UNSTEADY MHD MEMORY FLOW WITH OSCILLATORY SUCTION, VARIABLE FREE STREAM AND HEAT SOURCE

S. Mustafa[1], Rafiuddin[2] and M. V. Ramana Murthy[3]
[1]Department of Mathematics, Younis Sultan College of Engineering R.R District Hyderabad, India
[2]Department of Mathematics, N.S.A.K Center for P.G Studies and Research, AUC Campus, Hyderabad, India
[3]Department of Mathematics, Osmania University, Hyderabad, India
E-Mail: syed1.mustafa@gmail.com

ABSTRACT

Ohmic disspitaion effect on unsteady boundary layer flow and heat transfer of an incompressible electrically conducting memory fluid over a continuous moving horizontal non-conducting surface in the presence of transverse magnetic field, an oscillating free stream and volumetric rate of heat generation (or absorption) is investigated, neglecting induced magnetic field in comparison to the applied magnetic field. The velocity and temperature distributions are obtained numerically and presented in graphical form. The expressions of skin friction coefficient and rate of heat transfer in terms of Nusselt number at the surface are derived, numerically and their numerical values for various values of physical parameters are presented in Tabular form.

Keywords: memory fluid, skin friction, nusselt number.

INTRODUCTION

A study of boundary layer (Lachmann, 1961; Schlitchtinh, 1968 and Bansal, 1977) behaviour on continuous solid surface has attracted the attention of researchers because such flows find application in different areas such as aerodynamics extrusion of plastic sheets, the boundary layer along material handling conveyers, the cooling of an infinite metallic plate in a cool bath and the boundary layer along liquid film in condensation processes (Skiadis, 1961) studied theoretically the boundary layer flow on a continuous semi-infinite sheet moving steadily through otherwise quiescented fluid environment. Sparrow and Cess, 1961 presented temperature dependent heat sources or sink in a stagnation point flow. Heat transfer in laminar flow flow of the Newtonian heat generating fluids was treated by (Foraboschi and Federice, 1964). Skin-friction and heat transfer on a continuous flat surface moving in a parallel free stream was discussed by (Abdel Hafez, 1965). Flow and heat transfer in the boundary layer on a continuous moving surface was discussed by (Tsou et. al, 1967). (Raptis and Tzivanidis, 1981) discussed the flow of a Walters liquid B′ model in the presence of constant heat flux between the fluid and the plate and taking into account the influence of the memory fluid on the energy equation. (Sarangi and Sharma, 2002) studied unsteady laminar flow of an electrically conducting incompressible fluid between two non-conducting wavy walls in the presence of transverse magnetic field. Veena *et. al*, 2006 studied the heat transfer in a viscoelastic fluid past a stretching sheet with viscous dissipation and heat generation.(Sharma *et. al*, 2004) studied unsteady MHD flow and heat transfer over a continuous porous moving horizontal surface in the presence of an oscillating free stream and heat source. Noushima *et. al*, 2008 had extended the above problem to viscoelastic fluid. The aim of the present study was to extend the work of (Noushima *et. al*) with variable suction.

The constitutive equation for the rheological equation of state for a memory fluid liquid B′ model given by (Walter, 1960 and 1962). The mixture of polymethyl mehacrylate and pyridine at 25^{o}C containing 30.5g of polymer per litre behaves very nearly as the (Walter liquid B′).

FORMULATION OF THE PROBLEM

Consider unsteady flow and heat transfer of an incompressible electrically conducting memory fluid over a continuous moving horizontal non-conducting surface in the presence of an oscillating free stream, ohmic dissipation and volumetric rate of heat generation (or absorption). The x-axis is taken along surface in flow direction and y-axis is normal to the surface. Transverse magnetic field B0 (const) is applied and induced magnetic field in comparison to the applied magnetic field is neglected.

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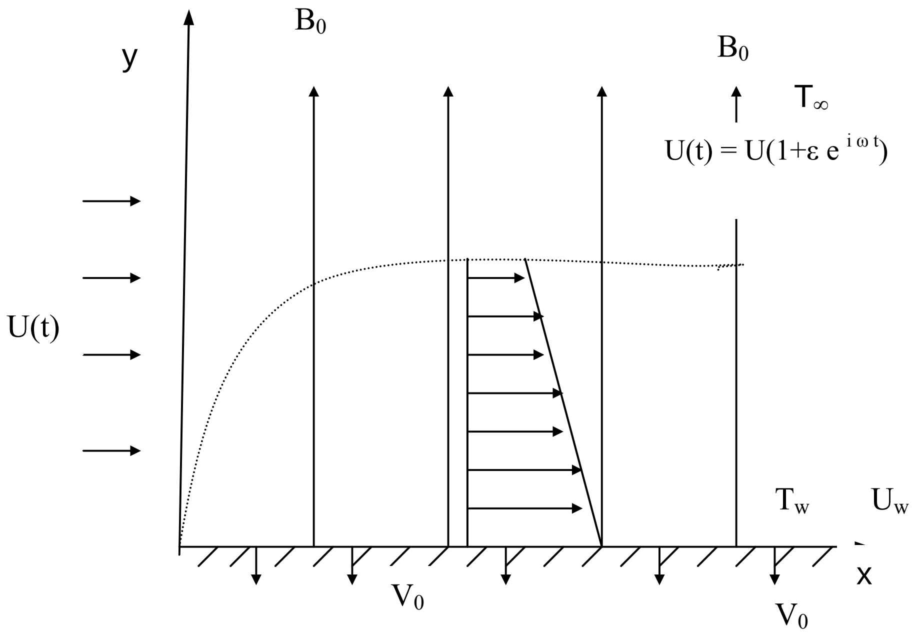

Figure-1. Physical Model.

The governing equations of continuity, motion and energy are:

$$\partial v / \partial y = 0 \Rightarrow \quad v = -v_0 (1+ \varepsilon e^{i \omega t}) \tag{1}$$

$$\rho (\partial u / \partial t + v\, \partial u / \partial y) = \rho\, \partial U(t) / \partial t + \mu\, \partial^2 u / \partial y^2 - B_1 (\partial^3 u / \partial t\, \partial y^2 + v\, \partial^3 u / \partial y^3) - \sigma B_0^2 (u - U(t)) \tag{2}$$

$$\rho C_p (\partial T / \partial t + v\, \partial T / \partial y) = \kappa\, \partial^2 T / \partial y^2 + \mu (\partial u / \partial y)^2 + Q (T-T_\infty) + \sigma B_0^2 (u - U(t))^2 \tag{3}$$

where u is the fluid velocity component along x - axis, T the fluid temperature, t the time, v_0 the cross-flow velocity, ρ the density, μ the coefficient of viscosity, C_p the specific heat at constant pressure, κ the thermal conductivity, B_0 the applied magnetic field, B_1 kinematic viscoelasticity, Q the volumetric rate of heat generation (or absorption), σ the electrical conductivity of the medium and U(t) the uniform free stream.

The boundary conditions are:

$$\left.\begin{aligned} y = 0 &: \quad u = U_w ,\ T = T_w \\ y \to \infty &: \quad u = U(t) ,\ T \to T_\infty \end{aligned}\right\} \tag{4}$$

where U_w is the surface velocity, T_w the surface temperature, T_∞ the free stream temperature and ω the frequency.

Introducing the following non-dimensional quantities:

$$\bar{y} = y\, v_0 / \nu,\quad \bar{t} = t\, v_0^2 / 4\nu,\quad \bar{u} = u / U,\quad \theta = (T-T_\infty) / (T_w - T_\infty),\quad \beta = U_w / U,\quad \bar{\omega} = \omega\, 4 \nu / v_0^2,$$

$$Pr = \mu C_p / \kappa ,\quad \alpha = \nu^2 Q / \kappa v_0^2 ,\quad R_m = B_1 v_0^2 / \rho \nu^2 ,\quad M = \sigma B_0^2 \nu / \rho v_0^2,\quad \bar{U}(t) = U(t) / U,$$

$$Ec = U^2 / C_p (T_w - T_\infty) \tag{5}$$

into the equations (2) and (3) , we get

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$$(\partial u/\partial t)/4 - (1+\varepsilon e^{i\omega t})\,\partial u/\partial y = (\partial U/\partial t)/4 + \partial^2 u/\partial y^2 - R_m((\partial^3 u/\partial t\,\partial y^2)/4 - (1+\varepsilon e^{i\omega t})\,\partial^3 u/\partial y^3) - M(u - U(t)) \quad (6)$$

$$Pr((\partial\theta/\partial t)/4 - (1+\varepsilon e^{i\omega t})\,\partial\theta/\partial y) = \partial^2\theta/\partial y^2 + Pr\,Ec\,(\partial u/\partial y)^2 + \alpha\theta + M\,Pr\,Ec\,(u - U(t))^2 \quad (7)$$

where M is the Hartmann Number,α the heat generation (or absorption) parameter, Pr the Prandtl number, Ec the Eckert number, and R_m is Magnetic Reynolds number.

The corresponding boundary conditions in non – dimensional form are:

$$\left.\begin{aligned} &y = 0: \quad u = \beta, \quad \theta = 1 \\ &y \to \infty: \quad u \to U(t),\ \theta \to 0 \end{aligned}\right\} \quad (8)$$

Assuming,
$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y)$,
$\theta(y,t) = \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y)$

and for free stream velocity $U(t) = 1 + \varepsilon e^{i\omega t}$ (9)

Now, using (9) into the equations (6) and (7) and equating the coefficients of O(ε), we get :

Zero-Order Equations

$$R_m u_0^{111} + u_0^{11} + u_0^{1} - M u_0 = -M \quad (10)$$

$$\theta_0^{11} + Pr\,\theta_0^{1} + \alpha\,\theta = -Pr\,Ec\,(u_0^{1})^2 - M\,Pr\,Ec\,(u_0 - 1) \quad (11)$$

First-Order Equations

$$R_m u_1^{111} - (R_m i\omega/4 - 1)\,u_1^{11} + u_1^{1} - (i\omega/4 + M)\,u_1 = -u_0^{1} - R_m u_0^{111} - (i\omega/4 + M) \quad (12)$$

$$\theta_1^{11} + Pr\,\theta_1^{1} + (-Pr\,i\omega/4 + \alpha)\,\theta_1 = -Pr\,\theta_0^{1} - 2\,Pr\,Ec\,u_0^{1}u_1^{1} - 2\,M\,Pr\,Ec\,(u_0 - 1)(u_1 - 1) \quad (13)$$

Here, the prime denotes differentiation w.r.t ‘y’.
The corresponding boundary conditions are reduced to

$$\left.\begin{aligned} &y = 0: \ u_0 = \beta,\ u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 0 \\ &y \to \infty: \ u_0 \to 1, u_1 \to 1, \theta_0 \to 0,\ \theta_1 \to 0 \end{aligned}\right\} \quad (14)$$

The zero -order and first-order equations correspond to steady flow and unsteady flow, respectively. Since (10) and (12)are coupled nonlinear third order differentiation equation due to presence of elasticity of the fluids. Since the magnetic Reynolds number R_m is very small, therefore u_0,u_1, θ_0 and $\theta_1$can be expanded using (Beard and Walter rule 1964) in the equations (10), (11), (12) and (13) ,we get :

Zero-Order of R_m :

$$u_{00}^{11} + u_{00}^{1} - M u_{00} = -M \quad (15)$$

$$\theta_{00}^{11} + Pr\,\theta_{00}^{1} + \alpha\,\theta_{00} = -Pr\,Ec\,(u_{00}^{1})^2 - M\,Pr\,Ec\,(u_{00} - 1)^2 \quad (16)$$

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$$u_{10}^{11} + u_{10}^{1} - u_{10}(i\omega / 4 + M) = - u_{00}^{1} - (i\omega / 4 + M) \quad (17)$$

$$\theta_{10}^{11} + Pr\, \theta_{10}^{1} + (- i\omega Pr / 4 + \alpha)\, \theta_{10} = - Pr\, \theta_{00}^{1} - 2\, Pr\, Ec\, u_{00}^{1} u_{10}^{1} - 2\, M\, Pr\, Ec\, (u_{00} -1)(u_{10} - 1) \quad (18)$$

First-Order of R_m:

$$u_{01}^{11} + u_{01}^{1} - M\, u_{01} = - u_{00}^{111} \quad (19)$$

$$\theta_{01}^{11} + Pr\, \theta_{01}^{1} + \alpha\, \theta_{01} = -2\, Pr\, Ec\, u_{00}^{1} u_{01}^{1} - 2\, M\, Pr\, Ec\, u_{01}(u_{00}-1) \quad (20)$$

$$u_{11}^{11} + u_{11}^{1} - u_{11}(i\omega / 4 + M) = - u_{10}^{111} - u_{01}^{1} + i\omega\, u_{10}^{11} / 4 - u_{00}^{111} \quad (21)$$

$$\theta_{11}^{11} + Pr\, \theta_{11}^{1} + (- i\omega\, Pr / 4 + \alpha)\, \theta_{11} = - Pr\, \theta_{01}^{1} - 2\, Pr\, Ec\, (u_{00}^{1} u_{11}^{1} + u_{01}^{1} u_{10}^{1}) - 2\, M\, Pr\, Ec\, [u_{11}(u_{00}-1) + u_{01}(u_{10}-1)] \quad (22)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} y = 0 \;:\; u_{00} = \beta, u_{01} = u_{10} = u_{11} = 0, \theta_{00} = 1, \theta_{10} = \theta_{01} = \theta_{11} = 0 \\ y \to \infty : u_{00} = u_{10} = 1,\ u_{01} = u_{11} = 0\ ,\ \theta_{00} = \theta_{01} = \theta_{10} = \theta_{11} = 0 \end{array}\right\} \quad (23)$$

SOLUTION OF THE PROBLEM

The equations (15) to (22) are ordinary linear second-order differential equations with the boundary conditions (23). Through straight forward Algebra the solution of $u_{00}(y)$, $u_{01}(y)$, $u_{10}(y)$, $u_{11}(y)$, $\theta_{00}(y)$, $\theta_{01}(y)$, $\theta_{10}(y)$ and $\theta_{11}(y)$ are known. The expression of velocity distribution is:

$$u (y , t) = (\beta-1) \exp(-a_2 y) + R_m\, y\, a_{20} \exp(-a_2 y) + \varepsilon (\cos (\omega t) M_r - \sin (\omega t) M_i)$$

$$M_r = -\exp (a_3\ y) (\cos a_4 y + a_5 \sin a_4 y) + 1 + R_m [\exp(-a_3 y) (a_{23} \cos(a_4 y) + a_{24} \sin(a_4 y)) + \exp(-a_2 y) a_{25} - \exp (-a_3 y) (a_{27} \cos (a_4 y) + a_{28} \sin (a_4 y))]$$

$$M_i = -\exp(a_3\ y)(a_5 \cos a_4 y - \sin a_4 y) + a_5 \exp (-a_2 y) + R_m [\exp (-a_3 y) (a_{24} \cos (a_4 y) - a_{23} \sin(a_4 y)) + \exp(-a_2 y) (a_{21} m_2 y - a_{26}) - \exp (-a_3 y) (a_{28} \cos (a_4 y) - a_{27} \sin (a_4 y))] \quad (24)$$

and temperature distribution is

$$\theta = A_5 \exp (-a_{29} y) + A_4 \exp (-2 a_2 y) + R_m [-K_6 \exp (-a_{29} y) + K_6 \exp (-2 a_2 y) - K_5 y \exp (-2 a_2 y)] + \varepsilon (\cos (\omega t)\ M_r - \sin (\omega t) M_i)$$

$$M_r = \exp (-K_7\ y) (a_{48} \cos K_8 y + a_{49} \sin K_8 y) - a_{50} \exp(-2\ a_2\ y) + \exp(-(a_2 + a_3)\ y) (a_{52} \cos a_4 y + a_{53} \sin a_4 y) + R_m [-\exp (-K_7 y) (N_{55} \cos (K_8 y) + N_{56} \sin (K_8 y)) + \exp (-a_2 y) a_{57} - \exp (-2 a_2\ y) N_{57} + \exp (-2 a_2\ y) N_{59} y + \exp(-(a_2 + a_3) y)(N_{61} \cos(a_4 y) + N_{62} \sin(a_4 y)) - \exp(-(a_2 + a_3) y)\, y\, (N_{63} \cos(a_4 y) + N_{64} \sin(a_4 y))]$$

$$M_i = \exp(-K_7\ y)(a_{49} \cos K_8 y - a_{48} \sin K_8 y) - a_{51} \exp(-2\ a_2\ y) + \exp (-(a_2 + a_3)\ y) (a_{53} \cos a_4 y - a_{52} \sin a_4 y) + R_m [-\exp(-K_7 y)(N_{56} \cos (K_8 y) - N_{55} \sin (K_8 y)) + \exp (-a_2 y) a_{58} - \exp (-2\ a_2 y) N_{58} + \exp (-2 a_2\ y) N_{60} y + \exp(-(a_2 + a_3) y)(N_{62} \cos(a_4 y) - N_{61} \sin(a_4 y)) - \exp(-(a_2 + a_3) y)\, y\, (N_{64} \cos(a_4 y) - N_{63} \sin(a_4 y))] \quad (25)$$

SKIN-FRICTION AND NUSSELT NUMBER

The coefficient of skin -friction at the surface is given by:

$C_f = \tau_{xy} / \rho U v_0 |_{y=0} = (\partial u / \partial y)_{y=0}$,

where $\tau_{xy} = \mu (\partial u / \partial y)_{y=0}$ (26)

$C_f = a_2 + R_m (a_{20} - a_2) + \varepsilon\ (\cos \omega t\, N_r - \sin \omega t\, N_i)$

$N_r = a_3 - a_5 a_4 - R_m [a_3 a_{23} - a_4 a_{24} + a_2 a_{25} - a_3 a_{27} + a_4 a_{28}]$

$N_i = a_3 a_5 + a_4 - a_5 a_2 - R_m [a_3 a_{24} + a_4 a_{23} - a_2 a_{26} - a_2 a_{21} - a_3 a_{28} - a_4 a_{27}]$ (27)

The rate of heat transfer in terms of Nusselt number at the surface is given by :
$Nu = q \nu / v_0 \kappa (T_w - T_\infty) |_{y=0} = - \partial \theta / \partial y |_{y=0}$,

where $q = - \kappa (\partial T / \partial y)_{y=0}$ (28)

$Nu = - A_5 a_{29} - 2 a_2 A_4 + R_m (K_6 a_{29} - 2 a_2 K_6 - K_5) + \varepsilon\ (\cos \omega t\, N_r - \sin \omega t\, N_i)$

$N_r = - K_7 a_{48} + K_8 a_{49} + 2 a_2 a_{50} - (a_2 + a_3) a_{52} + a_4 a_{53} + R_m [K_7 N_{55} - N_{56} K_8 - a_2 a_{57} + 2 a_2 N_{57} + N_{59} -$

$(a_2 + a_3) N_{61} + N_{62} a_4 - N_{63}]$

$N_i = - K_7 a_{49} - K_8 a_{48} + 2 a_2 a_{51} - (a_2 + a_3) a_{53} - a_4 a_{52} + R_m [K_7 N_{56} + N_{55} K_8 - a_2 a_{58} + 2 a_2 N_{58} + N_{60} -$

$(a_2 + a_3) N_{62} + N_{61} a_4 - N_{64}]$ (29)

where a_1…are constants and their expressions are not presented here for the sake of brevity.

DISCUSSIONS AND CONCLUSIONS

Table-1 shows that the skin-friction coefficient at the surface decreases due to increase in the Hartmann number, velocity of surface, frequency, phase angle and magnetic Reynolds number.

Table-1. Values of skin-friction coefficient at the surface when ε = 0.05

ω	M	ω t	β	R_m	C_f
5.0	1.0	π/6	2.0	0.4	-2.3758
5.0	2.0	π/6	2.0	0.4	-3.0666
5.0	2.0	π/6	4.0	0.4	-9.2000
5.0	2.0	π/3	2.0	0.4	-3.0667
10.0	2.0	π/6	2.0	0.4	-2.7537
5.0	2.0	π/6	2.0	0.5	-3.3333
5.0	2.0	π/6	2.0	0.0	-2.0002
5.0	2.0	π/6	0.0	0.4	3.0606

It is seen from Table-2 that the Nusselt number at the surface increases with the increase in the Prandtl number, while it decrease due to increase in the Hartmann number the Eckert number, frequency, heat generation parameter, phase angle or magnetic Reynolds number.

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Table-2. Values of Nusselt Number at the surface when $\varepsilon = 0.05$

ω	M	ω t	β	R_m	Pr	Ec	α	Nu
5.0	1.0	π/6	2.0	0.4	5.0	0.01	1	4.7248
5.0	2.0	π/6	2.0	0.4	5.0	0.01	1	4.6503
5.0	3.0	π/6	2.0	0.4	5.0	0.01	1	4.2275
5.0	2.0	π/6	4.0	0.4	5.0	0.01	1	3.8945
5.0	2.0	π/3	2.0	0.4	5.0	0.01	1	4.6506
10.0	2.0	π/6	2.0	0.4	5.0	0.01	1	4.6600
5.0	2.0	π/6	2.0	0.4	5.0	0.02	1	3.4024
5.0	2.0	π/6	2.0	0.4	5.0	0.01	2	4.3720
5.0	2.0	π/6	2.0	0.4	7.0	0.01	1	6.7421
5.0	2.0	π/6	2.0	0.5	5.0	0.01	1	4.6352
5.0	2.0	π/6	2.0	0.0	5.0	0.01	1	4.7121
5.0	2.0	π/6	0.0	0.4	5.0	0.01	1	4.7736

It is observed from Figure-2 that the fluid velocity decreases due to increase in the Hartmann number, frequency, phase angle, magnetic Reynolds number and it become asymptotic in y-direction when the surface is in motion. It is also observed that fluid velocity is more for viscous case than the viscoelastic one. The fluid velocity increase with the increase of y and it tends to asymptotic when the surface is at rest.

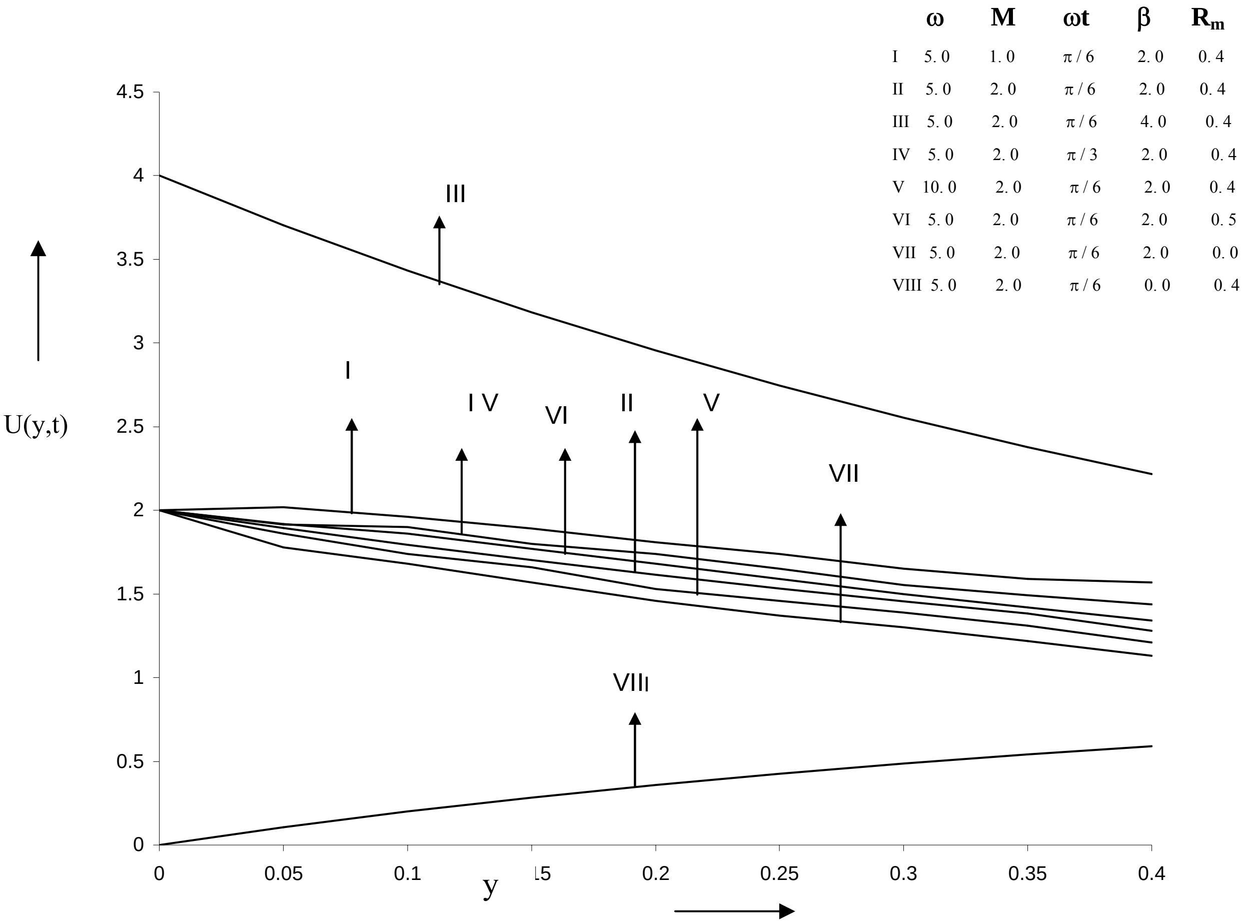

Figure-2. Variation of velocity distribution versus y.

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It is seen from Figure-3 that the fluid temperature increases with the increase in heat generation parameter, the Eckert number, frequency, phase angle, magnetic Reynolds number, velocity of the surface or the Hartmann number, while it decreases due to increase in the Prandtl number. Here, also fluid temperature is more for viscous case than viscoelastic one.

	ω	M	ωt	β	R_m	Pr	Ec	α
I	5.0	1.0	π / 6	2.0	0.4	5.0	0.01	1
II	5.0	2.0	π / 6	2.0	0.4	5.0	0.01	1
III	5.0	3.0	π / 6	2.0	0.4	5.0	0.01	1
IV	5.0	2.0	π / 6	4.0	0.4	5.0	0.01	1
V	5.0	2.0	π / 3	2.0	0.4	5.0	0.01	1
VI	10.0	2.0	π / 6	2.0	0.4	5.0	0.01	1
VII	5.0	2.0	π / 6	2.0	0.4	5.0	0.02	1
VIII	5.0	2.0	π / 6	2.0	0.4	5.0	0.01	2
XI	5.0	2.0	π / 6	2.0	0.4	7.0	0.01	1
X	5.0	2.0	π / 6	2.0	0.5	5.0	0.01	1
XI	5.0	2.0	π / 6	0.0	0.4	5.0	0.01	1
XII	5.0	2.0	π / 3	2.0	0.0	5.0	0.01	1

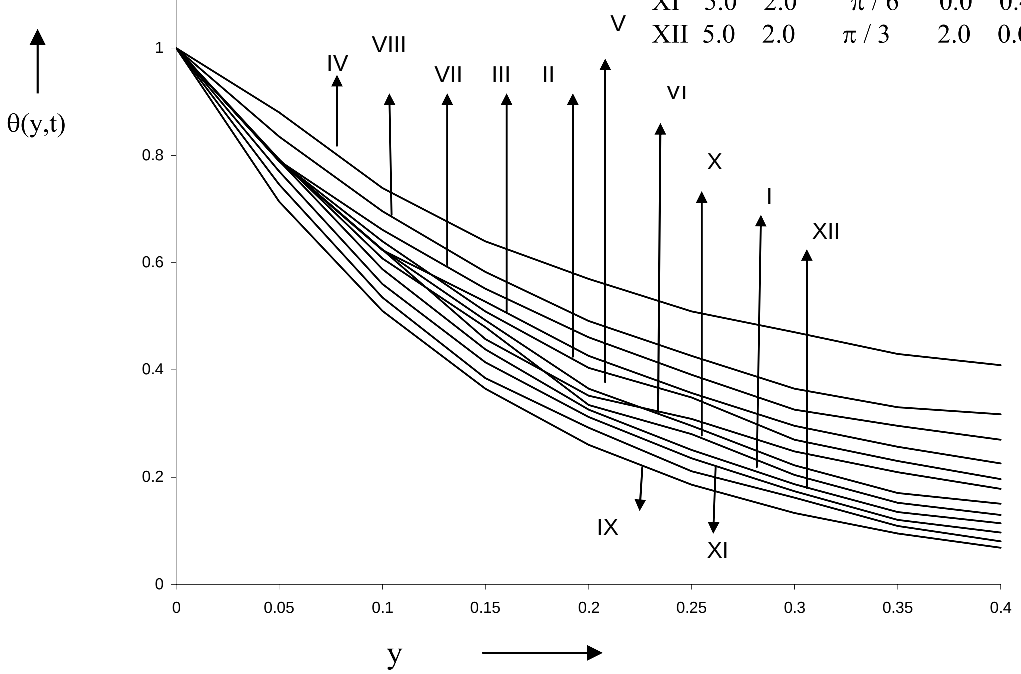

Figure-3. Variation of temperature distribution versus y.

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