.0	0.1	-0.05

Figure-2. Effects of M, Pr, JI, Ec and S on mean temperature.

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