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DETERMINING A COMMON PRODUCTION CYCLE TIME FOR AN EPQ MODEL WITH NON INSTANTANEOUS DETERIORATING ITEMS ALLOWING PRICE DISCOUNT USING PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT

This paper considers the Economic Production Quantity (EPQ) for non instantaneous deteriorating items allowing price discount with constant production and demand rate extending the facility of permissible delay in payments. It is assumed that a single machine produces single product over an infinite planning horizon. The optimal production cycle time is derived under conditions for continuous review, deterministic demand and no shortage.

Keywords: EPQ, production, quantity, deterioration, price, discount, payment, delay.

1.0. INTRODUCTION

An economical production quantity (EPQ) model [1] is an inventory control model that determines the quantity to be produced on a single facility so as to meet a deterministic demand over an infinite planning horizon. An assumption, common to many inventory models, is that products generated have indefinitely long lives. In general, almost all items deteriorate over time. Often the rate of deterioration is low and there is little need to consider the deterioration in the determination of economic lot sizes. Nevertheless, there are many other products in the real world that are subject to a significant rate of deterioration. Hence, the impact of product deterioration should not be neglected in the decision process of production lot size. Deterioration can be classified as age-dependent on-going deterioration and age-independent on-going deterioration. Blood, fish and strawberries are some examples of commodities facing age-dependent on-going deterioration. Volatile liquids such as alcohol and gasoline, radioactive chemicals, and grain products are examples of age-independent on-going deteriorating items. Legally these products do not have an expiry date; they can be stored indefinitely, though they suffer natural attrition while being held in inventory. In general, perishableness or deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero.

More recently, the supplier offers the retailer a trade credit period in a competitive market environment, to pay the cost of the supplied material. Usually, there is no charge if the outstanding amount is settled within the permitted fixed settlement period. Beyond this period, interest is charged. In recent research, the extensive use of trade credit as an alternative has been addressed by Goyal [2] who developed an EOQ model under the conditions of permissible delay in payments. Chung [3] then developed an alternative approach to the problem. Chand and Ward [4] analyzed Goyal's problem under assumptions of the classical EOQ model, obtaining different results. Next, Aggarwal and jaggi [5] extended Goyal's model to allow for deteriorating items. Jamal *et al.* [6] extended Aggarwal and Jaggi's model to shortages.

Misra [7] first studied the EPQ model for deteriorating items with the varying and constant rate of deterioration. Choi and Hwang [8] developed a model determining the production rate for deteriorating items to minimize the total cost function over a finite planning period. Raafat [9] extended the model, given in Park [10] to deal with a case in which the finished product is also subject to a constant rate of decay. Yang and Wee [11] considered a multi-lot-size production-inventory system for deteriorating items with constant production and demand rates.

Gary C. Lin, Dennis E. Kroll and C.J. Lin [12] obtained common production cycle time for an ELSP with deteriorating items. K. Jeyaraman, C. Sugapriya [13] developed an ELSP for non instantaneous deteriorating items using price discount. In this paper an EPQ model for a single product subject to exponential deterioration under production-inventory policy using permissible delay in payments is discussed. The product is provided with price discount for its deterioration rate. In the next section, assumptions and notations for the development of the model are given. The cycle time of the model is derived in the Section 3. An illustrative numerical example and final concluding remarks are given in the subsequent sections.

2.0. BASIC ASSUMPTIONS AND NOTATION

The following assumptions are used in the development of the model:

1. The demand rate for the product is known and finite.
2. No shortages are allowed.

3. An infinite planning horizon is assumed.
4. The production rate of each product is finite. The machine has large enough capacity to produce all the items to meet the demand of all products.
5. Once a unit of a product is produced, it is available to meet the demand.
6. Once the production is terminated, the product starts deterioration and the price discount is considered.
7. The production rate of the product is independent of the production lot size.
8. The time to deterioration of the product follows an exponential distribution.
9. There is no replacement or repair for a deteriorated item.
10. The production lot size is unknown but it will not vary from one cycle to another.
11. Inventory holding cost is charged only to the amount of undecayed stock.
12. The cost of deterioration unit is known and includes any disposal cost or salvage value.
13. Customer is given credit period and the credit period is less than or equal to production cycle time.

The notation employed in this paper is given below:

p	production rate of the particular product given in the number of units per unit time.
d	actual demand of the product given in number of units per unit time.
A	cost of setting up of a production run for the product.
h	inventory carrying cost/unit/unit time for the product.
k	production cost price per unit of the product.
r	price discount per unit of the product.
Q	order quantity /Unit time.
M	permissible delay in payments (credit period).
I_E	interest lost per unit time due to credit period.
I_P	interest payable per unit time by the customer to the supplier for the exceeding credit period.
$T*$	optimal cycle time.
T_1	production period.
T_2	time during which there is no production of the product in a cycle i.e. $T_2 = T - T_1$.
$I_1(t_1)$	time varying inventory level for product in the cycle segment, $0 \le t_1 \le T_1$.
$I_2(t_2)$	time varying inventory level of the product in the cycle segment, $T_2 \le t_2 \le T$.
$I(M)$	maximum inventory level of the product.
$TVC(T)$	total cost/unit time.
θ	a constant deterioration rate (unit/unit time).

3.0. MODEL DEVELOPMENT

At start $t = 0$, the inventory level is zero. The production starts and increases to the maximum, $I_1(T_1)$. During this time period $[0, T_1]$, the inventory is built up at a rate p-d and there is no deterioration. After the maximum inventory is reached, production is terminated and the deterioration starts. From this point, the on-hand inventory diminishes to the extend of the demand plus the loss due to the deterioration. Production will be resumed when all on hand inventories are depleted at time T. Then an identical production run will begin. Since an exponential deterioration process is assumed, the inventory level of the system for the product at time t over period $[0, T]$ can be the represented by the differential equations:

$$\frac{dI_1(t_1)}{dt_1} = p - d \text{, for } 0 \le t_1 \le T_1. \quad \text{………… (1)}$$

$$\frac{dI_2(t_2)}{dt_2} + I_2(t_2)\theta = -d \text{ for } 0 \le t_2 \le T_2 \quad \text{…………. (2)}$$

The boundary conditions associated with these equations are: at $I_1(0) = 0, I_2(T_2) = 0$

$$I_1(t_1) = (p - d)t_1 \text{, for } 0 \le t_1 \le T_1 \quad \text{………….(3)}$$

$$I_2(t_2) = \frac{d}{\theta}\left(e^{\theta(T_2 - t_2)} - 1\right) \text{, for } 0 \le t_2 \le T_2 \quad \text{……...….. (4)}$$

Production cost: The production cost per unit time is given by:

$$PC = \frac{pkT_1}{T} \quad \text{..……..... (5)}$$

Setup cost: The setup cost per unit time is given by

$$OC = \frac{A}{T} \quad \text{..……….. (6)}$$

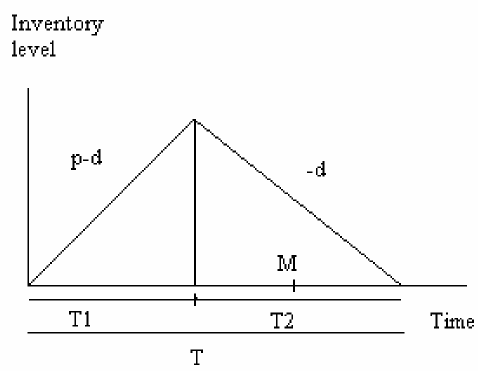

Figure-1

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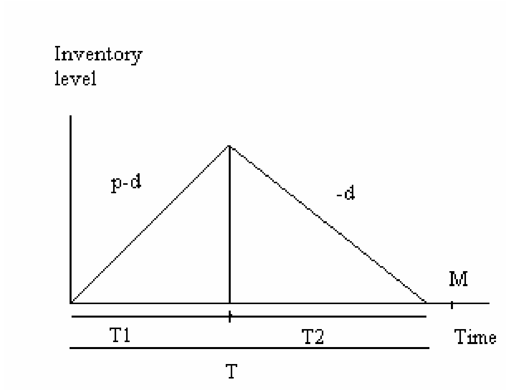

Figure-2

Holding cost: The holding cost per unit time is given by:

$$HC = \left[\frac{h}{T}\left(\int_0^{T_1} I_1(t_1)dt_1 + \int_0^{T_2} I_2(t_2)dt_2\right)\right] = \sum_{j=1}^{n}\left[\frac{h}{T}\left(\int_0^{T_1}(p-d)t_1dt_1 + \int_0^{T_2}\frac{d}{\theta}\left(e^{\theta(T_2-t_2)}-1\right)dt_2\right)\right] \quad (7)$$

Assuming θt_2 <1, an approximate solution by neglecting those terms of degree greater than or equal to 2 in θt_2 in Tailors expansion of the exponential functions yields.

$$HC=\frac{h}{2}\left(\frac{pT_1^2}{T}+dT-2dT_1\right) \quad \text{.......... (8)}$$

Deterioration cost: The number of deteriorating units occurring in a cycle is the difference between the maximum inventory and the number of units used to meet the demand. Hence the deterioration cost per unit time is given as:

$$DC=\frac{k}{T}\left(I_2(0)-\int_0^{T_2} d.dt_2\right)=\frac{k\theta T_2^{\,2}d}{T} \quad \text{........... (9)}$$

Price discount: Price discount per unit time is:

$$PD=\frac{kr}{T}\int_0^{T_2} d.dt_2=\frac{krdT_2}{T} \quad \text{...... (10)}$$

Therefore the total profit per unit time is given by, If we use (5), (6), (8), (9) and (10) to express T_2 and T_1 in terms of T in equation (A.1) and (A.2), neglect the third and higher powers of θT terms for small values of θT.

$$TVC(T)=+\frac{A}{T}+\frac{h}{2}\left(dT-\frac{d^2T}{P}\right)+\frac{cd\theta T}{2}\left(\frac{p-d}{d}\right)^2 + kr\left(p-d\right) \quad \text{..........(11)}$$

Case 1: M < T

The optimal production cycle time is greater then the credit period, the loss of revenue to the supplier by way of interest

$$I_{E1}=\frac{kI_e d}{T}\left(\int_0^{T}(T-t)dt\right)=\frac{kI_E dM}{T} \quad \text{........... (12)}$$

The interest payable by the customer to the supplier per unit time on the assumption that the customer has to pay the entire amount before the next production.

$$I_P=\frac{kI_p}{T}\int_M^{T_2} I_2(t_2)dt_2=\frac{kI_p d}{2}\left[T+\frac{M^2}{T}-2M\right] \quad (13)$$

$$TVC_1(T)= kd + \frac{A}{T}+\frac{h}{2}\left(dT-\frac{d^2T}{P}\right)+\frac{kd\theta T}{2}\left(\frac{p-d}{d}\right)^2 + kr(p-d)-\frac{kI_E dM}{T}+\frac{kI_p d}{2}\left[T+\frac{M^2}{T}-2M\right] \quad (14)$$

To minimize the total cost per unit time TVC_1 (T), differentiate TVC_1 (T) with respect to T and set the result equal to zero then we get

$$T=\sqrt{\frac{2A+2kI_e dM^2}{hd\left(1-\frac{d}{p}\right)+\frac{k\theta(p-d)^2}{d}-kI_p d\left(\frac{p-d}{d}\right)^2+dk\left(I_p-I_E\right)}} \quad (15)$$

$$\frac{\partial^2 TVC(T)}{\partial T^2}=\frac{2A}{T^3}+\frac{dkI_e M^2}{T^3}>0.$$

And the second derivative is found to be positive.

Case 2: M > T

In this study, the customer need not pay interest. The loss of revenue by the supplier

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$$I_{E2} = \frac{pI_e d}{T}\left(\int_0^T (T-t)dt + (M-T)\int_0^T dt\right)$$

$$= pI_e d\left[M - \frac{T}{2}\right] \quad \text{...........(16)}$$

$$\text{TVC}_2\,(\text{T}) = kd + \frac{A}{T} + \frac{h}{2}\left(dT - \frac{d^2T}{P}\right) + \frac{kd\theta T}{2}\left(\frac{p-d}{d}\right)^2 + kr(p-d) - pI_e d\left[M - \frac{T}{2}\right] \quad \text{.....(17)}$$

To minimize the total cost per unit time TVC_2 (T), differentiate TVC_2 (T) with respect to T and set the result equal to zero then we get

$$T = \sqrt{\frac{2Ad}{hd^2\left(1-\frac{d}{p}\right) + k\theta(p-d)^2 + kI_e(p-d)^2}} \quad (18)$$

$\frac{\partial^2 TVC_2(T)}{\partial T^2} = \frac{2A}{T^3} > 0$ and the second derivative is found to be positive.

Example 1

A = \$50/set up, P = 70 units/unit time, D = 30units/unit time, k = \$15/ unit time, M = 2 unit time, $I_p = 0.15$/ unit time, $I_E = 0.12$/ unit time, h = 3 units/unit time, θ = 0.08, r = 2/ unit time, T = 2.1382Unit time, TVC (T) = \$1088, Order Quantity = 7.762.

Example 2

A = \$50/set up, P = 70 units/unit time, D = 30units/unit time, k = \$15/ unit time, M = 2 unit time, $I_E = 0.12$/ unit time, h = 3 units/unit time, s = \$100/unit time, r = 2/ unit time, T = 0.7683 Unit time, TVC (T) = \$1672.2, Order Quantity =30.73.

Figure-3 shows the graph of total cost verses deterioration rate. It shows when deterioration rates are small; the impact on total cost is insignificant .In case1 and case 2 total costs increase as the deterioration rates decreases.

Figure-4 shows the graph of total cost VS production cycle time. It appears that the production cycle time decreases as the deterioration rate of product increases.

Figures-5 and 6, the graph production run time VS order quantity shows that production run time increases as order quantity increases as expected.

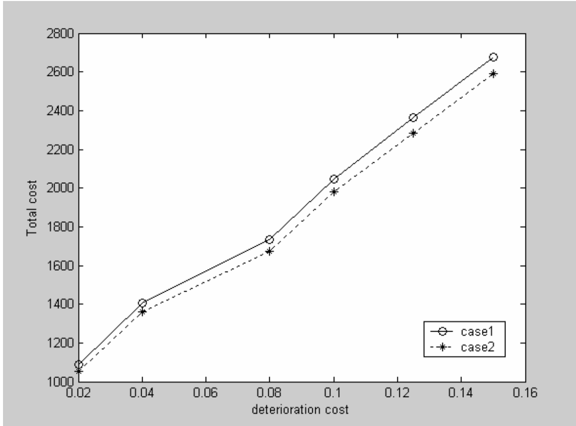

Figure-3. Total cost verses deterioration rate.

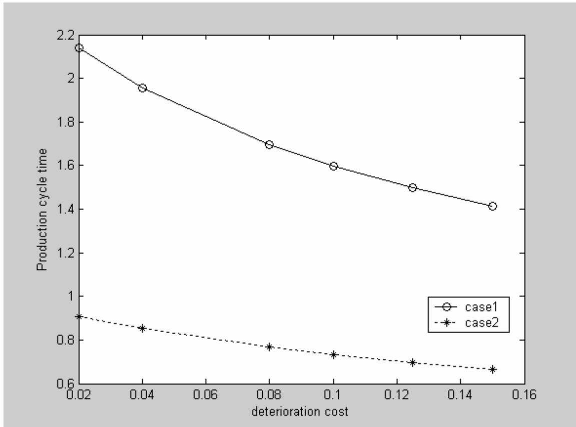

Figure-4. Total cost VS production cycle time.

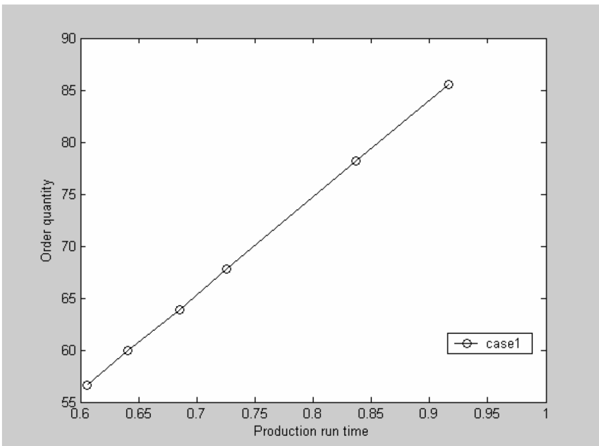

Figure-5. Production run time VS order quantity (case 1).

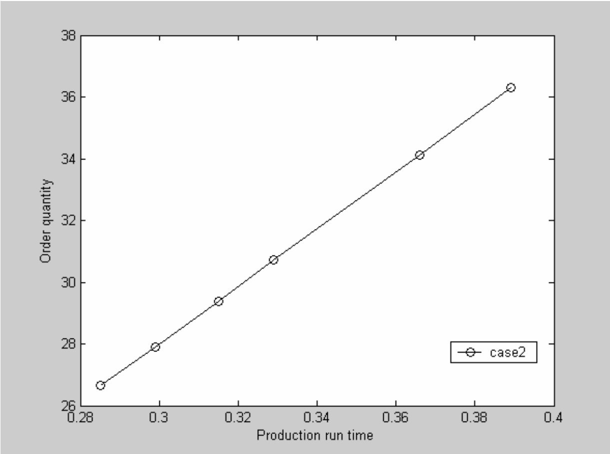

Figure-6. Production run time VS order quantity (case 2).

CONCLUSIONS

In this study, an EPQ model for a single machine single product system in which the product deteriorates non-instantaneously receives the price discount, the purchaser receives credit period from the supplier has been developed. This assumption is more realistic. A common cycle time for the model is arrived. It is possible to obtain a relatively simple expression as was shown in this paper. The model developed in this paper reduces the production cycle time and maximize the total profit.

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Appendix-A

To minimize the total cost per unit time TVC to express in terms of T in eqn. (11) so that there is only one variable T in the equation. At the moment when production run is terminated during a cycle ∴

$$I_1(T_1) = I_2(0)\,(p-d)\,T_1 = \frac{d}{\theta}\left(e^{\theta T_2}-1\right)$$

Applying Tailor's expansion and approximation

$$(p-d)\,T_1 = d\left(T_2 + \frac{\theta T_2^{\,2}}{2}\right) \quad T_2 = \frac{p-d}{d}\,\mathrm{T} \qquad \text{….. (A.1)}$$

$$T = T_1 + T_2 = \frac{p}{d}T_1 \qquad \text{………..(A.2)}$$

And $e^{\theta T_2} = 1 + \theta T_2 + \frac{(\theta T_2)^2}{2}$

Order quantity

Q $= I_1(T_1) = I_2(0) = (p-d)T_1$ ………..(A.3)