INNOVATIVE THINNING AND GRADIENT ALGORITHM FOR BINARY AND GREY TONE IMAGES USING FIRST IN FIRST OUT LINEAR DATA STRUCTURE

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ABSTRACT

Homotopic Thinning algorithms have long been investigated in pattern recognition and image analysis. However, they are order dependent in the sense that the output depends upon the order used for processing the image pixels as well as the order in which homotopic structuring elements are applied for algorithms based on sequential homotopic thinning. In the present study an algorithm will be proposed to tackle the disadvantage by introducing an effective notion for order independent homotopic thinning by using First in First out Linear Data Structure. A critical study will be made for analyzing the algorithms by considering appropriate anchor points for skeletonization and by homotopic marking.

Keywords: homotopy, thinning, skeleton, anchor point, crest line, mathematical morphology.

1. INTRODUCTION

In pattern recognition and shape analysis, methods for extracting object features are needed. A classical approach consists in working on a contour representation of the object. Another possibility is to thin the object to a set of lines considering the information of the original object while preserving its homotopy. The resulting thin lines are called the skeleton or medial axis of the input pattern. The corresponding transformations are called skeletonization or medial axis transformation. The detection of end points, multiple points, and closed loops of a skeleton is important for many shape recognition techniques such as those used for Optical Character Recognition.

Several formal definitions for the skeleton of a Euclidean set are available depending on whether wave front propagations, distance transforms, maximal disks, minimal paths, or morphological openings are considered (Soille, 1999). Fortunately, all these approaches, when applied to Euclidean sets, output similar thinned patterns. However, the extension of the notion of skeleton to discrete sets is not straightforward. Indeed, notions such as wave propagations or disks have no direct and unique discrete equivalent. Moreover, a discrete skeletal line is not infinitely thin because it has a thickness of at least 1 pixel. In addition, one-pixel thick skeletal lines cannot be centered in situations where the width of the input pattern corresponds to an even number of pixels. For example, the skeleton of a 2 pixels thick horizontal line can not be both centered and 1 pixel thick. It follows that adaptations of the definitions presented in the continuous space lead to a wide variety of discrete skeletons which do not share the same properties. Accordingly, a vast amount of skeletonization algorithms have been published. A survey on thinning methodologies referring to 138 publications has been produced by Lam et al., (1992). All surveyed algorithms are order dependent in the sense that, in sequential algorithms, the result depends on the order in which pixels are examined, and in parallel algorithms that remove one or two types of border points in each sub-iteration, the resulting skeleton depends on the order of the sub-iterations (Lam et al., 1992, pp. 882). This has motivated the present study, to develop an innovative and simple algorithm avoiding any kind of order dependence. We present an efficient algorithm based on FIFO data structure able to compute order independent anchor skeletons on large data sets. Algorithms for three types of thinning are homotopic marking, notion of background homotopy and anchored skeletonization. In section 2 basic definitions and notations are provided, in section 3 notion of independence, independence w.r.t. a single pixel, strict independence, non strict independence discussed, in section 4 proposed thinning algorithm for order independent homotopic thinning using FIFO data structures, in section 5 order independent grey tone homotopic thinning algorithm, in section 6 implementation and results and concluded in section 7.

2. BASIC DEFINITIONS AND NOTATIONS

The definition of basic concepts such as neighborhood and connectivity, homotopy, discrete images, distances, and introduction to digital topology can be found in (Kong and Rosenfeld, 1989).

2.1 Neighborhood and connectivity

We denote $I_b$ a discrete 2-D binary image, i.e. the value of the image $I_b$ at a given pixel $P$ equals either 0 or 1: $I_b(P)$€\{0,1\}. The letter $\chi$ for denoting connectivity, i.e. $\chi$ equals either 4 or 8 depending on whether we consider the 4- or 8-connectivity. We call $\chi$-path a sequence of pixels such that each pixel of the sequence is a $\chi$-connected neighbor of the next one. The first and last pixels of this sequence are called the end points of the path.
2.2 Homotopy

The adjacency tree (Buneman, 1969) or homotopy tree (Serra, 1982) of a binary image is a graph whose vertices represent the background and foreground connected components, whose root is So and where the children of a vertex V are vertices corresponding to the connected components embedded by and adjacent to V. Two 2-D binary images \( I_{b1} \) and \( I_{b2} \) are homotopic if and if only they have the same homotopy tree (Serra, 1982). A pixel is called deletable (Rosenfeld, 1970) or simple (Kong, 1995) if and only if the image obtained by removing this pixel by changing from 1 to 0 is homotopic to original image. Depending on whether a pixel is simple or not, the simpleness test will return 1 or 0.

3. BASIC CONCEPTS OF DEPENDENCE/INDEPENDENCE ON THINNING ALGORITHM

3.1 Dependence and independence

An Order Dependent homotopic thinning algorithm applied to a binary image \( I_{b} \) is as follows:
For each pixel P of \( I_{b} \)
If P is simple in \( I_{b} \)
Add P to the set Simpleset of simple pixels
For each pixel P\( \in \) Simpleset (Raster scan order)
If P is simple in \( I_{b} \)
\( I_{b}(p) = 0 \)
This algorithm will be referred as Ordep1; the input image \( I_{b} \) contains a thinned image which is homotopic to the original image. The output depends on the scanning order of the set of simple pixels.

An order independent homotopic thinning algorithm can be obtained by setting to 0 those pixels of the original image which are set to 0 by Ordep1 whatever the considered scanning order. We call these pixels the order independent simple pixels and they are said to be independent.

3.2 Order independence w. r. t. a pixel

In the present study has defined a simple pixel P is independent of a simple pixel Q1 if it has one of the following configurations.

In Figure-1a simple pixel P is dependent on simple pixel Q1, if it has one of the above configurations. For first two configurations (Figure-1a and Figure-1b) P is simple. This implies that is 8-connected. The other four configurations (Figure-1c to Figure-1f) we denote by P1 and P2 the two pixels with value 1, P is simple; this implies at least one of its 4 neighbors has value 0.

A simple pixel P is independent of a simple pixel Q1 if and only if it has one of the following configurations

\[
\begin{array}{ccc|ccc}
* & Q1 & 1 & Q1 & * & 1 \\
P & P & P & * & 0 & 0 \\
\end{array}
\]

Figure-1a Figure-1b Figure-1c

In Figure-1a simple pixel P is dependent on simple pixel Q1, if it has one of the above configurations. For first two configurations (Figure-1a and Figure-1b) P is simple. This implies that is 8-connected. The other four configurations (Figure-1c to Figure-1f) we denote by P1 and P2 the two pixels with value 1, P is simple; this implies at least one of its 4 neighbors has value 0.

A simple pixel P is independent of a simple pixel Q1 if and only if it has one of the following configurations

\[
\begin{array}{ccc|ccc}
1 & * & 0 & 0 & 1 & * \\
P & Q1 & P & Q1 & * & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Figure-2a Figure-2b Figure-2c

Figure-2. Representation of simple pixel p which is independent of simple pixel Q1.

In Figure-2a simple pixel P is independent of a simple pixel Q1, if and only if it has one of the above configurations. We denote the pixel with * by P1 and the one with value 1 by P2. For all other configurations the simpleness becomes false as soon as Q1 is set to 0.

3.3 Strict independence

We define a strict independence between P and Q1 in the sense that the simpleness test of P is independent of Q1 whatever the values of the other simple neighbors of P. As a consequence, if P is strictly independent of all its simple neighbors’ q1, P will always be removed by Ordep1.

3.4 Non-strict independence

\[
\begin{array}{ccc|ccc}
0 & Q1 & 0 & Q1 & 0 & Q2 \\
P & Q2 & P & Q3 & 0 & 0 \\
\end{array}
\]

Figure-3a Figure-3b Figure-3c
4. ORDER INDEPENDENT HOMOTOPIC THINNING ALGORITHM

4.1 Thinning algorithm properties

This thinning algorithm has the following properties:

- Structuring homotopic elements are not used
- Independent of the order in which the pixels processed
- Outputs a result invariant through \( \pi/2 \) rotation
- Naturally adopted to a parallel implementation
- Polynomial implementation in \( O(n^2) \), \( n \) is no of pixels of input image

4.2 Order independent simple pixels characterization

A simple pixel \( P \) is characterized as follows:

If \( P \) is dependent of one of its simple neighbors, Scanning order exists, \( P \) will not be removed Else if \( P \) is strictly independent of all its simple neighbors, \( P \) will be removed by scanning orders

Else if \( P \) and \( Q_1 \) are not in one of the four configurations Sec 3.4, Exists an order, \( P \) is not removed
Else \( P \) will be removed by all scanning orders.

4.3 Order independent thinning algorithm

The classical order independent thinning algorithm iterated until idempotence as follows:

While Pixels are deleted
Detect the simple pixels
Delete the independent simple pixels

We present an algorithm based on FIFO data structure able to compute order independent anchor skeletons on large data sets. The condition for order independent removable pixels has been proposed above. In this algorithm three types of thinning are described. First based on simple pixels, second on b-simple pixel and third on anchored skeletonization. The consumption time of proposed algorithm is much shorter than the classical algorithm with multiple scans of the whole image. Thinning requires multiple scans of the image to find and remove the appropriate pixels until scalability is reached. The proposed algorithm is applicable to any of the described thinning methods. A two stage iterative process is usually applied. For each iteration, first all removable pixels are initially detected (detection phase) and are subsequently removed (removal phase). Usually in the detection phase, the entire image is scanned to find removable pixels requiring multiple time-consuming scans. However, the full image scans can be performed only once at the beginning to detect an initial set of removable pixels. Full scans are not compulsory because new removable pixels can appear only within the neighborhoods of already modified pixels. So it has been implemented with FIFO data structure.

Proposed thinning algorithm using FIFO data structure

/* input image \( I_b \), temporary image \( I_{mp} \), pixel \( p \), FIFO queues \( Q_r, Q_n \) and \( Q_l \) foreground and background connectivity g.g."; thinning method is Removable( ), modified boolean variable */
For each pixel \( p \) do
Begin
If is Removable\((p) = true \) then add\((p, Q_r)\); \( I_{mp}(p) \leftarrow 1 \)
Else \( I_{mp}(p) \leftarrow 0 \)
End
Do
Begin
Modified \leftarrow false
While empty \((Q_{C_l}, false)\) do
Begin
\( P \leftarrow \) retrieve \((Q_r)_1; \)
\( I_{mp}(p) \leftarrow 0 \);
Modified \leftarrow true;
\( I_b(p) \leftarrow 0 \);
Add \((p, or)_1; \)
End
If (modified = true) then
Begin
While empty \((Q_l, false)\) do
\( P \leftarrow \) retrieve \((Q_r)_1; \)
For all \( q \in Ng \) \( (p) \) do
Begin
If is Removable \((q) \) and \( I_{mp}(q) =0 \) then
Add \((q, Q_{b_l})\); \( I_{mp}(q) \leftarrow 1 \);
End
End
\( Q_c \leftarrow Q_{ma} \)
End
While modified = true;
The algorithm proceeds in two stages. In first stage the entire image is scanned which is removable pixels are inserted in the queue. In the second stage pixels are successively retrieved from the queue and removed from input image \( I_b \) in consecutive iterations. The loop is controlled by modified Boolean variable. The detection and removal phases are performed in two internal loops. At the end of the loop the content of queue \( Q_n \) is copied to the queue \( Q_c \) by exchanging the pointers in practice.

5. CONCEPTS RELATED TO GREY TONE IMAGES AND ALGORITHM

5.1 Definitions and notations

In case of the values of the pixels of a grey tone image is not restricted to \{0, 1\} but is extended to a finite set of non-negative integers i.e. 0 to \( t_{max} \), \( t_{max} \) is the maximum image value. Two grey tone images are homotopic if and only if their cross-sections (Serra, 1982; Bertrand 1997), accordingly a transformation is said to be homotopic if and only if, for every image, the homotopy of the transformed image is identical to that of the input image. A pixel is said to be simple if and only if decreasing its value by 1 is a homotopic transformation. Thus a pixel \( P \) of a grey tone image \( I_g \) is simple if and only if \( P \) is simple in the cross section. The simpleness property of a pixel \( P \) is local in the sense that it is fully defined by the value equal to \( I_g (P) - 1 \), once \( I_g (P) \) has been decreased by 1; \( P \) remains simple in the output image.

5.2 Order independent homotopic algorithm for grey tone images

For each pixel \( P \) of \( I_g \)
If \( P \) is simple in \( I_g \)
Add \( P \) to the set \( \text{Simpleset} \) of simple pixels
For each pixel \( P \in \text{Simpleset} \) (raster scan order)
If \( P \) is simple in \( I_g \)
\( I_g (P) \) assigns to \( \text{max} \{ I_g (P) \} \)

An order independent thinning can be obtained by first applying the above algorithm with all possible sequential scanning orders; second set the value of each pixel \( P \) to the maximum output value at \( P \).

Lower Skeleton of an image \( I_g \) is obtained by thinning \( I_g \) until idempotence. Our thinning can be used to compute a lower skeleton same as binary skeleton, its form is
Repeat
For each pixel \( P \) of \( I_b \)
If \( P \) is independently simple in \( I_b \)
Add \( P \) to \( \text{DeletableSet} \) of deletable pixels
Assign \( I_b^{\text{temp}} \) to \( I_b \)
Until pixel values are not changed
For each pixel \( P \in \text{DeletableSet} \)
\( I_b (P) = \text{max} \{ I_b^{\text{temp}} (P') \} \)

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6. IMPLEMENTATION AND RESULTS

We have introduced a homotopic thinning algorithm which uses linear data structure in the form of a queue which does not require the choice of a family of homotopic structuring elements because, implicitly, it allows for all homotopic structuring elements to be considered simultaneously. In addition the procedure is parallel so that it is independent of the order used for processing the image pixels. All developments extend directly to grey tone images. When performed until idempotence, order homotopic thinning leads to binary and grey tone homotopic marking.

The above algorithm is applied Lincon and Monalisa Images collected from text book of digital image processing by Rafael C. Gonzalez. The Figures 4a, 4b and 4c shows the original images of Monalisa, Lincon and the characters T and I. The Figures 4a.1, 4b.1 and 4c.1 show the borders of the original images, respectively by applying homotopic property.
extracted, in Figure-5.d the resulting anchored skeleton is shown. Compared to homotopic marking in Figure-5.a, the final shape is much better modeled and shown all important elongated details.

![Figure-6.a](image1)

Figure-6.a

![Figure-6.b](image2)

Figure-6.b

![Figure-6.c](image3)

Figure-6.c

Application of the proposed algorithm to compute the skeleton of a Swarnamukhi river network. Figure-6.a represents the input image, Figure-6.b represents homotopic marking and Figure-6.c represents the anchored skeleton. Grey marks relates to water areas, north pointing towards top.

The efficiency of the proposed algorithm also tested. The linear data structure FIFO queue shortens the computing time of the thinning algorithm. In the proposed algorithm, multiple images scanning, where every pixel is checked for its simpleness, is avoided because only those pixels that could possibly become simple are tested once the queue has been initialized. The proposed algorithm was compared to the classical iterative approach where in each iteration the entire image was scanned twice, first when simple pixels were detected and second when they were removed. Depending on the test image size and complexity, the proposed algorithm is much faster than the classical one.

7. CONCLUSIONS

In this paper we proposed an innovative thinning algorithm implemented of order independent by using FIFO linear type of data structure. It speeds up the computation when compared to the classical algorithm, because it scans the whole image only once to initialize the queue and its computations for different connectivity schemes. Time and space complexity confirms the superiority of the proposed algorithm over the other types of implementations.

REFERENCES


