PREDICTION OF ELASTIC PROPERTIES OF FRP COMPOSITE LAMINA FOR LONGITUDINAL LOADING

Syed Altaf Hussain, B. Sidda Reddy and V. Nageswara Reddy
Department of Mechanical Engineering, Rajeev Gandhi Memorial College of Engineering and Technology, Nandyal, India
E-Mail: s_a_hussain1@rediffmail.com

ABSTRACT
A structural composite is a material system consisting of two or more phases on a macroscopic scale, whose mechanical performance and properties are designed to be superior to those of constituent materials acting independently. FRP composites are slowly emerging from the realm of advanced materials and are replacing conventional materials in a variety of applications. However, the mechanics of fiber-reinforced composites is complex owing to their anisotropic and heterogeneous characteristics. The mechanical properties \( E_1, E_2, \nu_{12}, \) and \( G_{12} \) are determined for three different types of FRP (Glass/Epoxy composite, Graphite/Epoxy composite and Kevlar/Epoxy composite) unidirectional continuous fiber lamina at different fiber volume fractions using the finite element method. A finite element model incorporating the necessary boundary conditions is developed and is solved using commercially available FEA package. The results are compared with analytical solution possible.

Keywords: model, FRP, micro-mechanics, finite element method.

Notations
- \( E_1 \) Longitudinal Young’s Modulus along
- \( E_2 \) Transverse Young’s Modulus along
- \( G_{12} \) Shear Modulus along
- \( \nu_{12} \) Major Poisons ratio along
- \( \nu_{21} \) Minor Poisons ratio along
- \( V_f \) Volume fraction of fiber
- \( V_m \) Volume fraction of matrix
- \( E_f \) Young’s Modulus of fiber
- \( E_m \) Young’s Modulus of matrix
- \( \eta \) packing factor
- \( \xi \) Reinforcing factor
- \( r \) Radius of fiber
- \( a \) Edge length of square unit cell
- \( G_f \) Shear Modulus of fiber
- \( G_m \) Shear Modulus of matrix

1. INTRODUCTION
A Composite is a material system consisting of two or more phases on a macroscopic scale, whose mechanical performance and properties are designed to be superior to those of constituent materials acting independently. One of the phase is discontinuous, stiffer, and stronger and is called reinforcement. Where the less stiff and weaker phase is continuous and is called matrix. The low density, high strength, high stiffness to weight ratio, excellent durability and design flexibility of fiber-reinforced composite materials are the primary reasons for their extended use.

The properties of a fiber reinforced plastics can be controlled by the appropriate selection of the substrata parameters such as fiber orientation, volume fraction, fiber spacing, and layer sequence. The required directional properties can be achieved in the case of fiber reinforced composites by properly selecting fiber orientation, fiber volume fraction, fiber spacing, and fiber distribution in the matrix and layer sequence. As a result of this, the designer can have a tailor-made material with the desired properties. Such a material design reduces the weight and improves the performance of the composite. For example, the carbon-carbon composites are strong in the direction of the fiber reinforcement but weak in the other direction. A great number of micromechanical models have been proposed in the literature [1-4] for predicting various mechanical properties of composite materials. Several other models have been proposed such as numerical homogenization [5], FEM [6, 7] among others.

In this paper the finite element method is adopted for predicting various engineering properties of a FRP lamina and the results of \( E_1, E_2, \nu_{12}, \) and \( G_{12} \) are compared with the rule of mixtures and Halpin-Tsai criteria.

2. METHODOLOGY
The present research work deals with the evaluation of engineering properties by the elasticity theory based finite element analysis of representative volume elements of fiber-reinforced composites (square unit cell)
2.1 Numerical solution

Finite element method is an approximate numerical method which has been successfully used for solutions of problems in various fields, including solid mechanics, fluid mechanics and heat transfer. In the present work, the computational numerical analysis is done using ANSYS version 7.1 running on a Pentium IV system.

Assumptions made for the present analysis were

- Fibers are uniformly distributed in the matrix;
- Fibers are perfectly aligned;
- There is perfect bonding between fibers and matrix;
- The composite lamina is free of voids and other irregularities; and
- The load is within the linear elastic limit.

2.2 Finite element model

The 1-2-3 coordinate system shown in Figure-2 is used to study the behavior of a unit cell (The direction 1 is along the fiber axis and normal to the plane of the 2D plane given in (Figures 1 and 2). It is assumed that the geometry, material and loading of the unit cell are symmetrical with respect to 1-2-3 coordinate system. Therefore, a one fourth portion of the unit cell is modeled Figure-3 for the prediction of mechanical properties. The 3D Finite Element mesh on one fourth portion of the unit cell is shown in Figure-4.

2.2.1 Element type

The element SOLID95 of ANSYS V10.0 used for the present analysis is based on a general 3D state of stress and is suited for modeling 3D solid structure under 3D loading. The element has 20 nodes with three degrees of freedom per node (UX, UY and UZ).

2.2.2 Geometry

The dimensions of the finite element model are taken as X = 200 units Y = 100 units Z = 100 units. The radius of the fibre is varied corresponding to the volume fraction.

\[
V_f = \frac{\text{C/S Area of fibre}}{\text{C/S Area of unit cell}}
\]

\[
V_f = \frac{\pi r^2}{a^2}
\]  \hspace{1cm} (1)

where

- \(V_f\) = Volume fraction of fibre
- \(r\) = Radius of fibre
- \(a\) = Edge length of square unit cell

2.2.3 Boundary conditions

Due to the symmetry of the problem the following symmetric boundary conditions are used

At \(x = 0\), \(U_x = 0\)

At \(y = 0\), \(U_y = 0\)

2.3 Analytical solution

The mechanical properties of the lamina are calculated using the following expressions of Theory of elasticity approach and Halpin - Tsai’s formulae.

Young’s Modulus in the fiber direction and transverse direction

\[
E_1 = \frac{\sigma_1}{\varepsilon_1}
\]

\[
E_2 = \frac{\sigma_2}{\varepsilon_2}
\]  \hspace{1cm} (2)

(3)

Major Poison’s ratio

\[
\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}
\]  \hspace{1cm} (4)

where

- \(\sigma_1\) = Stress in x-direction
- \(\varepsilon_1\) = Strain in x-direction
- \(\sigma_2\) = Stress in y-direction
- \(\varepsilon_2\) = Strain in y-direction
2.3.1 Rule of mixtures

Longitudinal Young’s Modulus: \( E_1 = E_f V_f + E_m V_m \)  
(5)

Transverse Young’s Modulus: \( E_2 = E_f V_f + E_m V_m \)  
(6)

Poisson’s Ratio: \( \nu_{12} = \nu_f V_f + \nu_m V_m \)  
(7)

In-Plane Shear Modulus: \( G_{12} = G_f V_f + G_m V_m \)  
(8)

2.3.2 Semi-empirical model (Halpin-Tsai’s)

The values obtained for transverse young’s modulus and in-plane shear modulus through equations (6) and (8) do not agree with the experimental results. This establishes the need for better Modeling techniques, which include finite element method, finite difference method and boundary element methods. Unfortunately, these models are available for complicated equations. Due to this semi-empirical models have been developed for the design purposes. The most useful of these semi-empirical models includes those of Halpin and Tsai, since they can be used over a wide range of elastic properties and fiber volume fractions.

Halpin and Tsai developed their models as simple equations by curve fitting to results that are based on elasticity. The equations are semi-empirical in nature since involved parameters in the curve fitting carry physical meaning.

Longitudinal Young’s Modulus

The Halpin-Tsai equation for the longitudinal Young’s modulus is the same as that obtained through the strength of materials approach, that is,

\[ E_1 = E_f V_f + E_m V_m \]  
(9)

Transverse Young’s Modulus

The transverse Young’s modulus, \( E_2 \), is given by

\[ E_2/E_m = (1+\xi\eta V_f)/(1-\eta V_f) \]  
(10)

where

\[ \eta = ((E_f/E_m)-1)/((E_f/E_m) + \xi) \]  
(11)

The term \( \xi \) is called the reinforcing factor and depends on the following:

- Fiber geometry
- Packing geometry
- Loading conditions

Major Poisson’s Ratio

The Halpin and Tsai equation for the Major Poisson’s ratio is the same as that obtained using the strength of materials approach, that is,

\[ \nu_{12} = \nu_f V_f + \nu_m V_m \]  
(12)

In-Plane Shear Modulus

The Halpin and Tsai equation for the in-plane shear Modulus \( G_{12} \) is

\[ G_{12}/G_m = (1+\xi\eta V_f)/(1-\eta V_f) \]  
(13)

where

\[ \eta = ((G_f/G_m)-1)/((G_f/G_m) + \xi) \]  
(14)

The value of reinforcing factor \( \xi \) depends on fibre geometry, packing geometry and loading conditions.

2.3.3. Materials

Three different types of composite materials are taken for the analysis.

- Glass/ Epoxy composite
- Graphite/ Epoxy composite
- Kevlar/Epoxy composite

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Graphite</th>
<th>Glass</th>
<th>Aramid(or)Kevlar</th>
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</thead>
<tbody>
<tr>
<td>Axial modulus</td>
<td>Gpa</td>
<td>230</td>
<td>85</td>
<td>124</td>
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<tr>
<td>Transverse modulus</td>
<td>Gpa</td>
<td>22</td>
<td>85</td>
<td>8</td>
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<tr>
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<td>-</td>
<td>0.30</td>
<td>0.2</td>
<td>0.36</td>
</tr>
<tr>
<td>Transverse Poisson’s ratio</td>
<td>-</td>
<td>0.35</td>
<td>0.2</td>
<td>0.37</td>
</tr>
<tr>
<td>Axial shear modulus</td>
<td>Gpa</td>
<td>22</td>
<td>35.42</td>
<td>3</td>
</tr>
<tr>
<td>Axial coefficient of thermal expansion</td>
<td>( \mu m/\mu oc )</td>
<td>-1.3</td>
<td>5</td>
<td>-5</td>
</tr>
<tr>
<td>Transverse coefficient of thermal expansion</td>
<td>( \mu m/\mu oc )</td>
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<td>5</td>
<td>4.1</td>
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<tr>
<td>Axial tensile strength</td>
<td>Mpa</td>
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<td>1550</td>
<td>1379</td>
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<tr>
<td>Axial compressive strength</td>
<td>MPa</td>
<td>1999</td>
<td>1550</td>
<td>276</td>
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<tr>
<td>Transverse tensile strength</td>
<td>MPa</td>
<td>77</td>
<td>1550</td>
<td>7</td>
</tr>
<tr>
<td>Transverse compressive</td>
<td>MPa</td>
<td>42</td>
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<td>7</td>
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<tr>
<td>Shear strength</td>
<td>MPa</td>
<td>36</td>
<td>35</td>
<td>21</td>
</tr>
<tr>
<td>Specific gravity</td>
<td></td>
<td>1.8</td>
<td>2.5</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Table-2. Typical properties of matrices (SI system of units).

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Epoxy</th>
<th>Aluminium</th>
<th>Polyamide</th>
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<tr>
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<td>Gpa</td>
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<td>71</td>
<td>3.5</td>
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<tr>
<td>Transverse modulus</td>
<td>Gpa</td>
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<td>71</td>
<td>3.5</td>
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<tr>
<td>Axial Poisson’s ratio</td>
<td></td>
<td>.3</td>
<td>.3</td>
<td>.35</td>
</tr>
<tr>
<td>Transverse Poisson’s ratio</td>
<td></td>
<td>.3</td>
<td>.3</td>
<td>.35</td>
</tr>
<tr>
<td>Axial shear modulus</td>
<td>Gpa</td>
<td>1.308</td>
<td>27</td>
<td>1.3</td>
</tr>
<tr>
<td>Axial coefficient of thermal expansion</td>
<td>µm/m/°c</td>
<td>63</td>
<td>23</td>
<td>90</td>
</tr>
<tr>
<td>Transverse coefficient of thermal expansion</td>
<td>µm/m/°c</td>
<td>0.33</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>Axial tensile strength</td>
<td>Mpa</td>
<td>72</td>
<td>276</td>
<td>54</td>
</tr>
<tr>
<td>Axial compressive strength</td>
<td>Mpa</td>
<td>102</td>
<td>276</td>
<td>108</td>
</tr>
<tr>
<td>Transverse tensile strength</td>
<td>Mpa</td>
<td>72</td>
<td>276</td>
<td>54</td>
</tr>
<tr>
<td>Transverse compressive</td>
<td>Mpa</td>
<td>102</td>
<td>276</td>
<td>108</td>
</tr>
<tr>
<td>Shear strength</td>
<td>Mpa</td>
<td>34</td>
<td>138</td>
<td>54</td>
</tr>
<tr>
<td>Specific gravity</td>
<td></td>
<td>1.2</td>
<td>2.7</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Glass/Epoxy composite

Figure-5. Variation of E1 with fiber volume fraction.

Figure-6. Variation of E2 with fiber volume fraction.

Figure-7. Variation of $\nu_{12}$ with fiber volume fraction.

Figure-8. Variation of $G_{12}$ with fiber volume fraction.

Graphite/Epoxy composite.
Figure-9. Variation of $E_1$ with fiber volume fraction.

Figure-10. Variation of $E_2$ with fiber volume fraction.

Figure-11. Variation of $\nu_{12}$ with fiber volume fraction.

Figure-12. Variation of $G_{12}$ with fiber volume fraction after Kevlar/Epoxy composite.

Figure-13. Variation of $E_1$ with fiber volume fraction.

Figure-14. Variation of $E_2$ with fiber volume fraction.

Figure-15. Variation of $\nu_{12}$ with fiber volume fraction.

Figure-16. Variation of $G_{12}$ with fiber volume fraction.
4. ANALYSIS OF RESULTS

Figures 5-16 show the variation of various properties with the fiber volume fraction. The observations made from the plots are:

1. The longitudinal young’s modulus $E_1$ and transverse young’s modulus $E_2$ increases linearly with increase in fiber volume fraction for all the three types of composite materials;
2. The Poisson’s ratio $\nu_{12}$ increase linearly with increase in fiber volume fraction up to 50% followed by Rapid decrease for higher fiber volume fraction;
3. In-plane shear modulus $G_{12}$ gradually increases with increase in fiber Volume fraction up 60% thereafter it increases rapidly for higher fiber volume fraction for all three types of composite materials; and
4. Comparison of elastic properties of three different composite materials reveals that Kevlar/Epoxy composite are stronger than the other two composites.

5. CONCLUSIONS

Several elastic properties of three different FRP composite lamina have been evaluated for various fiber volume fractions with the help of FEA software, ANSYS. The results of elastic moduli $E_1$, $E_2$, $G_{12}$ and $\nu_{12}$ are compared with the results obtained by using the Rule of Mixture and Semi-empirical Model (Halpin- Tsai’s). It is seen that the results from the Finite Element simulation are little bit deviating with the analytical results. Also several properties for which simple and accurate analytical methods are not available are evaluated. Hence finite element method provides with a large property set to perform better analysis for fiber reinforced composite materials.

REFERENCES


