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TRANSVERSE FAR-FIELD DISTRIBUTION IN QUANTUM CASCADE LASER

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ABSTRACT

In this paper, we perform a transverse far-field calculation for a quantum cascade laser treated as a rectangular waveguide. An analytical method for the solution of integral diffraction equation that describes the transverse far-field in a quantum cascade laser is presented. The equations permitting to calculate the full width at half maximum in both directions, parallel and perpendicular to the growth are determined and compared with reported experimental results.

Keywords: quantum cascade laser, Fresnel diffraction, transverse far-field, full width at half maximum.

1. INTRODUCTION

Quantum cascade (QC) lasers [1] are unipolar complex devices in which the laser transition occurs between quantized energy levels within the conduction band. A fundamental property of the QC laser is that the wavelength of light is not controlled by the material band-gap but by the layer thickness. For this reason, the emission wavelength of such a laser can be changed without using different semiconductor materials. Many operating schemes for the QC laser have been proposed, and some of them also realized [2-12]. Since their appearance, QC laser emitting in the mid-to-far infrared opening a new windows for applications such trace gas analysis, free space communications, medical applications, and imaging. In QC lasers the emitted beam is strongly divergent. Therefore, study of the transverse field is important to asses the applicability of this type of light. The theory of Fresnel diffraction in optics at plane apertures was developed long ago [13]. In this paper, we present a simple method to study the transverse far-field distribution in QC laser.

2. FORMULATION OF PROBLEM

We consider a cleaved mirror of QC laser with width W and height l, as shown in Figure-1, and we calculate the transverse field $U(\xi,\eta)$ at point P of coordinates (ξ,η) for different distances z' from the cleaved mirror by applying the Fresnel two-dimensional integral to the transverse field $u(x,y)$ inside the cleaved mirror (here l depends on the number of stages of QC laser and x parallel to the growth direction).

$$U(\xi,\eta)=\frac{e^{ikz'}}{i\lambda z'}e^{\frac{i\pi}{\lambda z'}(\xi^2+\eta^2)}\int_{-\frac{W}{2}}^{\frac{W}{2}}\int_{-\frac{l}{2}}^{\frac{l}{2}}u(x,y)e^{\frac{\pi i}{\lambda z'}(x^2+y^2)}e^{\frac{-2i\pi}{\lambda z'}(\xi x+\eta y)}dxdy \quad (1)$$

In above equation, λ is the wavelength of laser light and $k=2\pi/\lambda$ is the wave vector.

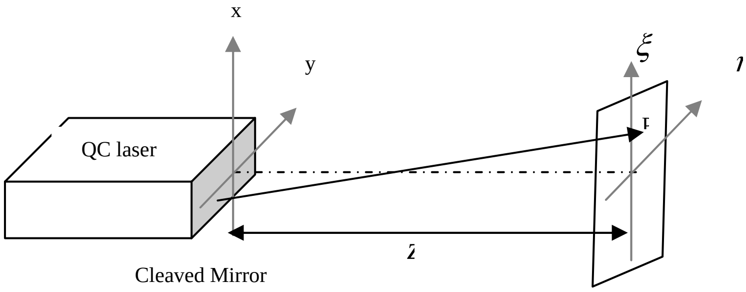

Figure-1. Block diagram for the diffraction of a QC laser by a cleaved mirror.

It was seen in Eq. (1) that, in the region of Fresnel diffraction, the transverse field $U(\xi,\eta)$ at P can be found from a Fourier transform of the product of the cleaved mirror $u(x,y)$ and a quadratic phase function $\exp((i\pi / z'\lambda)(x^2+y^2))$.

If $z' \gg (W^2+l^2)/(4\pi\lambda)$ is satisfied, then the quadratic phase factor under the integral sign in Eq. (1) is approximately unity over the entire aperture, and the transverse field at P can be found directly from a Fraunhofer diffraction. For QC laser, the last condition is verified, and then we can apply the Fraunhofer diffraction in QC laser:

$$U(\xi,\eta) = \frac{e^{ikz'}}{i\lambda z'} e^{\frac{i\pi}{\lambda z'}(\xi^2+\eta^2)} \int_{-\frac{W}{2}}^{\frac{W}{2}} \int_{-\frac{l}{2}}^{\frac{l}{2}} u(x,y) e^{\frac{-2i\pi}{\lambda z'}(\xi x+\eta y)} dxdy \qquad (2)$$

Suppose that a cleaved mirror is normally illuminated by a plane wave of two singles sinusoidal humps amplitude at $z'=0$. The transverse field immediately behind the cleaved mirror is

$$u(x,y,z=0) = u_0 \cos(\frac{\pi x}{l}) \cos(\frac{\pi y}{W}) \, rect(\frac{2x}{l}) \, rect(\frac{2y}{W}) \qquad (3)$$

where u_0 is the maximum amplitude and the amplitude transmittance of cleaved mirror is given by:

$$rect(\chi) = 0 \text{ for} |\chi| \rangle 1 \text{ and } 1 \text{ for } |\chi| \le 1 \qquad (4)$$

Substituting into Eq. (2), we obtain

$$U(\xi,\eta) = \frac{e^{ikz'}}{i\lambda z'} e^{\frac{i\pi}{\lambda z'}(\xi^2+\eta^2)} \iint_{\Omega} u_0 \cos(\frac{\pi x}{l}) \cos(\frac{\pi y}{W}) \, e^{\frac{-2i\pi}{\lambda z'}(\xi x+\eta y)} dxdy \qquad (5)$$

Expression (5) can be separated into the product of two one-dimensional integrals,

$$U(\xi,\eta) = \frac{e^{ikz'}}{i\lambda z'} e^{\frac{i\pi}{\lambda z'}(\xi^2+\eta^2)} u_0 F_1(\xi) F_2(\eta) \qquad (6)$$

where the integrals in Eq. (6) can be solved analytically. This yields for the field in ξ - direction and η -direction, respectively:

$$F_1(\xi) = \int_{-\frac{l}{2}}^{+\frac{l}{2}} \cos(\frac{\pi x}{l}) \, e^{\frac{-2i\pi}{\lambda z'}\xi x} dx = \frac{2l}{\pi} \frac{\cos(\frac{\pi l}{\lambda z'}\xi)}{\left(1-(\frac{2l}{\lambda z'}\xi)^2\right)} \qquad (7.a)$$

$$F_2(\eta) = \int_{-\frac{W}{2}}^{+\frac{W}{2}} \cos(\frac{\pi y}{W}) \, e^{\frac{-2i\pi}{\lambda z'}\eta y} dy = \frac{2W}{\pi} \frac{\cos(\frac{\pi W}{\lambda z'}\eta)}{\left(1-(\frac{2W}{\lambda z'}\eta)^2\right)} \qquad (7.b)$$

Substitution of Eqs. (7.a) and (7.b) in (6) and using the angular coordinates $\theta_x = \xi / z'$ and $\theta_y = \eta / z'$, the Fresnel number $N_x = l^2/(4\lambda z')$ and $N_y = W^2/(4\lambda z')$, in x-direction and y-direction, respectively, yields a complex field distribution

$$U(\theta_x,\theta_y)=u_0\frac{16}{\pi^2}e^{i\pi\left(-\frac{1}{2}+\frac{z'}{\lambda}(2+\theta_x^2+\theta_y^2)\right)}N_xN_y\frac{\cos(\frac{\pi\, l}{\lambda}\theta_x)}{\left(1-(\frac{2l}{\lambda}\theta_x)^2\right)}\frac{\cos(\frac{\pi W}{\lambda}\theta_y)}{\left(1-(\frac{2W}{\lambda}\theta_y)^2\right)} \quad (8)$$

Thus, the transverse intensity $I=UU^*$ at θ_x and θ_y is, in which case, U^* is the conjugate-complex of U .

$$I(\theta_x,\theta_y)=I_0\left(\frac{\cos(\frac{\pi\, l}{\lambda}\theta_x)}{\left(1-(\frac{2\,l}{\lambda}\theta_x)^2\right)}\frac{\cos(\frac{\pi W}{\lambda}\theta_y)}{\left(1-(\frac{2W}{\lambda}\theta_y)^2\right)}\right)^2 \quad (9)$$

where $e^{i\pi(-\frac{1}{2}+\frac{z'}{\lambda}(2+\theta_x^2+\theta_y^2))}$ was suppressed and $I_0=\left(u_0\frac{16}{\pi^2}N_xN_y\right)^2$ being the intensity across the cleaved mirror.

The full width at half maximum (FWHM) diameters of the central lobe of the intensity distribution (9), in x-direction and y-direction, respectively, are calculated from:

$$FWHM_x=2\delta\theta_{x,1/2} \quad (10.a)$$

$$FWHM_y=2\delta\theta_{y,1/2} \quad (10.b)$$

where $\delta\theta_{x,1/2}$ and $\delta\theta_{y,1/2}$ are the points where the intensity is half its maximum value.

The intensity becomes maximum if the conditions $\theta_x=2m\lambda/l$ and $\theta_y=2m\lambda/W$ are satisfied, where *m* is an integer, and the points where the intensity is half its maximum value are at

$$\theta_x=\frac{2m\lambda}{l}+\delta\theta_{x,1/2} \quad (11.a)$$

$$\theta_y=\frac{2m\lambda}{W}+\delta\theta_{y,1/2} \quad (11.b)$$

Thus (9) gives

$$\frac{\cos(\pi\frac{l}{\lambda}\delta\theta_{x,1/2})}{\left(1-(2\frac{l}{\lambda}\delta\theta_{x,1/2})^2\right)}=\frac{\sqrt{2}}{2} \quad (12.a)$$

$$\frac{\cos(\pi\frac{W}{\lambda}\delta\theta_{y,1/2})}{\left(1-(2\frac{W}{\lambda}\delta\theta_{y,1/2})^2\right)}=\frac{\sqrt{2}}{2} \quad (12.b)$$

3. NUMERICAL RESULTS AND DISCUSSIONS

In the following discussion we study the effect of cleaved mirror on the transverse far-field distribution for the structure of QC laser described in [14]. We use in our calculations the parameters taken from [14], $\lambda=4.3$ µm, $l=10$ µm and $W=24$ µm. In Figure-2, we present the transverse far mid-infrared field distribution in both directions, perpendicular and parallel to the growth. In the direction along the waveguide, we observed, due to the wide aperture, a narrow far-field angle of about 12.2° FWHM, whereas in the other, perpendicular direction, the FWHM is 29.3°. These values are in good agreement with measured values reported in [14].

Figure-3 shows the variation of the FWHM as a function of normalized width W (height l) by the wavelength λ. The curves obtained from Eq. (12). The FWHM is inversely proportional to normalized width W (height l). We notice also that this variation has an exponential shape; it presents a maximal value of FWHM for a normalized width W (height l) close to unity and decreases considerably when the width W (height l) increases. The good agreement between theoretical and experimental results reported in [14] achieved in the case of $l/\lambda=10/4.3$ and $W/\lambda=24/4.3$ brings the validity of the proposed model.

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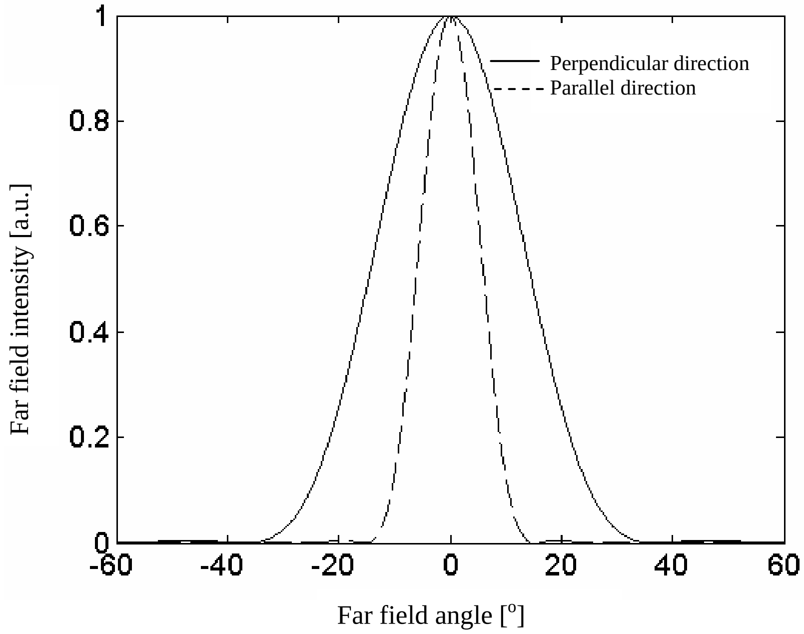

Figure-2. Far field distribution of a QC laser in the directions both parallel and perpendicular to the waveguide.

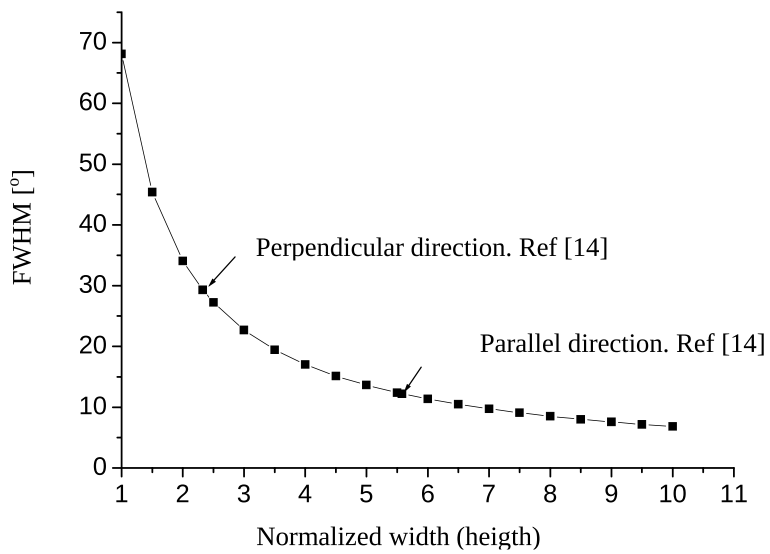

Figure-3. Theoretical variation of FWHM of a QC laser as a function of normalized width (height) by the wavelength λ .

4. CONCLUSIONS

In conclusion, we have proposed a simple compact model in order to calculate the full width at half maximum of the transverse far-field distribution by a cleaved mirror of QC laser. Our model is based on the assumption that the cleaved mirror is normally illuminated by a plane wave of two singles sinusoidal humps amplitude. The good agreement obtained between our model and experiments supports the validity of these assumptions.

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