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PERMUTATION AND COMBINATIONS APPROACH TO PROGRAM EVALUATION AND REVIEW TECHNIQUE

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ABSTRACT

The global market today is very competitive. Completion of projects on time and estimating the project completion time is still difficult in some areas like in the printing industry, where skill also plays a predominant role. The print production activities require human judgment and skill to accomplish the job. The raw materials properties used in the printing process influence runability of the printing machine. Controlling these factors in the production environment is very difficult. Estimating the stochastic job completion time by Program Evaluation and Review Technique (PERT) uses approximate calculation of standard deviation σ and the activity expected time T_{exp}. The project scheduled time T_{sch} is greatly influenced by standard deviation. The most popular N-method of standard deviation adopted in calculating the T_{sch} in two conditions where T_{min} and T_{max} are closer to T_{maen} and in extreme time estimate condition to T_{min} and T_{max}. Permutation and combinations of the T_{min}, T_{mean} and T_{max} are used to calculate T_{sch} of the project by a software developed for the purpose. The T_{sch} obtained in these three methods studied with the actual projects completion time over a period of time.

Keywords: program, evaluation, review, PERT, stochastic, permutation, time, runability.

1. INTRODUCTION

Every project has some specific purpose, it starts at some point of time and completes when its objectives have been full-filled. Many techniques often are enhanced by stochastic treatment of the time required to perform some activity. Expert's knowledge is alternative quantitative information. As the nature of job, material behavior to different conditions and involvement of skill and unpredictable behavior of labor makes the prediction to be difficult especially where skill and manpower requirement is essential like in the printing industry. The prediction of project completion time by Program Evaluation and Review Technique (pert) is not sufficient. In pert the activity three estimated times were not given equal chance of occurring, which is against the principle of probability. Pert has inherent technological draw backs [1]. The formula to calculate the expected times, standard deviations is an approximation [2]. The technique is enhanced with stochastic treatment of time required to perform each activity equal chances and calculated the project duration time required. The effect of standard deviation calculated by two different methods are sudieded. Five different volume print production projects and five jobs under each volume on four different machines were examined. The project duration times were well suited with the stochastic treatment given to activity times by permutation and combination method.

2. PROBABILISTIC PERT

Probabilistic PERT is a technique used to find the probable project completion time. The technique is probabilistic in nature, it regards the time required to complete an activity is stochastic [10]. This stochastic time is based on expert estimates of the minimum (optimistic) T_{min}, mode (most likely) T_{mean}, and maximum (pessimistic)T_{max} time required to complete each activity have beta distributed. The expected time T_{exp} approximate mean is calculated as:

$$T_{exp} = \frac{T_{min} + 4\,T_{mean} + T_{max}}{6} \quad (1)$$

These transformed estimates are assumed to be mean value. The T_{exp} represents approximate average value, while the T_{mode} represents the mode of the beta distribution [3]. The expected time T_{exp} represents a particular value on distribution curve, that has a both 50-50 chance of exceeding and 50-50 chance of being met. It means there is equal chance for an activity to take more time or less time than calculated T_{exp}. Hence, the curve will be symmetrical and approaches normal curve with standard deviation

$$\sigma = \frac{T_{max} - T_{min}}{6} \quad (2)$$

$$\text{Variance } \sigma^2 = \left[\frac{T_{max} - T_{min}}{6}\right]^2 \quad (3)$$

$$\sigma = \sqrt{ó2} \quad (4)$$

3. RANDOM PERT

Most of the distributions can be approximated by normal distribution by simple transformation of variables [4]**.** The time estimates in pert has beta distribution. The equation (1) used for calculating T_{exp} transforms the beta distribution to normal distribution with parmeters T_{mean} and variance $\sigma^{2.}$ The density function is given by

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\frac{(Tmean - Texp)^2}{2\sigma 2}$$

This equation shows that $f_x(x)$ reaches maximum at $T_{mean} = T_{exp}$ and has points of inflection at $T_{exp} = T_{mean} \pm \sigma$. The expected time T_{exp} computed using equation (1). When the expected time T_{exp} is equal to $T_{mean,}$ the distributed curve will be symmetrical [5]. Time estimates for optimistic time T_{min} and pessimistic time T_{max} are randomly generated within the ± 3 standard deviations to T_{mean}. Optimistic time estimate T_{min} is calculated by $T_{mean} - 3$ standard deviations. The pessimistic time estimate is calculated as $T_{mean} + 3$ standard deviations [10]. Within this range T_{min} and T_{max} are generated randomly by using MATLAB software. The expected activity time T_{exp} is calculated for each activity by using equation (1). The T_{sch} found out by summing up the T_{exp} of critical activities.

4. PERMUTATION AND COMBINATIONS IN PERT

An activity when influenced with several conditions will not give uniform results, but it may result in any one of the several possible outcomes [6]. The probability of expected time T_{exp} depends on occurrence of one of the three estimates. The three time estimates are considered by giving equal chances. The permutation and combination method gives the complete stochastic treatment in estimating T_{exp} of an activity.

In pert, estimated activity most likely time T_{mean} and approximate variance are insufficient basis for stochastic calculations. However this can be used as a basis for calculating the critical path. The permutation and combinations in pert method the three time estimates of an activity i.e. minimum (optimistic) T_{min}, mean (most likely) T_{mean}, and maximum (pessimistic) T_{max} time were given equal chances of occurrence with unrestricted repetition. Activity time estimates 'n' kinds unlimited number of each kind after being chosen is replaced in the data. Hence, 'r' permutations will be filled in 'n' different ways, with repetition. By the rule of product, the number of permutation is 'r' permutation with repetition of 'n' things is [7]:

$$U = (n, r) = n^r \quad (5)$$

Software program is developed to calculate the critical path of a network. It generate permutation and combinations of critical activities T_{sch} using the three estimated times.

5. CHI-SQUARE TEST

To find the significance or the discrepancy between the two methods Chi-square test is conducted. The scheduled estimated time $T_{sch.}$ calculated by using random pert standard deviation (4) and n-method standard deviation (7) are tested. The following Chi-square formula used:

$$\text{Chi-square} = \sum_{i=1}^{n} \left[\frac{Oi-Ei}{Ei}\right]^2 \quad (6)$$

O_i (i = 1,2,…,n) is a set of observed results and E_i (i = 1, 2, …, n) is the corresponding set of expected hypothetical results [8].

6. METHODOLOGY

In probabilistic program evaluation and review technique the optimistic time estimate $T_{min,}$ pessimistic time estimate T_{max} and most likely time estimate T_{mean} are not given equal chances of occurrence in calculation of the activity estimated time $T_{exp.}$ The weighted average factor considered for the time estimates in calculating T_{exp} are 0.17, 0.17 and 0.66 respectively. In this weighted method the T_{mean} plays a predominant role. Therefore in most cases T_{mean} approaches the activity estimated time $T_{exp.}$ In this kind of situations where the skill of personnel and human psychology also plays a vital role, like in the printing industry, this method of calculating T_{exp} gives wrong assumptions. The standard deviation is the most common measure of statistical dispersion, measuring how widely spread the values in a data set. Standard deviation of each activity duration time is calculated in pert method by considering each activity's optimistic and pessimistic time estimates T_{min} and T_{max} only. This is an approximate method of standard deviation calculation. This is compared and analysed with the most popular n-method and by introducing permutation and combinations in the pert method in different conditions.

In random condition the most likely time is the average observed time of an activity for the project volume T_{mean}. The optimistic time T_{min} and, pessimistic time T_{max} of an activity are estimated within the 3 standard deviations from $T_{mean.}$ The expected time T_{exp} is calculated using equation (1). The probability of occurrence T_{mean} is considered as 0.66. T_{min} and T_{max} probabilities of occurrence are considered as 0.17 each. In this condition T_{exp} approaches $T_{mean.}$ Therefore the curve will be symmetrical [5]. The optimistic T_{min} and pessimistic T_{max} were randomly generated in MAT Lab within ± 3 standard deviations to T_{exp}. The sum of all activities T_{exp} on the critical path is the T_{sch}. On Web, Miller, HMT and Eagle machines, five different projects with five different volumes are considered. T_{maen} of each activity is calculated by gathering data of five similar jobs in all aspects. The project duration time for different T_{exp} values and Z value calculated. Z is the number of standard deviations by which T_{sch} exceeds T_{exp}. The scheduled time of project completion time T_{sch} of both the methods with the two different standard deviations is compared.

$$Z = \frac{T_{sch} - T_{exp}}{\sigma} \quad (7)$$

$$T_{sch} = \sigma Z + T_{exp} \quad (8)$$

In N-method, the standard deviation is calculated by

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2} \quad (9)$$

It is introduced in pert in place of the approximate standard deviation.

In second condition extreme values are given to the optimistic T_{min} and pessimistic T_{max}. The projects duration time are calculated by pert and N-methods.

Permutations and combinations method in pert, the T_{exp} of the project calculated by the software developed. The time estimates $T_{min,}$ T_{mean} and T_{max} of each activity selected randomly and by permutation and combinations technique and the T_{exp} of the project is calculated. The scheduled time T_{sch} is calculated by using two different standard deviation equations (4) and (9). The results obtained by the three methods are studied in the real life situation.

Significance of the test methods calculated in Chi-square for validity.

7. RESULTS AND DISCUSSIONS

In probabilistic program evaluation and review technique the optimistic time estimate $T_{min,}$ pessimistic time estimate T_{max} and $_{most}$ likely time estimate T_{mean} are not given equal chances of occurrence in calculation of the activity estimated time $T_{exp.}$ The weighted average factor considered for the time estimates in calculating T_{exp} is 0.17, 0.17 and 0.66, respectively. In this weighted factor the T_{mean} plays a predominant role. Therefore in most cases T_{mean} approaches the activity estimated time $T_{exp.}$ In this kind of situations where the skill of personnel and human dependence also plays a vital role, like in the printing industry, this method of calculating T_{exp} gives wrong assumptions. The standard deviation is the most common measure of statistical dispersion, measuring how widely spread the values in a data set. Standard deviation of each activity duration time is calculated in pert method by considering each activity's two time estimates T_{min} and T_{max} only. This is an approximate method of standard deviation calculation. The project duration time of printing projects of different volumes are calculated and observed the results. Results obtained by using equation (2) show much lower project completion time, T_{sch} for all projects and with 100, 200, 300, 400 and 500 page volumes on Web, Miller, HMT and Eagle machines than results obtained by N-method of standard deviation which is widely accepted in statistical methods for the same activity durations. Therefore the project scheduled times are different for the same project. In situations where the prediction of activity time depends on human factors like skill, psychological the activity duration time may take any of the three time estimates. Hence, permutation and combinations method used in which all the activity time estimates are considered of equal chances an event happening. This method is more realistic than pert in this kind of situations. The method is applied to the same printing projects. The results obtained showed middle values to both the pert approximation and N-method of standard deviations.

In extreme time estimates condition the project completion $T_{sch\ calculated}$ by using N-method of standard deviation showed lower results on Web and Miller machine projects. On HMT machine projects also the trend was similar up to 300 pages volume. For above 300 page volumes, pert method showed lower values than N-method. Standard deviations in both methods are almost equal. For 400 and 500 pages volumes the difference in standard deviation obtained in both the method is 1.9 and 11.19%, respectively. This difference makes the pert method to give lower values than n-method. Projects completed on Eagle machine showed the lower project duration time in pert method than in N-method because the standard deviation values are higher in the N-method than pert. Permutation and combination results showed higher values in all the extreme situations. Random, extreme conditions and permutation and combination methods are validated in Chi-square testing method for any significant difference. The Chi-square test showed that there is no significant difference in the methods except in condition where the standard deviation values are difference is very large. Over a period of two years the methods are practically evaluated in the printing organizations and showed the permutation and combination method results are behaving very closer to real life situations.

The results obtained for various projects' duration times with different standard deviations are given in Tables 1 to 4.

Table-1. T_{sch} for pert SD, permutation and combinations pert and N-methods in random and extreme conditions on web.

WEB	**Random**			**Extreme**		
Volume	Pert SD	PC Pert	N- method	Pert SD	PC Pert	N- method
100	52.77	53.25	55.05	94.24	105	60.68
200	68.36	67.73	76.18	121.6	129	81.17
300	97.05	96.73	109.69	169.1	184	118.4
400	114.88	116.89	132.59	158.1	197	148.5
500	138.30	142.47	162.44	200.3	308	187.2

Table-2. T_{sch} for pert SD, permutation and combinations pert and N-methods in random and extreme conditions on miller.

Miller	**Random**			**Extreme**		
Volume	Pert SD	PC Pert	N- method	Pert SD	PC Pert	N- method
100	65.52	68.71	76.81	93.35	141	84.14
200	92.97	95.01	116.92	135.1	200	131.9
300	127.83	130.92	166.4	195.8	285	185.4
400	157.29	160.87	209.18	238.6	341	231.9
500	200.28	204.87	269.87	337.3	500	312.3

Table-3. T_{sch} for pert SD, permutation and combinations pert and N-methods in random and extreme conditions on HMT.

HMT	**Random**			**Extreme**		
Volume	Pert SD	PC Pert	N- method	Pert SD	PC Pert	N- method
100	73.8	76.1	94.25	114.3	178	107.6
200	114.9	117.14	160.64	201.4	303	186.7
300	161.76	164.67	233.77	338.9	500	293.5
400	206.06	208.9	307.51	376.9	582	379.4
500	255.25	258.8	381.54	424.9	624	441.6

Table-4. T_{sch} for pert SD, permutation and combinations pert and N-methods in random and extreme conditions on Eagle.

Eagle	**Random**			**Extreme**		
Volume	Pert SD	PC Pert	N- method	Pert SD	PC Pert	N- method
100	121.37	123.54	184.94	217.2	328	222.2
200	205.6	208.39	339.93	361.9	495	390
300	293.39	296.13	499.48	541.4	722	588.1
400	383.59	388.2	655.0	662.4	846	725.9
500	423.2	428.08	706.02	737.9	968	809.8

Chi-square Test results

Chi-square calculated values for pert SD and N-methods under the two conditions.

Machine	**Chi-square calculated value** (random pert SD and N-method)	**Chi-square calculated value** (extreme pert SD and N-method)
Web	0.336	8.903
Miller	0.629	0.331
HMT	1.002	3.559
Eagle	0.817	0.474

Chi-square calculated values for pert SD, N-method and Permutation and combinations pert in random.

Machine	**Chi-square calculated value** (random pert SD and PC pert)	**Chi-square calculated value** (extreme pert SD and PC pert)
Web	0.062	2.585
Miller	0.018	2.989
HMT	0.010	0.336
Eagle	0.003	1.034

Chi-square calculated values for pert SD, N-method and Permutation and combinations pert in random.

Machine	**Chi-square calculated value** (random N-method and PC pert)	**Chi-square calculated value** (extreme N-method and PC pert)
Web	0.214	2.864
Miller	0.856	3.607
HMT	1.240	4.221
Eagle	0.930	2.948

Graphical representation of machine performance for various projects with the parameters total scheduled time vs number of pages of projects in Figures 1 to 8.

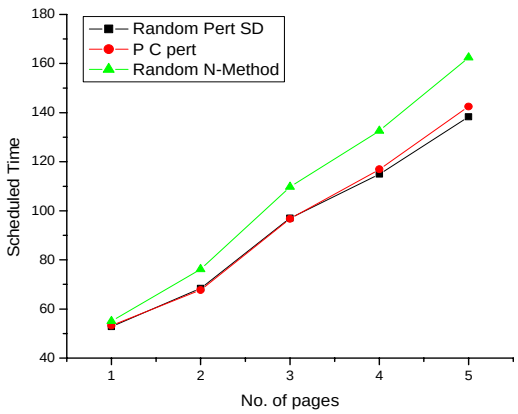

Figure-1. T_{sch} values in three different methods in random on Web.

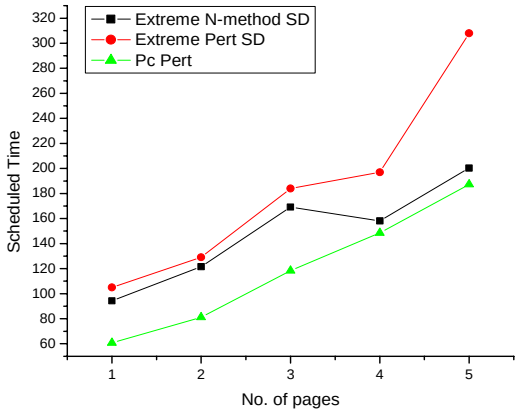

Figure-2.T_{sch} values in three different methods in extreme conditions on Web.

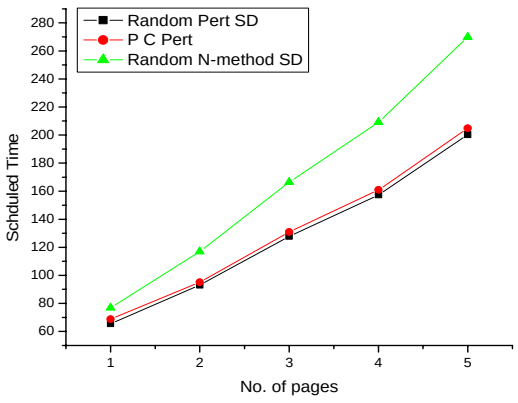

Figure-3. T_{sch} values in three different methods in random on Miller.

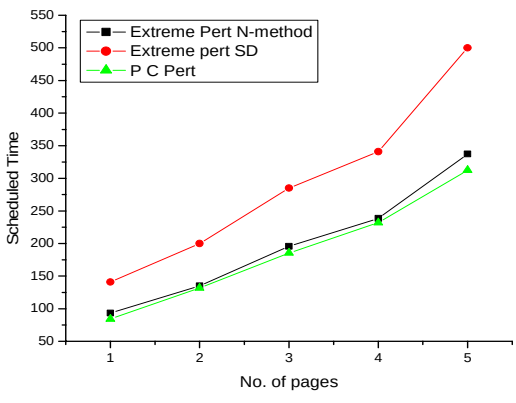

Figure-4. T_{sch} values in three different methods in extreme conditions on Miller.

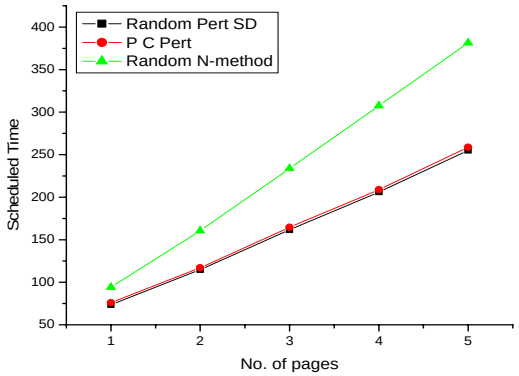

Figure-5. T_{sch} values in three different method in random on HMT.

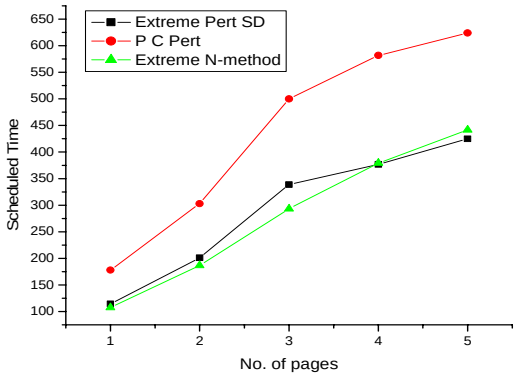

Figure-6. T_{sch} values in three different method in extreme conditions on HMT.

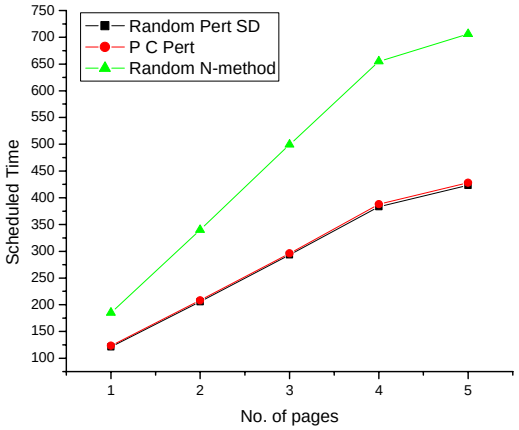

Figure-7. T_{sch} values in three different methods in random on Eagle.

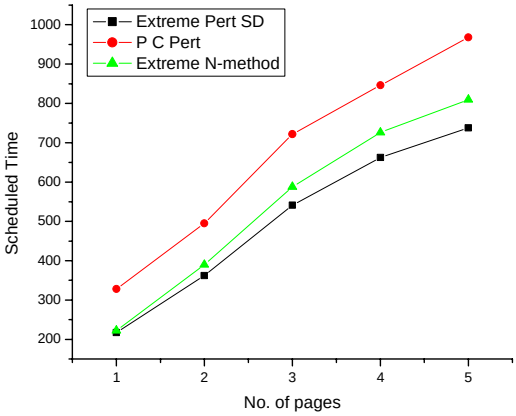

Figure-8. T_{sch} values in three different methods in extreme conditions on Eagle.

In pert the time estimates T_{min} and T_{max} are within in the three standard deviations calculated by approximate method of standard deviation showed lower project duration time in all volumes and all machines. The permutation and combination method in pert showed project duration values moderate. The pert method showed lower duration time. The difference ranges from 1.20% to 2.5%. The most popular method N-method in pert showed the higher values among the three methods, among all volumes and on all machines. The difference in the project completion time was ranging from 12.34% to 65.15% higher than permutation and combination method. The standard deviation calculated in N-method considers the complete set of data. Therefore the standard deviation calculated showed higher values than pert method. This caused the project duration times are higher than pert method. In permutation and combination method all the three time estimates T_{min}, T_{mean}, and T_{max} are given equal opportunity which reflected complete stochastic in nature. The project duration time obtained in this method is approaching the real life situations.

The Chi-square value for random T_{sch} in pert standard deviation and N-methods at 5% level of significance at 4 degrees of freedom table the value is 9.488. The calculated values are 0.336, 0.629, 1.002 and 0.817 on Web, Miller, HMT and Eagle, respectively. The significance between pert and permutation and combinations pert are 0.062, 0.18, 0.010, 0.003 on Web, Miller, HMT and Eagle machines, respectively. The significance between N-method and permutation and combinations pert are 0.214, 0.856, 1.240 and 0.930 on Web, Miller, HMT and Eagle machines, respectively. All test procedures showed that there is no significant difference in the methods. However the T_{sch} calculated values are different. In real life situations in the press the permutation and combinations pert results are approximately equal.

In extreme condition, the time estimate T_{min} and T_{max} are given extreme values. Therefore T_{mean} can not be equal to $T_{exp.}$ These conditions prevailed in the press when there was no control over the production and various other factors adversely affected the production schedules. In these conditions permutation and

combinations pert results are approximate to the actual results in the press. The permutation and combinations pert showed, higher project duration time than the other two methods. The N-method showed lower values than the probabilistic pert. The difference in the project completion time was ranging from 19.46% to 33.41% less than permutation and combination method. The N-method showed the difference of range from 18.55% to 35.58%. The area under the normal curve at T_{mean} is 50% and at plus one standard deviation the area under the curve is about 68%. The results obtained in extreme condition by N-method showed only 0.68 probability of completing the projects on time.

The Chi-square value for extreme T_{sch} in pert standard deviation and N-methods at 5%level of significance at 4 degrees of freedom table the value is 9.488. The calculated values are 8.903, 0.331, 3.559 and 0.474 on Web, Miller, HMT and Eagle, respectively. The significance between pert and permutation and combinations pert are 2.585, 2.989, 0.336 and 1.034 on Web, Miller, HMT and Eagle machines, respectively. The significance between N-method and permutation and combinations pert are 2.864, 3.607, 4.221 and 2.948 on Web, Miller, HMT and Eagle machines, respectively. All test procedures showed that there is no significant difference in the methods. However the T_{sch} calculated values are different. In real life situations in the press the permutation and combinations pert results are approximately equal in all conditions.

8. CONCLUSIONS

This paper describes the standard deviation calculated by weighted average method will produce different results in different conditions of the data. The popular N-method shows higher values in closer T_{min} and T_{max} conditions and lower values in the extreme conditions. The permutation and combination method considers all the three time estimates and gives complete stochastic treatment to the three time estimates. The project duration times obtained in this method are approximating to the real life situations where the activities involved human skill and psychological factors play a predominant role. The new approach is simple and key to keep the printing firms competitive in the competitive environment by supporting better decisions in production that serve in time to the market.

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