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SELECTION OF OPTIMAL HEAT SINK DIMENSIONS USING EVOLUTIONARY STRATEGIES: A CASE STUDY

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ABSTRACT

The ever rising transistor densities and switching speeds in microprocessors have been accompanied a dramatic increase in the system heat flux and power dissipation. In this context the rising IC densities combined with even more stringent performance and reliability requirement have made thermal management issues ever more prominent in the design of sophisticated microelectronic systems. In order to achieve a higher degree of power dissipation extruded heat sinks have been a standard for many years. The objective of this paper is to use an evolutionary optimization method for the determination of the optimal heat sink dimensions, such that the optimized dimensions are within realistic manufacturing constraints and the heat dissipation capability is maximized.

Keywords: thermal management, heat sink design, evolutionary optimization.

NOMENCLATURE

L	Channel or plate length	m
S	Channel or plate width	m
W	Width of prime area	m
D	Plate thickness	m
B	Plate spacing	m
P	Density	kg/m^3
X	Length coordinate	m
M	Dynamic viscosity	kg/m-s
W	Mass flow rate per unit Width	kg/m-s
p	Pressure	N/m^2
g	Gravitational acceleration	m/s^2
T_f	Fluid temperature	ºC
T_o	Entrance temperature	ºC
T_w	Wall temperature	ºC
q	Heat flow rate	W
A	Plate area	m^2
k	Thermal conductivity	$W/m^2 {}^oC$
Q_T	Total Heat flow rate	W
Nu	Channel Nusselt number	
B	Volumetric Coefficient of thermal expansion K^{-1}	

INTRODUCTION

Heat sinks constitute the most common and cost effective way for the dissipation of heat in micro electronic equipment. Over the past thirty years, a wide variety of designs have evolved to meet the rising heat dissipation demands [1]. However, to date there are no well established design procedures that guarantee optimal power dissipation within certain manufacturing constraints? This is primarily because of the multivariable and nonlinear nature of the design space. Conventional optimization methods such as gradient descent methods and have limited applicability because of their tendency to reach local optima.

In this context evolutionary optimization methods provide a feasible solution since they are designed specifically for multivariable non-linear problems. These methods are better for the determination of global optima because they incorporate a certain degree of randomness in the search process.

The objective of this paper is to apply an evolutionary optimization method known as Evolutionary Strategies for determination of parallel plate fin heat sink dimensions such that the rate of heat dissipation is maximized. In addition the heat sink dimensions thus determined must lie within realistic manufacturing constraints as well.

The manufacturing technology of choice for this research is convoluted or folded fins attached to an extruded base. This method is now the standard in the manufacture of heat sinks. The remaining part of this paper is organized as follows; the first two sections explain the derivation of the analytical model for the heat sink and the fundamental concepts of Evolutionary Optimization algorithms. In the third section optimization procedure is utilized for the determination of the optimal spacing between the fins of the heat sink, and the results obtained are compared with optimal values reported earlier [5]. Section four illustrates the application of the evolutionary optimization procedure for the determination of optimal heat sink dimensions which lie within appropriate manufacturing constraints. The last section provides the conclusions of this research and future possibilities.

HEAT SINK MODEL

Although heat dissipation patterns vary widely, many heat sink configurations can be modeled by symmetric or asymmetric isothermal or isoflux plates. Before the commencement of any optimization procedure gradient descent or evolutionary, it is necessary to develop a mathematical model which relates the operating conditions of the heat sink and it's dimensions to the rate of heat dissipation. The model used for this paper was first reported by Bar and Cohen [5].

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It is assumed in this analysis that the channel between the plates is sufficiently long to allow a fully developed flow. In addition since the local temperature is not known explicitly, therefore the local heat transfer coefficient is assumed to be constant and expressed in terms of the inlet or ambient temperature. Analytic expressions for the Nusselt number values are derived for the isolated plate limit and the fully developed flow limit. The intermediate values can then be determined experimentally [6] or by using correlation procedures

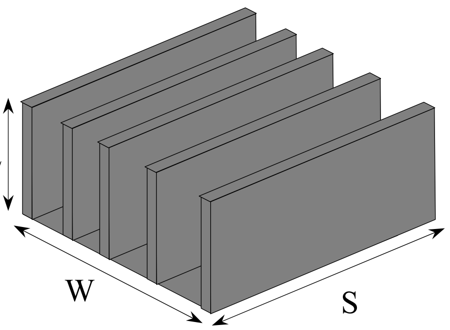

Figure-1a. Heat sink dimensions.

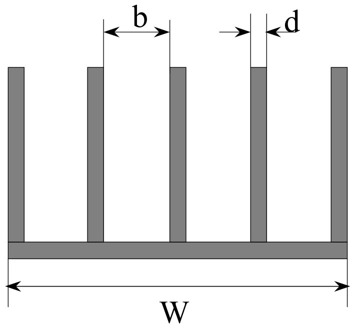

Figure-1b. Heat sink dimensions.

The remaining part of this section briefly illustrates the development of an analytical model which relates the rate of heat dissipation in terms of the temperature difference, heat sink dimensions and the convective heat transfer coefficient.

The flow rate through the channel can be determined from the fact that for a fully developed flow, the flow resistance is balanced by the buoyant resistance. The flow resistance as given by [7] is,

$$\left.\frac{dP}{dx}\right|_{loss} = \frac{-12\mu w}{\rho b^3} \tag{1}$$

Whereas the buoyant potential can be given as,

$$\left.\frac{dP}{dx}\right|_{buoy} = -\rho\beta g\left(T_f - T_o\right) \tag{2}$$

The flow rate w can be determined by equating the two expressions as,

$$w = \frac{\rho^2 g\beta b^3\left(T_f - T_o\right)}{12\mu} \tag{3}$$

The local fluid temperature can be determined by applying an energy balance on the differential volume as shown in Figure 1. By equating the heat transferred from two isothermal walls with that absorbed in the flow, the following relationship is obtained,

$$wC_p dT = 2h(T_w - T_f)dx \tag{4}$$

For continuity considerations the flow rate w, C_p and the local heat transfer coefficient h are considered constant. Therefore the ratio $wC_p/2h$
Denoted by Γ can also be considered constant.

The local fluid temperature can be obtained by integrating (4) as follows,

$$T_f = T_w - (T_w - T_o)e^{-\Gamma x} \tag{5}$$

The Nusselt number can be defined as,

$$Nu_o = \left[\frac{q/A}{T_w - T_o}\right]\frac{b}{k} \tag{6}$$

The transfer rate q, can be obtained by using the expressions (3) and (5) for the flow rate w and temperature rise in the channel. Expression must be evaluate by setting $x=L$ so that the exit temperature is determined. The final expression for the heat transfer rate q can then be determined as follows:

$$q = \left[\frac{C_p\rho^2 g\beta b^3 S}{12\mu}(T_w - T_o)(1 - \frac{1-e^{-\Gamma L}}{\Gamma L})\right]\left[(T_w - T_o)(1 - e^{-\Gamma L})\right] \tag{7}$$

Substituting the value of q into (6), and letting $A=LS$ the expression for the Nusselt number can be obtained as,

$$Nu_o = \frac{1}{24}\left[\frac{C_p\rho^2 g\beta b^4(T_w - T_o)}{\mu kL}\right]\left[(1 - \frac{1-e^{-\Gamma L}}{\Gamma L})(1 - e^{-\Gamma L})\right] \tag{8}$$

The combination of parameters in the left bracket is channel Rayleigh number $R_a^{'} \equiv R_a b/L$, where,

$$R_a = \rho^2 g\beta C_p L^3 \Delta T/\mu L$$

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For the fully developed limit $L \to \infty$ Nu_o approaches $R_a^{'} / 24$.

EVOLUTIONARY OPTIMIZATION

Evolutionary optimization procedures derive inspiration from the Darwinian Theory of evolution. The principle idea as opposed to classical optimization methods is that, instead of using the analytical model of a function *f(x)* and the corresponding gradients for guiding the search along suitable directions a stochastic procedure is used. This procedure is initiated by randomly generating a set of possible solutions. Each solution is referred to as an *individual* and the set itself is referred to as the *population*. For reasonable performance of the optimization procedure it is necessary that these individuals be scattered through the entire solution space. Once initialized, these individuals are evaluated using the function *f(x)*.The value thus returned from the function is referred to as the *fitness* value of the individual. After evaluation, the individuals with the lowest fitness values are selected to form the next generation of individuals and the remaining are discarded. This process is then repeated until a stopping criterion is achieved. This could either be a fixed number of generations or a minimum value of the fitness function.

The evolutionary optimization method of choice for this investigation is Evolutionary Strategies. Figure-1 shows the flow chart of this method. The first step is to generate a random population of individuals each representing a potential solution. These individuals are also referred to as the *parent* individuals, and their number is denoted by μ. In the *recombination* step, λ pairs of these parent individuals are chosen and their values are recombined to produce λ offspring individuals. Recombination can be done either by taking average values or by exchanging randomly chosen components of the parent pair [4]. Once generated, Gaussian noise is added to the solutions represented by these offspring individuals in the *mutation* step. This step is necessary to allow stochastic exploration of the design space. The degree of noise to be added is a subject of significant interest in Evolutionary Strategies [4].

After the mutation step is completed the fitness of each offspring individual is evaluated using the function *f(x)* and the μ best individuals are chosen as parent individuals for the next generation. This process is repeated until the predefined stopping criterion is attained.

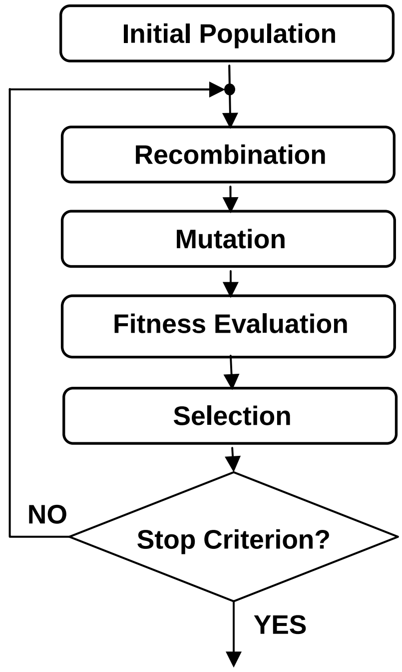

Figure.2. Evolutionary strategies flow chart.

OPTIMUM FIN SPACING

Analytical approach

The analytical model for the rate of heat dissipation in a heat sink as a function of the temperature difference and its dimensions can now be utilized to determine the optimal plate spacing b_{OPT}. The amount of heat dissipated by the heat sink can be given as

$$Q_T = (2LS\Delta T_o)(m)(Nu_o k / b) \tag{9}$$

Differentiating (9) with respect to b and setting the derivative to zero the following expressions are obtained,

$$\frac{dQ_T}{db} = 0 \tag{10}$$

$$\frac{d}{db}\left[(2LS\Delta T)\left(\frac{W}{b+d}\right)\left(\frac{Nu_o k}{b}\right)\right] = 0 \tag{11}$$

$$\left[(2LS\Delta TWk)\right]\frac{d}{db}\left[\left(\frac{1}{b+d}\right)\left(\frac{N_u}{b}\right)\right] = 0 \tag{12}$$

Setting the derivative term to zero and substituting the expression for the Nusselt number we get,

$$\frac{d}{db}\left[\left(\frac{1}{b+d}\right)\left(\frac{1}{b}\right)\left(\frac{576}{K^2 b^8} + \frac{2.873}{\sqrt{K b^2}}\right)^{-0.5}\right] = 0 \tag{13}$$

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After cancellation of common terms and algebraic simplifications,

$$(2b + 3d - 0.005K^{1.5}b_{OPT}^{7}) = 0 \qquad (14)$$

Since in most practical heat sink dimensions the fin thickness in negligible in comparison to the fin spacing, (14) can be further simplified to

$$b_{OPT} = 2.714 / P^{0.25} \qquad (15)$$

Evolutionary optimization

This section illustrates the application of the above mentioned evolutionary optimization method for the determination of optimal spacing for parallel plate fin heat sinks.

Initialization

The population is initialized by generating individuals randomly such that they lie within specified manufacturing constraints. Each individual consists of a numerical value that represents the spacing between the plates. The number of parent and offspring individuals in each generation is fixed at five and thirty five respectively. Each simulation run of the evolutionary optimization procedure consist of fifty generations.

Fitness function

The objective of the optimization procedure is to determine fin spacing such that for given operating conditions the rate of heat dissipation Q_T is maximized. This can be achieved if the thermal resistance of the heat sink R_{Therm} is minimized. The fitness function is therefore defined simply as,

$$f = R_{Therm} \qquad (16)$$

Subject to,

$$b \in [b_{\min} \quad b_{\max}]$$

In this case the minimum plate spacing $b_{\min} = 0.1\,mm$ is determined by the manufacturing limits, whereas the higher limit $b_{\max} = 20\,mm$ is determined by the maximum permissible width W of the heat sink.

Recombination and mutation

Once generated the parent individuals are recombined using the intermediate recombination operator and the Gaussian noise is added to the individuals to enable stochastic exploration of the design space.
These procedures are utilized to generate thirty- five offspring individuals in each generation.

Fitness evaluation and selection

The offspring individuals thus generated are evaluated using the analytical model developed in the previous section, and best five of these are chosen as parents for the next generation.

For the purpose of this paper the dimensions L, S, W, d are chosen so that they are comparable to the dimensions being used in conventional heat sinks. In this case L= 180mm, S=24mm , W=120mm , d = 1.0mm. The material chosen for the heat sink is Aluminum Al-6063T. The plate temperature T_s and environment temperature T_∞ are 80ºC and 30ºC respectively. The properties of air at the film temperature $T_f = (T_s + T_\infty)/2$ = 55ºC are determined from Table A-15 provided in [8].

Figure-1 illustrates the convergence of the best fitness values with respect to the generations. It can be seen clearly that as the generations pass, the best fitness values of the individuals is minimized. Figure-2 illustrates the convergence of the individuals to the optimal value. Each individual is indicated by a blue cross. As illustrated in the Figure, the individuals are scattered through out the entire design space for the first five generations, after which they start to converge and reach the optimal value.

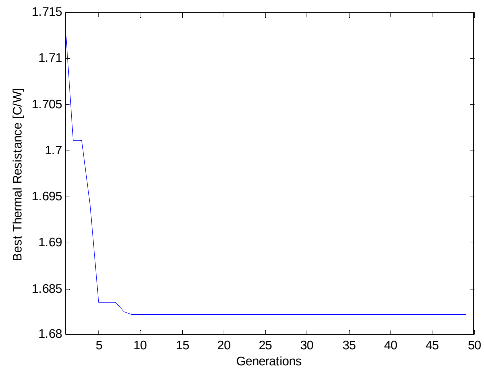

Figure-3a. Convergence of fitness with respect to generations.

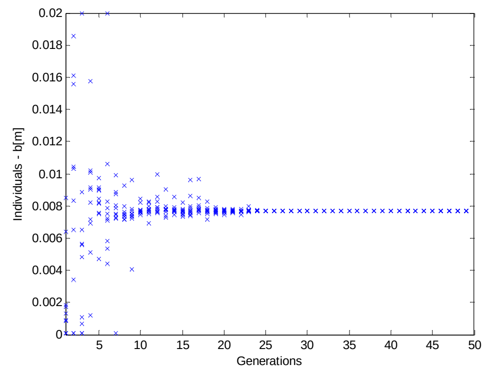

Figure-3b. Convergence of population with respect to generations.

The optimal spacing determined using the evolutionary procedure is 7.7 mm and the corresponding thermal resistance R_{Therm} is 1.683 W/C°. These results are exactly identical to those obtained using the analytical

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methods (15). This high degree of agreement between the two results can be explained due to the relative simplicity of the one dimensional optimization problem.

OPTIMUM HEAT SINK DIMENSIONS

The evolutionary optimization approach is now extended to five dimensions so that not just the thermal resistance can be minimized but in addition the amount of metal used for the heat sink can be constrained as well. Determination of the optimal dimensions using the analytical approach is difficult. This is because it involves the determination of partial derivatives of the thermal resistance with respect to each dimension and then application of a gradient descent method. The presence of a nonlinear constraint, i.e. the maximum permissible mass of the heat sink further complicates the analysis.

In order to ensure restriction of the heat sink mass the fitness function defined in the previous section is modified to include a *penalty* term *P(m),* where *m* is the mass of the heat sink. If during the course of the evolutionary optimization procedure, an individual is generated that has a mass greater than the maximum permissible limit, then P (m) has a positive value, otherwise it is always zero. In this case the maximum value of the penalty function is empirically chosen to be ten. The penalty function can therefore be defined as

$$p(m) = \begin{cases} 0 & m \le 100g \\ 10 & m > 100g \end{cases}$$

The modified fitness function can then be given as,

$$f = R_{Therm} + p(m)$$

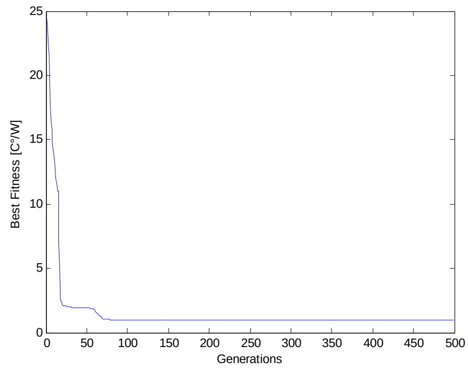
Subject to,

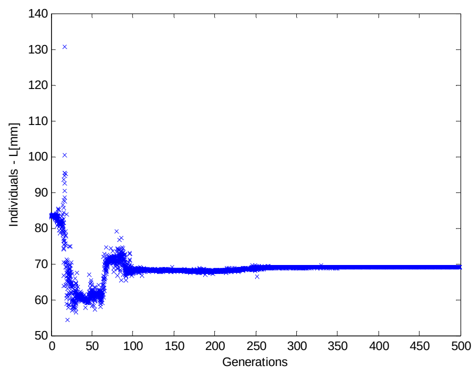
$$L \in [20\,mm \quad 150\,mm]$$
$$S \in [20\,mm \quad 150\,mm]$$
$$W \in [20\,mm \quad 150\,mm]$$
$$d \in [0.5\,mm \quad 1.5\,mm]$$
$$b \in [2.0\,mm \quad 20\,mm]$$

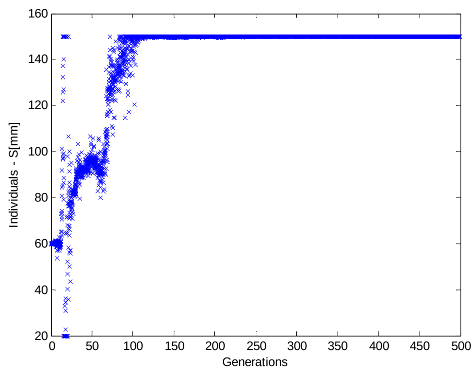
Figure-4a illustrates the convergence of the best fitness values with respect to the generations. The remaining Figures 4b to 4f show convergence of heat sink length, width, height, plate thickness and plate spacing respectively. Figure-4g illustrates convergence of heat sink mass. In all cases a greater portion of the design space is occupied initially and with the passage of generations convergence to the optimal values is achieved. The material used for heat sink manufacture is Al-6063T. The same optimization procedure was repeated for a Copper heat sink. These optimal heat sink dimensions are provided in Table-1. The minimum thermal resistance for the Aluminum heat sink is 0.9W/°C and the mass is constrained to 100 grams as desired. For the case of Copper heat sinks the minimal thermal resistance is 2.19 W/°C.

Table-1. Optimized heat sink dimensions.

	L (mm)	**S** (mm)	**W** (mm)	**d** (mm)	**b** (mm)
Al	70	150	56	0.5	7
Cu	20	148	58	0.5	7







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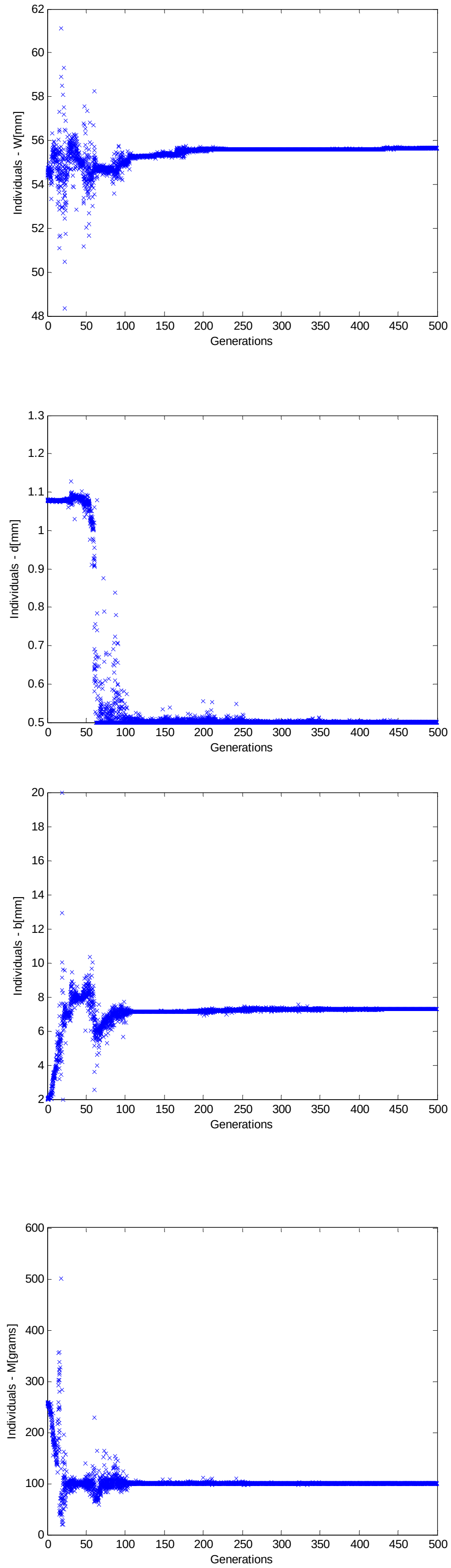

Figures 4a-4f. Convergence of best fitness and heat sink parameters.

CONCLUSIONS

The objective of this work is to apply the evolutionary optimization procedure for the determination of optimal heat sink dimension so that the thermal resistance is minimized. It has been clearly demonstrated that the results obtained for the one dimensional case are identical to those obtained using a purely analytical method. In addition the same method can be easily extended to the five dimensional case with a constraint upon the maximum permissible mass of the heat sink. No knowledge about the derivatives is needed and only a fitness function needs to be defined. It is planned that same approach be extended for the optimized design of heat exchangers using Finite Element Modeling methods.

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