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THE TARGETING PROBLEM FOR S-TYPE QUALITY CHARACTERISTICS

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ABSTRACT

The targeting problem generally identifies the process parameters such as the initial process means of a production process. The joint determination of the initial process means and the production run has been reported in the literature. Most of these studies considered a process with multiple independent *nominal-the-better* type (N-Type) of quality characteristics. In a real industrial situation, the quality characteristics may depend on each other. In addition, the process could have other types of quality characteristics. In this paper, a mathematical model is developed that jointly determines the optimum initial process means and production run of a process with multiple dependent *smaller-the-better* type (S-Type) of quality characteristics. A Numerical example is also provided to demonstrate the application of the proposed model.

Keywords: multiple dependent S-type quality characteristics, process mean, production run.

INTRODUCTION

The targeting problem is a classical problem which deals with the problem of selecting the optimal initial process means setting for a given product specification. The improper selection of this setting affects the expected total profit (or cost), the number of defective items, the inspection cost and the reprocessing cost. Generally, a loss function is used to quantify the quality loss when the quality characteristics deviate from target values. Thus it assists managers to evaluate the unobservable costs such as warranty cost, loss of market share, sales of return resulting from the customer's dissatisfaction. The traditional step loss function does not incorporate these unobservable costs into a quality loss computation. This approach assumes that the quality cost does not depend on the actual value of the quality characteristic as long as it is within the specification limit. In other words, the step loss function treats an item that falls exactly on the target value as the same as an item that falls just inside the specification limit. Therefore, this approach favours manufacturers by allowing them to produce items with a large variance. Dr. J. Taguchi [1] involves society loss in terms of customers' satisfaction and environment cost into the quality classification. He defined that *quality loss is what the product costs society from the time the product is released from shipment.* Afterwards, the Taguchi loss function is considered as a standard quality measurement system. Other quality loss functions [2] have been proposed which have a similar objective as the Taguchi loss function.

In reality, the process mean setting deviates from its initial value as a result of the presence of assignable causes which eventually moves the process to an out-of-control state. In this state, the increase of the number of defective items leads to an increase of quality loss. This situation can be accepted for a certain time above when it is necessary to take a restoration action. This action can include readjusting the process parameters to its optimum initial values and/or upgrading the tool. Such action is generally expensive and should be done from a viewpoint of the economic feasibility. Therefore, determining the production run is another critical factor to produce quality items. A joint determination of the optimum process mean and production run helps the manufacturer to produce items at a minimum production cost. A few researchers investigated the joint effect of process mean and production run in the process under different assumptions (for a review see [3]). These models can be summarised as follow.

First, most models worked with a *nominal-the-better* type (N-Type) of quality characteristics. Second, only a single quality characteristic was assumed in most of the studies and they used step loss function or Taguchi loss function as a quality measurement system. Third, few researchers [4-5] considered multiple quality characteristics, but they assumed that the quality characteristics were independent on each other.

However, the determination of initial process means setting and production run of a process with multiple *smaller-the-better* type (S-Type) and *larger-the-better* type (L-Type) quality characteristics has not been investigated in previous research. There are many situations where quality characteristics may depend on each other. These dependencies have not been explored in developing the process target models.

This paper addresses these issues and proposes a mathematical model to determine the optimum initial setting of process means and the optimum production run of a process with multiple *smaller-the-better* type (S-Type) of quality characteristics that incurs a minimum total cost. The dependency of the quality characteristics is considered in this model. To illustrate the application of the proposed model, a numerical example is provided.

MODEL DEVELOPMENT

A mathematical model for determining the optimal process means and the optimal production run is developed for a deteriorating process. The expected total production cost in this study consists of (a) the off target cost, which is modeled by the multivariate quality loss function, (b) the process adjustment cost and (c) the maintenance cost during the production run.

The following assumptions and nomenclatures are used in the model:

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Assumptions

(i) The production process has a known and constant variance for each of the dependent quality characteristics.
(ii) The quality characteristics are exponentially distributed with a mean of $\frac{1}{\lambda}$
(iii) At a random point in time, τ, the process changes from an in-control state to an out-of-control state as a result of a positive drift and a positive shift.
(iv) The production run is assumed to be the time difference between the starting of the process to the time when the process means are readjusted to the initial values.

Nomenclature

$y_i(t,\tau)$ The random variable denoting the quality measurement for the i-th quality characteristic at time t
k_{ii} Loss coefficient associated with quality characteristic i
k_{ij} Loss coefficient associated with quality characteristics i and j
μ_i Initial mean for i-th quality characteristic
μ_i^* Optimal initial mean for i-th quality characteristic
σ_i^2 Process variance for i-th quality characteristic
σ_{ij} Process covariance between i-th and j-th quality characteristic
ξ_i Target value for i-th quality characteristic
T Production run
T^* Optimal production run
$\mu_i(t,\tau)$ The process mean for i-th quality characteristic at time t

$$\mu_i(t,\tau)=\mu_i \qquad \tau \ge t$$
$$\mu_i(t,\tau)=\mu_i+W_i(t,\tau) \qquad \tau < t$$

$W_i(t,\tau)$ Deteriorating function

$$W_i(t,\tau)=0 \qquad \tau \ge t$$
$$W_i(t,\tau)=\delta+(t-\tau)\theta \qquad \tau < t$$

(For a shift and a drift)
C_A Adjustment cost
ρ Production rate per unit time
τ The elapsed time until the occurrence of the assignable cause. It is a random variable and is assumed to be exponentially distributed with a mean of $\frac{1}{\lambda}$
$f(\tau)$ The density function of the occurrence time of the assignable cause $=\lambda e^{-\lambda\tau}$
$M(t)$ Maintenance cost at time t

We consider a process with multiple *smaller-the-better* type (S-Type) of quality characteristics. For such a process the aim is to keep the quality characteristics as small possible, for instance: the surface roughness, the deviation from a design value, the emission of Carbon Monoxide gas. A traditional step loss function or a simple Taguchi loss function can not capture the quality loss in case of multiple quality characteristics. Therefore, a multivariate quality loss function should be applied to quantify the quality loss.

For this process where the quality characteristics are statistically independent, a multivariate quality loss function [12] can be used:

$$L(y)=k_{11}\,y_1^2+k_{22}y_2^2+k_{12}\,y_1y_2 \tag{1}$$

Here k_{11}, k_{22} and k_{12} are the quality loss coefficients, which can be determined by using the regression method or solving a system of simultaneous linear equations. While the univariate quality loss function only judges the individual quality characteristics, the multivariate approach considers the customer's perception of the quality characteristics. In this case, the quality characteristics are interdependent, leading to a cost (k_{12}) which accounts for the customer's reactions.

The quality loss at time t can be expressed as:

$$L(t,\tau)=k_{11}\left[y_1(t,\tau)\right]^2+k_{22}\left[y_2(t,\tau)\right]^2 + k_{12}\left[y_1(t,\tau)\right]\left[(y_2(t,\tau)\right] \tag{2}$$

When the quality characteristics depend on each other, a covariance term is included in the multivariate quality loss function. Since $E[X^2]=V[X]+\{E[X]\}^2$, where V[.] denotes variance, the expected quality loss can be given by:

$$E[L(t)]=k_{11}\left[\sigma_1^2(t)+(\mu_1(t))^2\right] + k_{22}\left[\sigma_2^2(t)+(\mu_2(t))^2\right] + k_{12}\left[\sigma_{12}(t)+(\mu_1(t))(\mu_2(t))\right] \tag{3}$$

We assume that the change in mean does not affect the process variability. Consider a deteriorating state where the effect of a positive shift and a positive drift is based on its random occurrence time τ (Figure-1). For example, a

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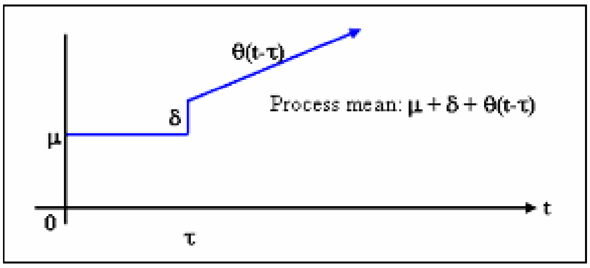
machine can experience wear out and leakage at the same time. Therefore this joint deterioration effect is important for some processes.



(a) Shift and Drift State

Figure-1. Deteriorating process states.

To represent this nature of the process, we assume:

If $t < \tau$ then

$$\sigma_1^2(t) = \sigma_1^2 \qquad \sigma_2^2(t) = \sigma_2^2 \qquad \sigma_{12}(t) = \sigma_{12}$$
$$\mu_1(t) = \mu_1 \qquad \mu_2(t) = \mu_2$$

If $t > \tau$ then

$$\sigma_1^2(t) = \sigma_1^2 \qquad \sigma_2^2(t) = \sigma_2^2 \qquad \sigma_{12}(t) = \sigma_{12}$$

$$\mu_1(t) = \mu_1 + W_1(t,\tau) \qquad \mu_2(t) = \mu_2 + W_2(t,\tau)$$
$$W_1(t,\tau) = \delta_1 + (t-\tau)\theta_1 \qquad W_2(t,\tau) = \delta_2 + (t-\tau)\theta$$

Thus, the expected loss at time t:

$$E[L(t)] = \int_t^\infty \left\{ \begin{matrix} k_{11}\left[\sigma_1^2 + \mu_1^2\right] + k_{22}\left[\sigma_2^2 + \mu_2^2\right] \\ + k_{12}\left[\sigma_{12} + \mu_1\mu_2\right] \end{matrix} \right\} f(\tau)d\tau$$
$$+ \int_0^t \left\{ \begin{matrix} k_{11}\left[\sigma_1^2 + \left(\mu_1 + \delta_1 + \theta_1(t-\tau)\right)^2\right] \\ + k_{22}\left[\sigma_2^2 + \left(\mu_2 + \delta_2 + \theta_2(t-\tau)\right)^2\right] + \\ k_{12}\left[\begin{matrix} \sigma_{12} + \left(\mu_1 + \delta_1 + \theta_1(t-\tau)\right) \\ \left(\mu_2 + \delta_2 + \theta_2(t-\tau)\right) \end{matrix}\right] \end{matrix} \right\} f(\tau)d\tau \qquad (4)$$

We assume the deterioration follows an exponential distribution i.e. $f(\tau) = \lambda e^{-\lambda\tau}$. After performing integration by parts and rearranging the terms, we find the total expected quality loss over the production run T is:

$$L(\mu,T) = \int_0^T E[L(t)]dt$$

$$L(\mu,T) = \rho \left\{ \begin{matrix} AT + (D + D')\left(T - \dfrac{1-e^{-\lambda T}}{\lambda}\right) \\ + (C+E)\left(\dfrac{T^2}{2} - \dfrac{T}{\lambda} + \dfrac{1-e^{-\lambda T}}{\lambda^2}\right) \\ + C'\left(\dfrac{T^3}{3} - \dfrac{T^2}{\lambda} + \dfrac{2T}{\lambda} - 2.\dfrac{1-e^{-\lambda T}}{\lambda^3}\right) \end{matrix} \right\} \qquad (5)$$

Where

$$A = \sum_{i=1}^{n} k_{ii}\left[\sigma_i^2 + \mu_i^2\right] + \sum_{i=1}^{n}\sum_{j=1}^{i} k_{ij}\left[\sigma_{ij} + \mu_i\mu_j\right]$$

$$B = \sum_{i=1}^{n} k_{ii}\left[2\mu_i\delta_i\right] + \sum_{i=1}^{n}\sum_{j=1}^{i} k_{ij}\left[\mu_i\delta_j + \mu_j\delta_i\right]$$

$$B' = \sum_{i=1}^{n} k_{ii}\delta_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{i} k_{ij}\delta_i\delta_j$$

$$C = \sum_{i=1}^{n} k_{ii}\left[2\mu_i\theta_i\right] + \sum_{i=1}^{n}\sum_{j=1}^{i} k_{ij}\left[\mu_i\theta_j + \mu_j\theta_i\right]$$

$$C' = \sum_{i=1}^{n} k_{ii}\theta_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{i} k_{ij}\theta_i\theta_j$$

$$E = \sum_{i=1}^{n} k_{ii}2\delta_i\theta_i + \sum_{i=1}^{n}\sum_{j=1}^{i} k_{ij}\left[\delta_i\theta_j + \delta_j\theta_i\right]$$

During resetting of the process parameters, an adjustment cost, C_A, will be incurred during this process. The maintenance actions are carried out during the production period, and the cost of maintenance accumulates up to time T. Thus, the total expected cost is as follows:

$$E[TC] = L(\mu,T) + C_A + \int_0^T M(t)dt \qquad (6)$$

The optimum process mean and production run can be found by minimising the expected cost per unit time as:

$$C[\mu^*, T^*] = \min\left\{ \tfrac{1}{T}\left[L(\mu,T) + C_A + \int_0^T M(t)dt \right] \right\} \qquad (7)$$

The total cost can be computed by determining the optimum initial mean settings of the two quality characteristics and the optimum production run. Theoretically, the optimal process parameters can be found by setting the derivatives with respect to each variable to

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zero (i.e. $\frac{\partial C(\mu,T)}{\partial \mu}=0$ and $\frac{\partial C(\mu,T)}{\partial T}=0$) and by solving the resulting equations. However, these derivatives are complicated and mathematically cumbersome. Therefore, pattern search techniques of Hooke and Jeeves [13] or Generalised Reduced Gradient (GRG) algorithms can be employed to determine the optimal process parameters.

NUMERICAL EXAMPLE

To illustrate the potential industrial application of the proposed model, let us consider a numerical example. Assume that a process with two dependent *smaller-the-better* type (S-Type) of quality characteristics (x and y) and they follow a bivariate normal distribution with given standard deviations (i.e.σ_1^2= 0.49, σ_2^2=0.25 and σ_{12}=0.25). The process adjustment cost is \$1000 and the production rate is 1000 units/month. Further, the loss coefficients, k_{11}, k_{22} and k_{12} are 3, 2 and 1 respectively.

Let us also assume the process moves to an out-of-control state as a result of a positive shift and a positive drift. The deterioration function is exponentially distributed at a failure rate, λ=0.5. The shift rates are considered to be $\delta_1 = 1$ and $\delta_2 = 2$. The drift rates are considered to be $\theta_1 = 0.4$ and $\theta_2 = 1$. The maintenance cost is $M(t) = 10 + 3t$.

We solve the cost model in (7) using Generalised Reduced Gradient, GRG. Table-1 summarises the results.

Table-1. Result of the numerical example.

	Symbol	Unit	Shift and Drift
Initial Mean of 1st QC	μ_1	inch	0.00
Initial Mean of 2nd QC	μ_2	inch	0.00
Production Run	T	month	3.25
Unit Cost	C	\$	4.98

A sensitivity analysis could also be carried out to show the effect of different input parameters on the output.

CONCLUSIONS

Targeting problem is one of the most studied problems in the field of combinatorial optimisation. In particular, this paper has made the following contributions. Firstly, the joint determination of the optimum initial setting of process means and the optimum production run of a deteriorating process with multiple *smaller-the-better* type (S-Type) of quality characteristics is developed. Secondly, the dependency of the multiple quality characteristics is considered.

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