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FINITE ELEMENT ANALYSIS OF CONSOLIDATION PROBLEM IN SEVERAL TYPES OF COHESIVE SOILS USING THE BOUNDING SURFACE MODEL

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ABSTRACT

Finite element analyses of consolidation problem in several types of saturated cohesive soils were performed using the elastoplastic bounding surface model. In this paper, the model and the finite element formulation were described and examples of model prediction and accuracy of the finite element formulation were given. The transient response of the saturated porous media is based on Biot's theory of consolidation. Transient analysis of a two-dimensional consolidation problem involving a flexible strip footing on a clay layer of finite thickness is then carried out which demonstrate the effects of consolidation process and model parameters on the pore pressure response and ground movements under the strip footing.

Keywords: model, finite element, consolidation, bounding surface, strip footing.

INTRODUCTION

The deformation and pore-water pressure responses of clayey soils are of great interest to civil engineers. The stability of foundations and earthworks in saturated fine-grained soils is a time-dependent process. This is because any change in total normal stress is initially resisted by pore pressures, which then dissipates over a period of time. In general, it is difficult to accurately predict or back-compute the pore-water pressure responses in clays in fields, especially in some cases in which, after the completion of structure construction, the pore-water pressure in the foundation soils continuously increased for a certain period of time (Kabbaj *el al.*, 1988).

Consolidation of soils has been an important subject studied for more than 5 decades. The theory of 3D consolidation was first formulated by Biot (1941). In special cases, such as strip, axisymmetric, or square footings with uniform load intensity resting on linear elastic porous material, analytic solutions have been found (Schiffman *et al.*, 1969). However, if the material is considered to be nonlinear elastic or elastic plastic or if the boundaries are complicated, numerical methods must be employed to find the solutions (Chang and Duncan, 1983). The most widely used method may be the finite-element (FE) method. So far a large number of studies using the FE method with either linear elastic, nonlinear elastic, or elastoplastic models have been performed for consolidation of soils. The FE method is a relatively matured method. The consolidation analysis using elastoplastic models has been an active research area in recent years (Adachi *et al.*, 1996 and Taiebat and Carter, 2001). In this paper, the consolidation behavior of a soft soil under strip footing is analyzed using the FE method with a bounding surface model which is one of the most sophisticated soil constitutive relations to model the behavior of soil. The model predicts amongst others the stress-strain relations and pore pressure changes when the soil is subjected to external loads as it has a prominent feature that inelastic deformations can occur for stress points within the surface.

In the following sections, the proposed model and the finite element formulation are described and examples of model prediction and accuracy of the finite element formulation are given. The consolidation behavior of a soft soil under a flexible strip footing in different cohesive soils is then studied using the model in order to show the influence of the consolidation process and model parameters on the behavior.

THE ELASTOPLASTIC BOUNDING SURFACE MODEL

Details of the elastoplastic formulation, the numerical implementation of the model and the parameters associated with the model are available elsewhere (Dafalias and Herrmann, 1986 and Herrmann *et al.*, 1987). Therefore, only the elastoplastic rate relations are given here.

The total strain rate is consisting of two parts: elastic strain and plastic strain:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{p}_{ij} \tag{1}$$

The inverse form of the constitutive relations is obtained as:

$$\dot{\sigma}_{ij} = D_{ijkl}\dot{\varepsilon}_{kl} \tag{2}$$

$$D_{ijkl} = G\left(\delta_{ki}\delta_{ij} + \delta_{kj}\delta_{ii}\right) + \left(K - (2/3)G\right)\delta_{ij}\delta_{kl}$$

$$\left[3KF_{,\bar{I}}\delta_{ij} + \frac{G}{J}F_{,\bar{J}}\delta_{ij} + \frac{\sqrt{3}G}{\cos 3\alpha}\frac{F_{,\alpha}}{bJ}\left(\frac{s_{in}s_{nj}}{J^2} - \frac{3S^3 s_{ij}}{2J^4} - \frac{2\delta_{ij}}{3}\right)\right]$$

$$-\frac{\bar{h}\langle L\rangle}{B}\left[3KF_{,\bar{I}}\delta_{ij} + \frac{G}{J}F_{,\bar{J}}\delta_{ij} + \frac{\sqrt{3}G}{\cos 3\alpha}\frac{F_{,\alpha}}{bJ}\left(\frac{s_{in}s_{nj}}{J^2} - \frac{3S^3 s_{ij}}{2J^4} - \frac{2\delta_{ij}}{3}\right)\right] \tag{3}$$

Where

$$L = \frac{1}{B}\left\{3KF_{,\bar{I}}\dot{\varepsilon}_{kk} + \frac{G}{J}F_{,\bar{J}}s_{ij}\dot{\varepsilon}_{ij}\right.$$
$$\left. + \frac{\sqrt{3}G}{\cos 3\alpha}\frac{F_{,\alpha}}{bJ}\left[\left(\frac{s_{ik}s_{kj}}{J^2} - \frac{3S^3 s_{ij}}{2J^4}\right)\dot{\varepsilon}_{ij} - \frac{2\dot{\varepsilon}_{kk}}{3}\right]\right\} \quad (4a)$$

$$B = K_p + 9K\left(F_{,\bar{I}}\right)^2 + G\left(F_{,\bar{J}}\right)^2 + G\left(F_{,\alpha}/bJ\right)^2 \quad (4b)$$

and where K and G represent the elastic bulk and shear moduli, respectively, δ_{ij} is the Kronecker delta, K_p the plastic modulus, I and J are the stress invariants, $1 \le b \le \infty$ and F represents the analytical expression of the bounding surface.

Required model parameters

The parameters in this category are determined from results of standard laboratory tests of short enough duration to ensure that viscoplastic effects are negligible. The material parameters used to operate the elastoplastic bounding surface model are (Kaliakin, 2005):

- $\lambda =$ slope of consolidation line,
- $\kappa =$ slope of swelling line,
- $N(\alpha) =$ slope of critical state line, $N_c = N$ in compression, $N_e = N$ in extension,
- $\upsilon =$ Poisson's ratio
- $R(\alpha) =$ $R > 1$ defines the point $I_1 = I_o / R$ (Figure-1), which together with point J_1 define the coordinates of point H which is the intersection of $F = 0$ and CSL, $R_c = R$ in compression, $R_e = R$ in extension,
- $A(\alpha) =$ parameter defines the distance $D = AI_o$ of apex H of the hyperbola from its center G intersection of the two asymptotes and thus pertains only to the composite form of the surface, $A_c = A$ in compression, $A_e = A$ in extension,
- $T =$ I_t / I_o parameter which determines the purely tensile strength of the material, and T is also pertains to the composite form of the surface, $C = 0 \le C < 1$ parameter which determines the center of the bounding surface $I_c = CI_o$.
- $s_p =$ parameter which determines indirectly "elastic nucleus". For $s_p = 1$ the elastic nucleus degenerates to point I_c center of bounding surface and as $s_p \to \infty$ the elastic nucleus expand towards the bounding surface.
- $h =$ slope-hardening factor, which is a function of lode angle (α), $h_c =$ for compression $(h_c = h(\pi/6))$, $h_e =$ for extension $(h_e = h(-\pi/6))$.

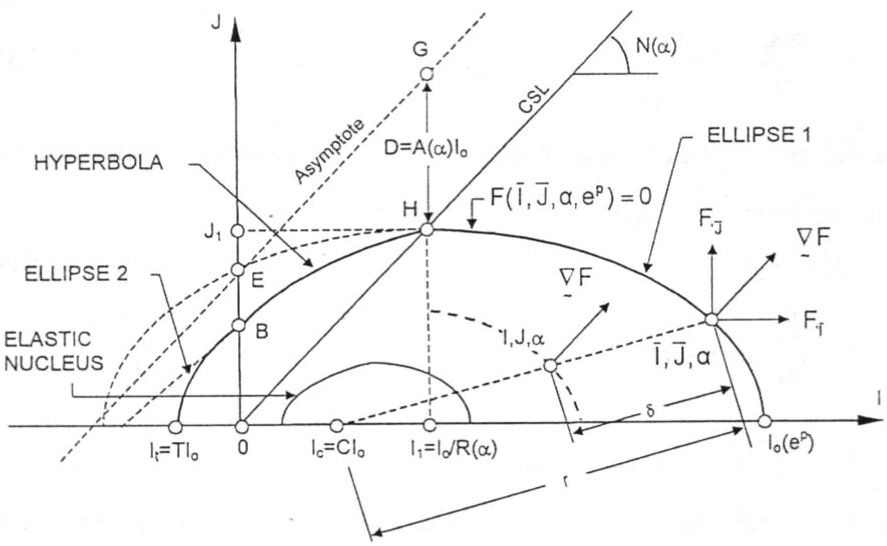

Figure-1. The bounding surface in the stress invariants pace (after Dafalias and Herrmann, 1986).

FINITE ELEMENT FORMULATION

The elastoplastic bounding surface model described above is incorporated in a finite element program, which has the feature of modeling two-dimensional (plane strain and axisymmetric) geotechnical problems such as consolidation, written by FORTRAN90 language. This program is primarily based on the programs presented by Smith and Griffiths (2004) for the analysis of one and a two-dimensional solid by finite element method utilizing elastic constitutive relationship and which is modified for the purpose of this study. So in addition to the elastoplastic bounding surface model, the program allows one to assign linear elastic behavior to any part of the problem geometry. Description of all of the program features is beyond the scope of this paper, and a brief summary of the feature relevant to this study is given below.

Transient formulation

In the case of a strip footing on saturated porous medium, the loading is time-dependent, so an incremental formulation was used in the following work producing the matrix version of the Biot equation at the element level presented below (Lewis and Schrefler, 1987).

$$\begin{bmatrix} K & L \\ L^T & S+\bar{\alpha}H\Delta t_k \end{bmatrix}\begin{Bmatrix} \bar{u} \\ \bar{p} \end{Bmatrix} = \begin{bmatrix} K & L \\ L^T & S-(1-\bar{\alpha})H\Delta t_k \end{bmatrix}\begin{Bmatrix} \bar{u} \\ \bar{p} \end{Bmatrix} + \begin{Bmatrix} dF/dt + C \\ \bar{F} \end{Bmatrix} \tag{5}$$

where: $K =$ element solid stiffness matrix, $L =$ element coupling matrix, $H =$ element fluid stiffness matrix, $\bar{u} =$ change in nodal displacements, $\bar{p} =$ change in nodal excess pore-pressures, $S =$ the compressibility matrix, $\bar{F} =$ load vector, $\Delta t =$ calculation time step, $\bar{\alpha} =$ time stepping parameter (= 1 in this work), $dF/dt =$ change in nodal forces.

VERIFICATION PROBLEMS

Consolidated undrained triaxial compression for normally consolidated clay

This problem has been drawn from Herrmann *et al.*, (1981), as reported by Dafalias and Herrmann (1986) and Kaliakin and Dafalias (1989). A laboratory prepared Kaolin clay was tested and the measured data are taken from the latter authors, whereas the composite bounding surface model parameters are taken from the former. The values of parameters are listed in Table-1.

The model behavior against the experimental data is illustrated in Figures 2a and 2b.The results of the above problem support the verification process of the used program, and indicate that the model successfully predicts results for soils under compression loadings.

Table-1. Bounding surface parameters value.

Parameters	Value	Parameters	Value
λ	0.15	A_e	____ [a]
κ	0.018	C	0.7
υ	0.3	s_p	1.0
M_c	1.25	h_c	50.0
M_e	____ [a]	h_e	____ [a]
R_c	2.5	T	-0.1
R_e	____ [a]	a	____ [b]
A_c	0.02	w	____ [b]

[a] Material response in extension was not simulated

[b] A bounding surface consisting of two ellipses

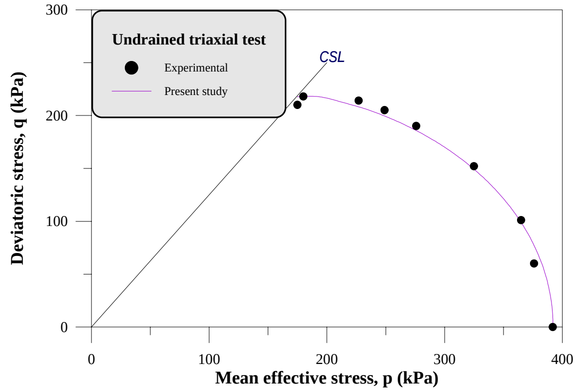

Figure-2a. Shear stress-strain curve for undrained triaxial compression of normally consolidated clay.

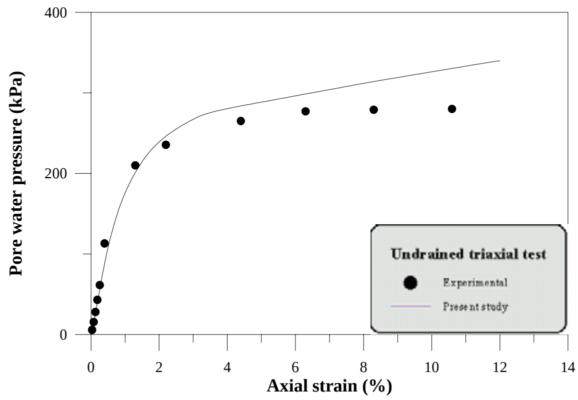

Figure-2b. Pore pressure-axial strain curve for undrained triaxial compression of normally consolidated clay.

Elastoplastic analysis of two-dimensional consolidation problem

Figure-3 shows the finite element mesh used; the width of the loaded area, b, is assumed equal to (3.05m). The problem is solved using the input material parameters shown in Table-2.

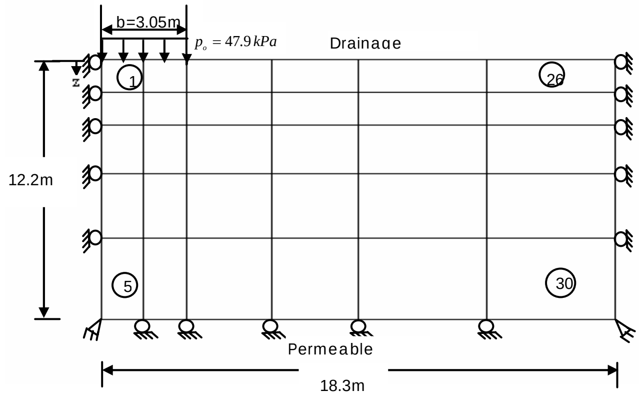

Figure-3. Finite element mesh for the two-dimensional consolidation problem.

The classical material parameters, which have been used by Siriwardane and Desai (1981), are taken as the same as reported by them. The other parameters are taken as

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typical values in the more limited ranges for practical applications (Kaliakin, 2005). The load applied at 25days or time factor of $T_v = 0.07$.

Table-2. Bounding surface parameters (after Kaliakin and Dafalias, 1991 and Siriwadane and Desai, 1981).

Parameters	Value	Parameters	Value
λ	0.14	A_e	0.08
κ	0.05	C	0.4
υ	0.4	s_p	1.0
M_c	1.05	h_c	4.0
M_e	0.89	h_e	4.0
R_c	2.72	h_o	4.0
R_e	2.18	m	0.02
A_c	0.1	$k_v = k_h$	1.22×10^{-5} m / day

Figure-4 shows time wise variation of surface settlements, using the modified Cam-clay and bounding surface models. It can be seen that settlement values obtained by the two models do not differ significantly at the early stage of time levels. However, at later times the bounding surface plasticity results show higher settlements but a smaller final settlement.

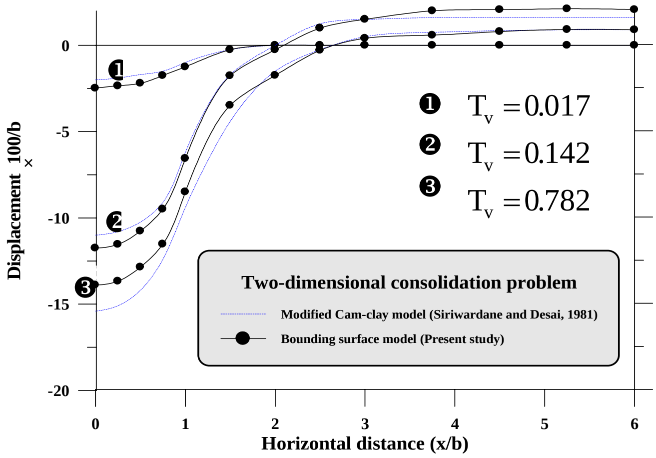

Figure-4. Surface settlements versus horizontal distance.

TIME-DEPENDENT BEHAVIOR OF A SOFT SOIL UNDER A FLEXIBLE STRIP FOOTING

Transient analysis of a two-dimensional consolidation problem involving a flexible strip footing on a clay layer of finite thickness is studied in this section using the bounding surface model. The finite element mesh representing the problem is illustrated in Figure-3. This is the same problem previously considered to show the ability of the bounding surface model to solve the consolidation problems but with different parameters of the model according to the type of cohesive soil (Kaliakin and Dafalias, 1991). The parameters are tabulated in Table-3, where the parameters S_p, a and w are fixed for all the types of soils as 1, 1.2 and 5, respectively. The loading will be applied in 25days or time factor of $T_v = 0.07$.

The results were obtained at the end of loading when the time factor $T_v = 0.07$ and after 100 days or time factor $T_v = 0.278$ .In the following sections, analyses are carried out in order to study the effects of the consolidation process and the bounding surface model parameters on the pore pressure response and ground movements under the strip footing.

In general, all types showed the same behavior but with a relative changes. This may be due to the variation of the parameters according to the type of cohesive soil.

The displacements during consolidation under the strip load $47.9\,kN/m^2$ are shown in Figures 5 and 7, exaggerated by a factor of 5 and plotted with the original finite element array. At all times the settlements are bowl-shaped and the initial displacements involve downward motion under the load and a general horizontal displacement away from the loaded area. The upward motion at the surface just outside the loaded area is increased somewhat by the rigid lateral boundaries. During consolidation the material settles further under the load and moves horizontally toward the load as the excess pore pressures dissipate with time and as shown in Figures-6 and 8, which draws the contours of excess pore pressure at the beginning and during the consolidation process. Figure-6 shows the excess pore water pressure contours at the end of applying strip loading for the five cohesive soils. It should be observed that all of pore pressures are positive reflecting the loading caused by strip footing and the largest pore water pressure lies directly below the bottom of footing where the loading is concentrated. These excess pore pressures dissipated with time as due to water flow away from the loading and as shown in Figure-8, which draws the excess pore pressure contours for the five cohesive soils and at time factor $T_v = 0.278$ causing larger effective stresses and thus increasing the displacements.

Also from Figures 5 and 7, in general it was observed that for cohesive soils that have higher values of the model parameters λ and κ and lower values of M, ν, R, h, A and C, higher surface settlements were predicted especially downward motion under the load and a general horizontal displacement away from the loaded area. Also, lower displacements under the strip load were predicted for Kaolin Mix that has the lowest values of λ and κ , and the highest values of M, ν, R, h and A with a larger amount of the projection center C.

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Table-3. Bounding surface parameters for the five cohesive soils (after Kaliakin and Dafalias, 1991).

Soil type / Proper	Kaolin mix	Kaolin	Marine silty	Grenoble	Umeda
λ	0.075	0.14	0.178	0.2	0.343
κ	0.011	0.05	0.052	0.1	0.105
υ	0.22	0.2	0.2	0.15	0.15
M_c	1.35	1.05	1.07	0.78	0.77
M_e	0.9	0.85	0.79	0.8	0.61
R_c	3.05	2.65	2.2	2.5	2.39
R_e	1.71	2.25	____ [a]	2	____ [a]
A_c	0.18	0.02	0.1	0.02	0.01
A_e	0.15	____ [a]	____ [a]	0.02	____ [a]
C	0.49	0.7	0.4	0.5	0.2
h_c	11	4	10	4.3	2
h_e	9.6	5.6	10	4.3	____ [a]

[a] Material response in extension was not simulated.

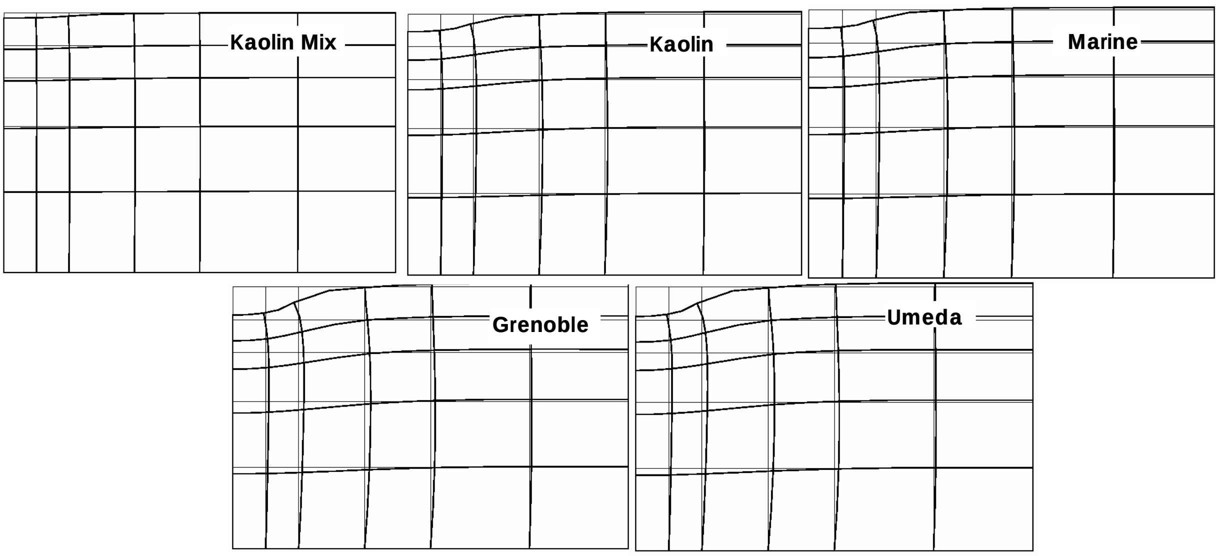

Figure-5. Deformed mesh at the end of loading in the five cohesive soils.

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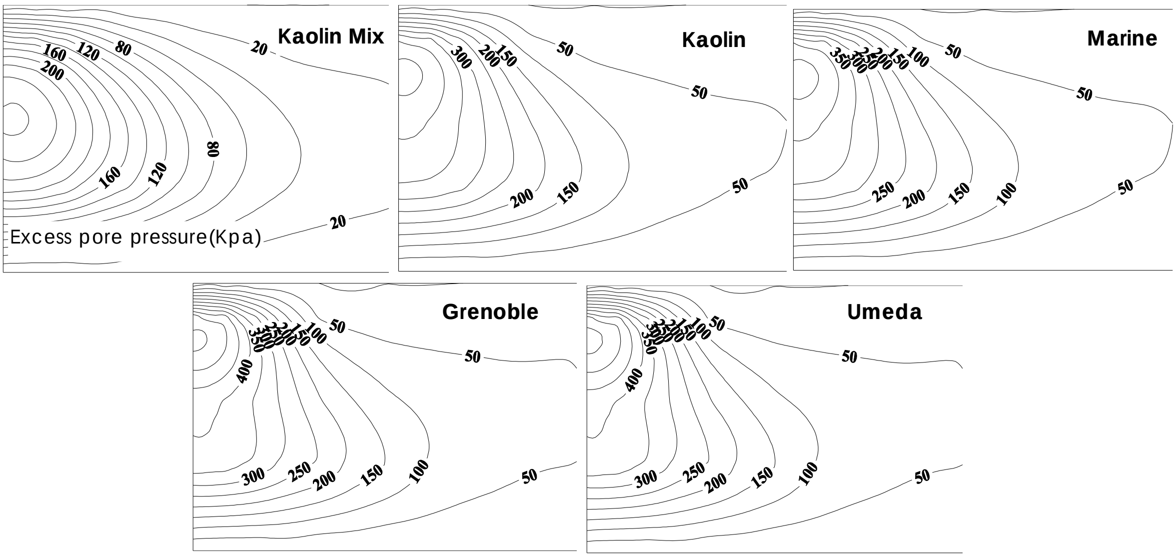

Figure-6. Contours of excess pore pressure at the end of loading in the five cohesive soils.

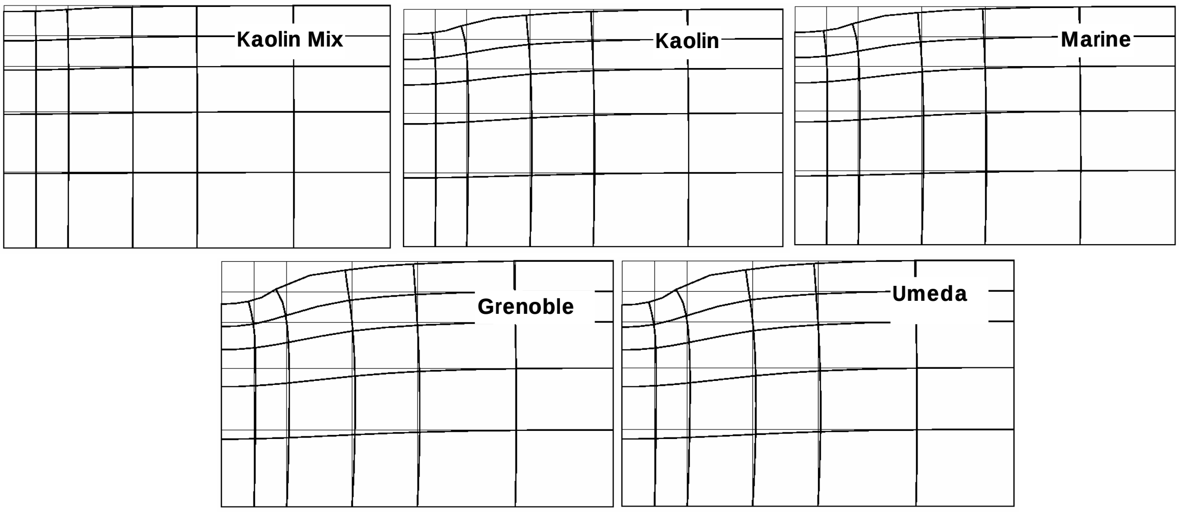

Figure-7. Deformed mesh at time factor (T_v=0.278) in the five cohesive soils.

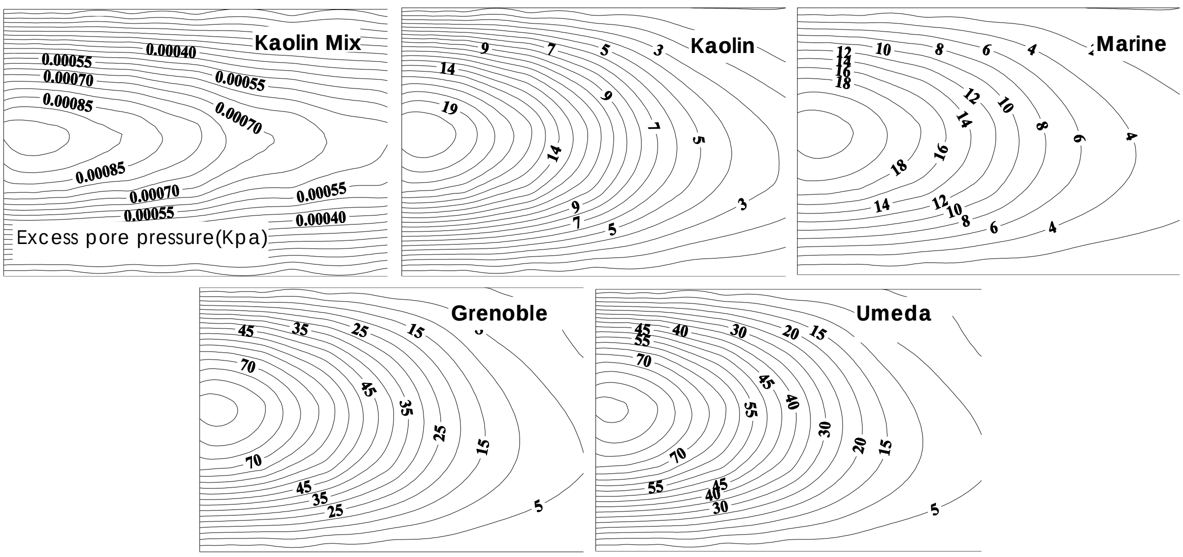

Figure-8. Contours of excess pore pressure at time factor (T_v= 0.278) in the five cohesive soils.

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CONCLUSIONS

A fully transient analysis of consolidation problems in saturated porous media is carried out. This was to allow the transition between the states of drained and undrained behavior to be investigated. An algorithm for carrying out such an analysis has been presented. The transient response of the saturated porous media was based on the theory of consolidation (Biot, 1941). Also it should be emphasized that the results presented herein were based on elastoplastic bounding surface soil model which has the feature that the inelastic deformations occur for stress points within the surface.

The elastoplastic bounding surface model implementation was verified using experimental and numerical results. Then the results of the elastoplastic analyses of consolidation problem involving a flexible strip footing on several cohesive soils are presented. The following conclusions were observed:

- At all times the settlements are bowl-shaped and the initial displacements involve downward motion under the load and a general horizontal displacement away from the loaded area. The upward motion at the surface just outside the loaded area is increased somewhat by the rigid lateral boundaries;
- During consolidation the material settles further under the load and moves horizontally toward the load as the excess pore pressures dissipate with time;
- For cohesive soils that have higher values of the model parameters λ and κ and lower values of M, ν, R, h, A and C, higher surface settlements were predicted especially downward motion under the load and a general horizontal displacement away from the loaded area; and
- Lower displacements under the strip load were predicted for Kaolin Mix that has the lowest values of λ and κ, and the highest values of M, ν, R, h and A with a larger amount of the projection center C.

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