CONSOLIDATION BEHAVIOR OF PILES UNDER PURE LATERAL LOADINGS

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ABSTRACT

The results of two-dimensional finite element analysis of piles under pure lateral loadings in saturated porous medium are described herein. The soils are characterized using the Mohr Coulomb model in which together with the finite element formulation are described and verified. Transient analysis of pile problem is then carried out, and the results of the analysis are presented which demonstrate the effects of the consolidation process on the lateral response of the pile under lateral loadings. Lower lateral loading gives lower differences in deformations with time and being significant with increasing load. Also lateral soil pressure and shear stress in soil increased with time and depth but in a rate lowered as the lateral load decreased.

Keywords: pile, load, consolidation, lateral, response, finite element method.

INTRODUCTION

Pile foundations are used to support the heavy structure and can act in the dual role of carrying the applied load to deeper, strong layers and also of reinforcing the soil. In the case of foundations of bridges, transmission towers, offshore structures and other type of huge structures, piles are also subjected to lateral loads. This lateral load resistance of pile foundations is critically important in the design of structures under loading from earthquakes, soil movement, waves etc. The maximum deflection of the pile is the major criterion in the design (Poulos and Davis, 1980).

The stability of foundations and earthworks such as piles in saturated soils is a time-dependent process. This is because any change in total normal stress is initially resisted by pore pressures, which then dissipates over a period of time.

It is generally accepted that the finite element method is the major technique used in numerical analysis of geotechnical problem particularly piles and soil consolidation. The finite element method is most widely used to perform the analysis of piles under lateral loads. As reported in Poulos and Davis (1980) the first attempts to study the lateral behavior of piles included two-dimensional finite element models in the horizontal plane. Many investigations occurring to study the behavior of pile under lateral load (Muqtadir and Desai, 1986; Yang and Jeremic, 2005 and Abbas *et al.*, 2008). Thus the analyses of laterally loaded piles are investigated by this approach in this study.

In view of this, the present paper focuses on the study of time-dependent behavior of piles subjected to lateral loads through 2D finite-element analyses.

LINEAR ELASTIC MODEL OF PILE (PERFECT-PLASTICITY)

This model represents Hooke's law of isotropic linear elasticity used for modeling the stress-strain relationship of the pile material. The model involves two elastic stiffness parameters, namely Young's modulus, E, and Poisson's ratio, ν, as shown in Figure-1. It is primarily used for modeling of stiff structural member for example piles in the soil (Brinkgreve and Broere, 2004).

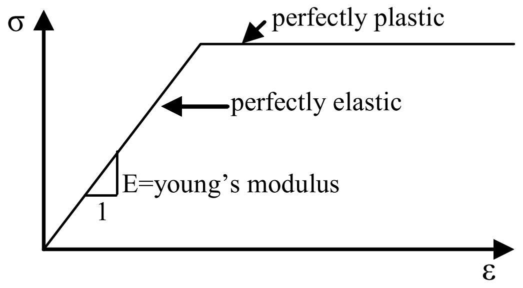

Figure-1. Stress-strain curve.
(Johnson *et al.*, 2006)

Hooke's law can be given by the Equation-1. Two parameters are used in this model, the effective Young's modulus, E', and the effective Poisson's ratio, ν'.

$$\begin{bmatrix} \sigma'_{xx} \\ \sigma'_{yy} \\ \sigma'_{zz} \\ \sigma'_{xy} \\ \sigma'_{yz} \\ \sigma'_{zx} \end{bmatrix} = \frac{E'}{(1-2\upsilon')(1+\upsilon')} \begin{bmatrix} 1-\upsilon' & \upsilon' & \upsilon' & 0 & 0 & 0 \\ \upsilon' & 1-\upsilon' & \upsilon' & 0 & 0 & 0 \\ \upsilon' & \upsilon' & 1-\upsilon' & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}-\upsilon' & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}-\upsilon' & \frac{1}{2}-\upsilon' \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} \qquad (1)$$

The relationship between Young's modulus E and other stiffness moduli, such as the shear modulus G, the bulk modulus K, and modulus elasticity E, is given by:

$$G = \frac{E}{2(1+\upsilon)} \qquad (2)$$

$$K = \frac{E}{3(1-2\upsilon)} \qquad (3)$$

$$E = \frac{(1-\upsilon)E}{(1-2\upsilon)(1+\upsilon)} \qquad (4)$$

MOHR-COULOMB SOIL MODEL

This elasto-plastic model is based on soil parameters that are known in most practical situations. The Mohr-Coulomb model is used to compute realistic bearing capacities and collapse loads of footings, as well as other applications in which the failure behavior of the soil plays a dominant role. The model involves two main parameters, namely the cohesion intercept, c and the friction angle, φ. In addition to three parameters namely Young's modulus, E, Poisson's ratio, ν, and the dilatancy angle, ψ need to calculate the complete σ-ε behavior. Mohr Coulomb's failure surface criterion is shown in Figure-2. According to Johnson *et al.* (2006) the failure envelope only depend on the principal stresses (σ1, σ3), and is independent of the intermediate principle stress (σ2). When mapped in three-dimensional stress space, Mohr-Coulomb criteria resolved into an irregular hexagonal pyramid. This pyramid forms the failure/yield envelope, which is turn governs how soil will behave. The material behaves elastically if the stress point lies within the failure envelope. However, if the stress reaches the yield surface the material will undergo a degree of the plastic deformation. In the Mohr-Coulomb model used herein, it is assumed that the soil has a linear elastic relation until failure.

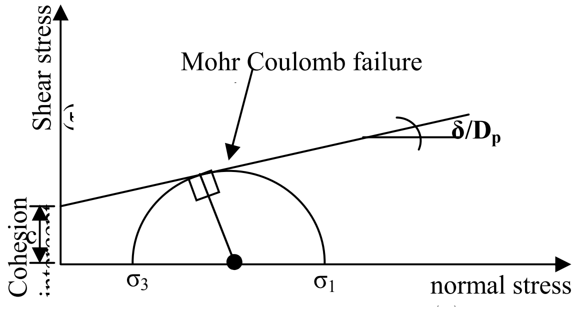

Figure-2. Mohr Coulomb's failure surface.
(Johnson *et al.*, 2006)

The usual definition of the equation of Mohr-Coulomb surface is (Smith and Griffith, 1982):

$$F=\frac{\sigma_1'+\sigma_3'}{2}\sin\phi'-\frac{\sigma_1'-\sigma_3'}{2}-c'\cos\phi' \qquad (5)$$

Which, when rewrite in terms of invariants and Lode angle θ becomes:

$$F=\frac{1}{3}I_1\sin\phi'+\sqrt{J_2}\left(\cos\theta-\frac{\sin\theta\sin\phi'}{\sqrt{3}}\right)-c'\cos\phi' \qquad (6)$$

where

$$I_1=\sigma_x'+\sigma_y'+\sigma_z' \qquad (7)$$

$$J_2=\frac{1}{6}\left\{(\sigma_x'-\sigma_y')^2+(\sigma_y'-\sigma_z')^2+(\sigma_z'-\sigma_x')^2\right\}+\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2 \quad (8)$$

FINITE ELEMENT FORMULATION

The models described above is incorporated in a finite element program, which has the feature of modeling two-dimensional (plane strain and axisymmetric) geotechnical problems such as consolidation written by FORTRAN90 language. This program is primarily based on the programs for the analysis of one and two-dimensional solid by finite element method (Smith and Griffiths, 1998) which is modified for the purpose of this study. So in addition to the Mohr Coulomb model, the program allows one to assign linear elastic behavior to any part of the problem geometry. Description of all of the program features is beyond the scope of this paper, and a brief summary of the consolidation theory relevant to this study is given below.

Transient formulation

In the case of a pile in saturated porous medium, the loading is time-dependent, so an incremental formulation was used in the following work producing the matrix version of the Biot equation at the element level presented below (Lewis and Schrefler, 1987).

$$\begin{bmatrix} K & L \\ L^T & S+\bar{\alpha}H\Delta t_k \end{bmatrix}\begin{Bmatrix}\bar{u}\\ \bar{p}\end{Bmatrix}=\begin{bmatrix} K & L \\ L^T & S-(1-\bar{\alpha})H\Delta t_k \end{bmatrix}\begin{Bmatrix}\bar{u}\\ \bar{p}\end{Bmatrix}+\begin{Bmatrix} dF/dt+C \\ \bar{F}\end{Bmatrix} \quad (9)$$

where: $K=$ element solid stiffness matrix, $L=$ element coupling matrix, $H=$ element fluid stiffness matrix, $\bar{u}=$ change in nodal displacements, $\bar{p}=$ change in nodal excess pore-pressures, $S=$ the compressibility matrix, $\bar{F}=$ load vector, $\Delta t=$ calculation time step, $\bar{\alpha}=$ time stepping parameter (=1 in this work), $dF/dt=$ change in nodal forces.

VALIDATION OF NUMERICAL MODELS

The lateral load- deflection response of bored piles in cemented sand was examined by field test on single pile under lateral load as reported by Ismael (1998). All piles were 0.3 m in diameter and had a length of 3 m or 5 m. A site was selected in Kuwait. The surface soil to depth of 3.5 m was characterized as having both component of shear strength, c and φ. The soil profile consists of a medium dense cemented silty sand layer to a depth 3 m. This is underlain by medium dense to very dense silty sand with cemented lumps to the bottom of the borehole. All properties of soil are listed in Table-1. Ground water was not encountered within the depth of the borehole. The same load sequence was apply on the pile after complete the whole geotechnical model for vertical and lateral pile tests.

In conclusion, the comparison between the finite element simulation and the reported in lateral data is shown in Figure-3. The piles were deflecting not in the same magnitude at the field test due to the variability of soil properties. Also the numerical simulation is reasonably accurate for the problem of laterally loaded piles and pile- soil interaction over a wide range of deformation for 3 m and 5 m piles long. The pile with length 5 m is highly resistance the lateral load from the second pile length value.

Table-1. Geotechnical properties of the soil layers.

Parameter	Name	Medium dense cemented silty sand layer	Medium dense to very dense silty sand with cemented lumps	Pile	Unit
Unsaturated soil weight	γ_{unsat}	18	19	25	kN/m^3
Saturated soil weight	γ_{sat}	18	19	-	kN/m^3
Young's modulus	E	1.300E+04	1.300E+04	2E+09	kPa
Poisson's constant	ν	0.3	0.3	0.15	-
Cohesion	c	20	1	-	kPa
Friction angle	φ	35	45	-	-

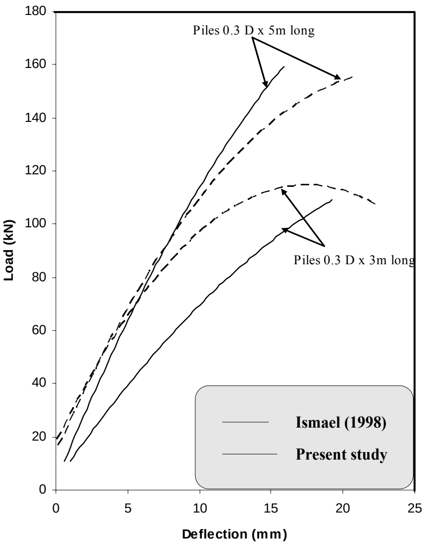

Figure-3. Comparison of finite element results with field test data.

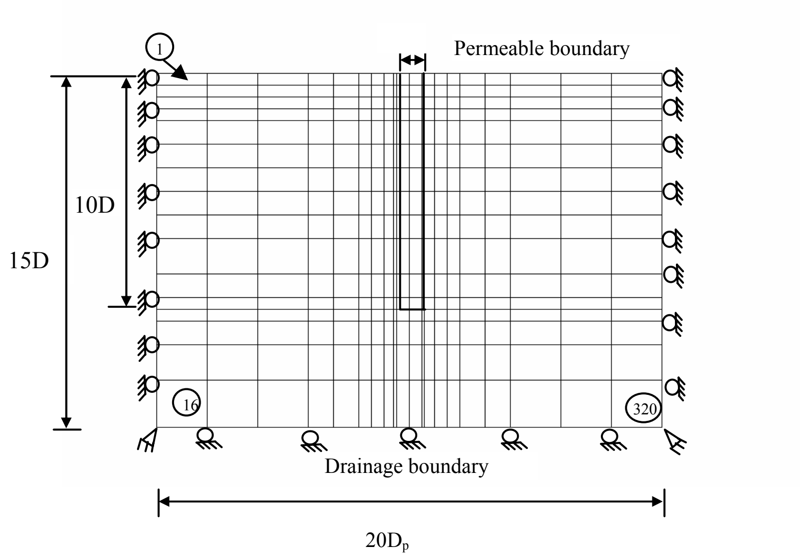

Figure- 4. Finite element mesh.

TIME-DEPENDENT BEHAVIOR OF PILE UNDER LATERAL LOADINGS

The time dependent behavior of a vertical pile embedded in a saturated elasto plastic soil and subjected to a lateral load applied at the mudline, is examined here. This problem was also studied by Carter and Booker (1983) for a consolidating soil with elastic skeleton.

A pile with diameter Dp is embedded in a layer of saturated cohesionless soil which obeys the Mohr-Coulomb failure criterion The friction angle of the soil is assumed to be φ = 30°. The soil is also assumed to have a submerged unit weight of γsub = 0.7 γw, where γw is the unit weight of pore water, a Young's modulus for fully drained conditions given by E's = 3000 γw and a Poisson's ratio v' = 0.30. The initial value of the coefficient of lateral earth pressure is K0 = 0.5. The Young's modulus of the pile material is Ep = 1000 E's. The problem was analyzed assuming elastic as well as elasto-plastic models for the soil. The finite element mesh and dimensions of the problem are defined in Figure-4. All elasto-plastic analyses have been carried out using 8-node quadrilateral finite elements.

To examine the time dependent consolidation behaviour of the pile, it is convenient to introduce a non-dimensional time factor T, defined as

$$T = \frac{k(1-\upsilon'_s)E'_s t}{\gamma_w (1-2\upsilon'_s)(1+\upsilon'_s)D_p^2} \qquad (10)$$

Where k is the coefficient of soil permeability and t represents time.

In the elastoplastic analysis, a horizontal load, H, varied from H = 5γw x D3p to H = 15γw x D3p are applied to the pile head during a total time of T = 0.0001. This loading was maintained constant with time and the analyses were continued, allowing excess pore pressures to dissipate, and thus for the soil to consolidate.

The predicted load-displacement curves for the pile head, for cases where the pile deforms under fully drained state and rapid loading (i.e., undrained) conditions, are presented in Figures 5 and 6. Cases are plotted for the elastoplastic soil model as well as for the elastic soil model. A significant dependence of the response of the pile on the assumed soil model can be observed in the drained case, that the displacement of the pile head predicting using Mohr Coulomb model is about four times that predicted using elastic model. While the response of the pile during rapid loading is almost linear and close to the elastic response of the pile with head displacement about twice that of elastic analysis. Good agreement was observed between present study and Carter and Booker (1984) results.

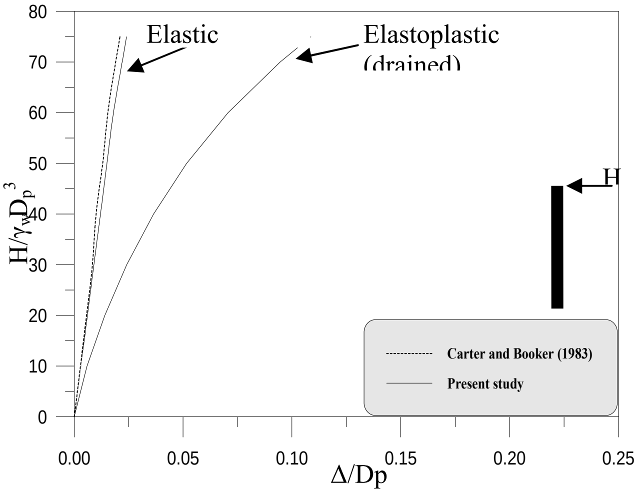

Figure- 5. Comparison of the pile response with different soil models, each deforming under fully drained conditions.

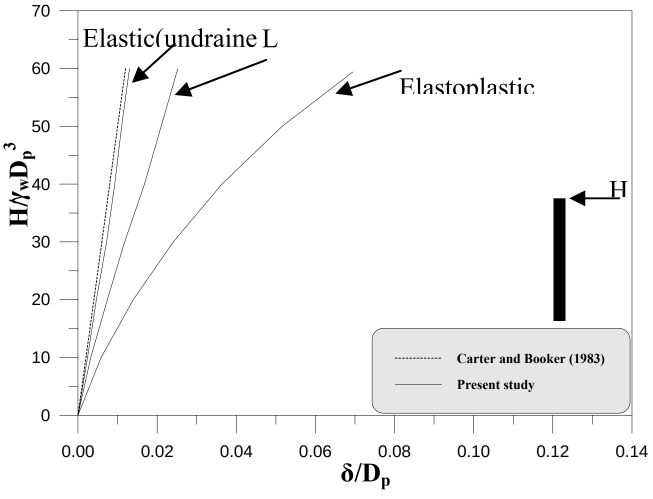

Figure- 6. Lateral displacement relationships for laterally loaded piles under drained and undrained conditions.

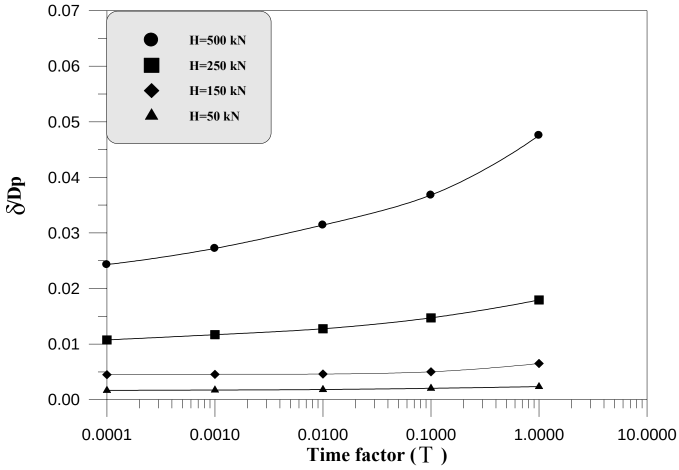

Figure- 7. Comparison of the lateral displacements of the pile head in elastoplastic soils.

The time-dependent lateral displacements of the pile head predicted by the elastoplastic analyses are plotted in Figure-7, for the case where the horizontal load is H = 5, 15, 25 and 50 (γ_wx D^3_p), respectively. Higher displacements were observed with time due to consolidation process and as it can be noticed the higher the lateral load applied the higher the percentage of increasing in displacements with time occurred. that at the early stage of consolidation and at lower lateral load insignificant dissipation of excess pore water pressures were conducted being more noticeable at higher time factors while at higher lateral load the dissipation of the excess pore water pressure is appeared clearly with time from the early stages of consolidation, causing higher effective stresses and thus higher displacements.

Figure-8 shows the lateral deformation of the pile with increasing the lateral load and time and as it can be noticed during the early stage of consolidation and at lower lateral load insignificant increasing in displacements were conducted being more noticeable at higher time factors while the increasing in displacements is appeared clearly with time and from the early stages of consolidation at higher load applied. Also Figure-8 shows that lateral displacements of the pile at time factor (T = 1.0) under lateral load (H = 50 kN) are close to that for time factor (T = 0.1) under lateral load (H = 500 kN). And the point of rotation becomes deeper with increasing the load and the time. These differences are shown clearly in Figure-9 which draws the displacements exaggerated by a factor of 5 and plotted with the original finite element mesh for the lowest to highest lateral load under different time factors.

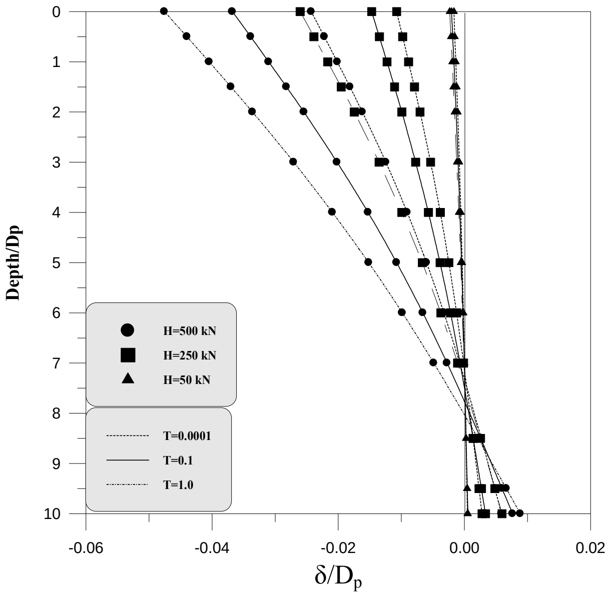

Figure-8. Lateral displacement of pile with depth under lateral loads.

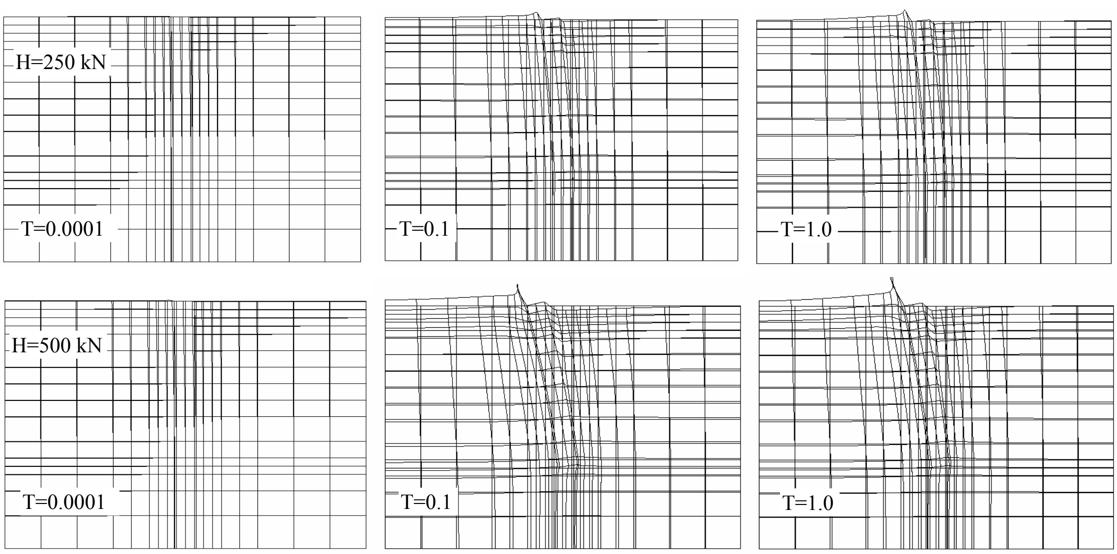

Figure-9. Deformed mesh for the two-dimensional pile problem following the time factors of T = 0.0001, 0.1 and 1.0 (Deformation exaggerated by a factor of 5).

The lateral soil pressures p and shear stresses C in soil resulting from the lateral loads are shown in Figures 10 and 11. It can be seen that the pressure and shear increased with time and depth but in a rate lowered as the lateral load decreased. Also higher values of pressures and shear stresses near the ground surface were predicted, this may be attributed to the boundary effects and as the point of measurement is close to the tip of the pile where the lateral load applied.

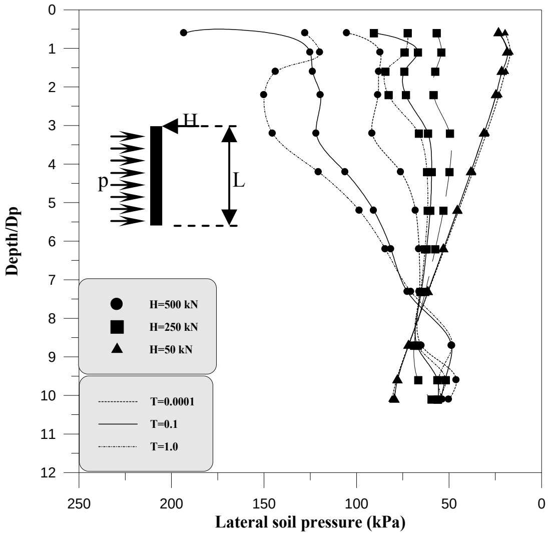

Figure-10. Lateral pressure of soil with depth under different lateral load and time factor.

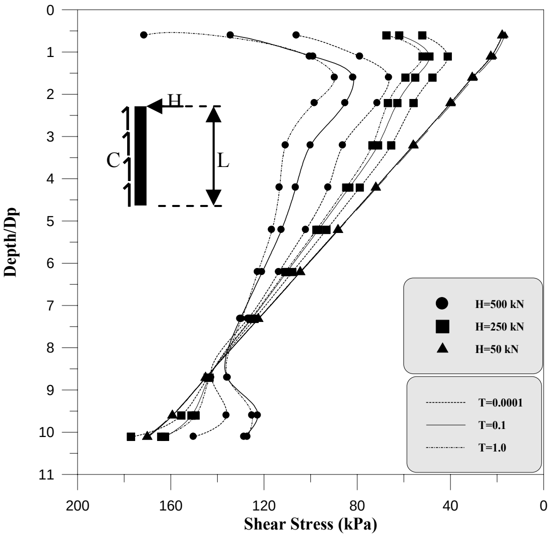

Figure-11. Shear stress of soil with depth under different lateral load and time factor.

CONCLUSIONS

A finite element analysis of the time-dependent behavior of piles subjected to pure lateral loadings in saturated porous medium is carried out in this paper. This was to allow the transition between the states of drained and undrained behavior to be investigated. An algorithm for carrying out such an analysis has been presented. The transient response of the saturated porous media was based on the theory of consolidation developed by Biot (1941). Also it should be emphasized that the results presented herein were based on elastic and elastoplastic soil models. The results of the elastoplastic analyses of pile problem in saturated soil under lateral loadings are presented and the following conclusions were drawn:

- In the drained case a higher dependence of the response of the pile on the assumed soil model can be observed, that the displacement of the pile head predicting using Mohr Coulomb model is about four times that predicted using elastic model, while the response of the pile during rapid loading is almost linear and close to the elastic response of the pile with head displacement about twice that of elastic analysis.
- With time and due to consolidation process higher displacements were observed, also the greater the percentage of increasing in displacements with time occurred as increasing the subjected lateral loading.
- With time the lateral soil pressure and shear stress in soil increased but in a rate lowered as the lateral load decreased.
- Time wise variation of lateral displacement of the pile tip show that at the early stage of consolidation and at lower lateral load insignificant increasing in displacements were conducted being more noticeable at higher time factors while at higher lateral load the increasing in displacements is appeared clearly with time from the early stages of consolidation.

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