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ROBUST STABILITY AND PERFORMANCE ANALYSIS OF UNSTABLE PROCESS WITH DEAD TIME USING Mu SYNTHESIS

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ABSTRACT

The design of the H^{∞} Controller was done with μ synthesis for the unstable processes with dead time. It is important to look at various issues like disturbance rejection and the robustness of the controller due to the uncertainties present in the system. While designing the controller, the weighing functions are chosen such that the system could meet the performance requirements, such as the peak value of the μ plot should be less than one [1]. The D-K iteration is also used to improve the performance of the H-infinity controller. Once the $\tilde{D}(s)$ is approximated, the plant is scaled appropriately and the H-infinity controller design is synthesized for the scaled plant. This procedure is repeated until; the μ calculation for robust performance yields a value less than 1 for all frequencies. The application of robust control was extended for such processes with uncertainties and disturbance as mentioned in [2, 3, 4, and 5].

1. INTRODUCTION

In most of the process industries, many processes are unstable with many uncertainties adding into the system. Those systems require a controller design, such that it should maintain its robust stability and robust performance even in the perturbed condition. In this paper, distillation column model [8] is considered as the unstable process with dead time for the robust controller design. In the survey many conventional controller designs have been there for the unstable processes. Normally in the conventional controller design, time domain specifications such as setting time, over shoot, performance indices plays a major role. Stability and performance criteria's are analyzed in the robust controller design.

1.1 Survey on conventional controller and robust control for the unstable process

Weidong Zhang *et al.* (1999) in their study [9] on "Quantitative performance design for integrating processes with time delay" discussed the control problem of integrating processes involving time delay and time constant. A novel H-infinity design method is presented, of which one main merit is that it does not need the co-prime factorization of the process and the controller is derived by analytical method instead of by numerical method. They also showed that the error introduced by the rational approximation will not cause stability problem if suitable controller parameter is selected. An important attribution of the proposed design procedure, as outlined, is that it can provide quantitative performance estimation.
E. Poulin and A. Pomerleau (1996) in [10] "PID tuning for integrating and unstable processes" developed a systematic PI and PID tuning method for integrating and unstable processes. The method, based on a maximum peak resonance specification, is graphically supported by the Nichols chart. The PI and PID parameters are adjusted such that the point G_{jw} (max) is tangent to the right-most point of the specified ellipse. For these types of processes, the M_r specification is more representative of the stability of the system and the desired response than phase or gain margins. In the case of unstable processes, stability conditions have been given. Charts that give the optimal specification according to the ITAE criterion for output and input step load disturbances were presented. The tuning method gave good responses for integrating processes and generally better results than those obtained by earlier workers for unstable processes.

Julio E. Normey-Rico *et al.* (1999) in [11] "Robust tuning of dead-time compensators for processes with an integrator and long dead-time" presented a simple criterion for tuning a dead time compensator for plants with an integrator and long dead time. A simple and effective robust tuning of a DTC is proposed. The controller has only three adjustable parameters that can be tuned manually or using some information about the plant and its uncertainties. If the dead time and velocity gain of the process are determined experimentally, then the controller has only one tuning parameter. The proposed controller is compared to others recently presented in the literature showing that it could offer similar robustness and better performance than the others, principally when plants with long dead times are considered.

Roy Smith [2], in their paper discussed about the application of robust control theory for the cart-spring pendulum system with uncertainties and disturbances. The objective is to design a controller that meets the specified robust performance criteria [3]. In the paper the author used the hinfsyn command to find the controller transfer function by defining all the input and output parameters. The partitioned matrix of the plant has to be known before using the hinfsyn command. Controller's robust performance was improved by using the D-K iteration. By this iteration method, plant has been scaled properly and the controller is found for the synthesized plant.

2. ROBUST CONTROL PROBLEM

The transfer function model of the distillation column is considered for the design of robust controller for the unstable process with dead time [8].The change in feed flow rate and the change in feed composition are

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considered as the major Disturbances as shown in the Figure-1. Partitioned matrix of the plant is obtained by considering the performance weight, disturbance weight along with the uncertainties present in the system as shown in Figure-2 which is given by

$$
\left[\begin{array}{cccccc|cc}
-3.3e\text{-}001 & -3.2e\text{-}004 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 1.0e{+}000 \\
3.1e\text{-}002 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 \\
0.0e{+}000 & 0.0e{+}000 & -2.0e\text{-}002 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 1.0e{+}000 \\
0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & -5.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 2.0e{+}000 & 0.0e{+}000 \\
-1.0e\text{-}001 & 1.1e{+}000 & 0.0e{+}000 & -4.9e{+}000 & -5.0e\text{-}001 & -2.0e{+}000 & 2.0e{+}000 & 4.0e{+}000 \\
0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & -3.0e{+}000 & 0.0e{+}000 & 2.0e{+}000 \\
\hline
0.0e{+}000 & 0.0e{+}000 & 7.9e\text{-}001 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 0.0e{+}000 & 4.0e\text{-}001 \\
-2.5e\text{-}002 & 2.7e\text{-}001 & 0.0e{+}000 & -1.2e{+}000 & 1.1e{+}000 & -5.0e\text{-}001 & 5.0e\text{-}001 & 1.0e{+}000 \\
-5.1e\text{-}002 & 5.4e\text{-}001 & 0.0e{+}000 & -2.5e{+}000 & 0.0e{+}000 & -1.0e{+}000 & 1.0e{+}000 & 2.0e{+}000
\end{array}\right] \quad \text{---------------(1)}
$$

From eq. (1) we can define the no of control inputs and no of measurand, which help to find the H-infinity controller for the plant using hinfsyn command.

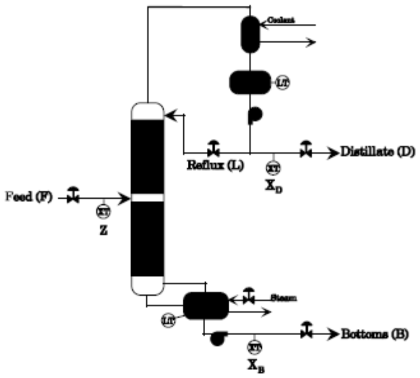

Figure-1. Functional block diagram of a distillation column.

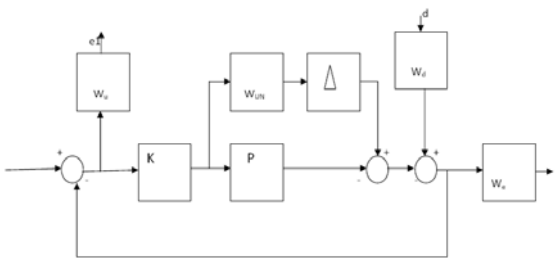

Figure-2. Robust controller design with weighting functions.

2.1 Design of H-infinity controller using D-K iteration

The objective of this paper was to design the robust H_∞ controller for the unstable process with dead time. In addition, the controller has to maintain its stability and performance, even in the perturbed condition of the plant.

3. METHODOLOGY

a. To design the lead-lag compensator for the unstable process (It helps to select the weighting function properly).
b. To design the H-infinity controller for the unstable process (Integral process with dead time).
c. The robust performance of the controller has to be checked in the presence of the uncertainty in the system.
d. D-K iteration (D-scaling) method can be used to improve the performance of the H-infinity controller design [6] for the system.
e. The weighting functions have to be selected based on the systems input requirements. By understanding the concepts of lead-lag compensator design, the selection of weighting functions can be made easily.
f. Robust performance can be analyzed in two ways [5]
   - $\| |W1S| + |W2T| \|_\infty < 1$.
   - Peak value of the μ (D-K iteration) bound should be less than one.
g. Order reduction method can be used to reduce the order of the controller, if some nonlinearity presents in the hankel singular value plot.
h. Loop shaping method:
   - In this section we again look at tracking a reference signal.
   - If L denotes the loop transfer function, as L=PC, then the transfer function from reference input r to tracking error e is given by $S = \dfrac{1}{1+L}$ called the sensitivity function, for the specified plant. Simply we can say the performance specification is $\|W1S\|_\infty < 1$.
i. The following conditions are used to check for Robust Stability, Nominal Performance and Robust Performance. The phrases robust stability, nominal performance and robust performance of the plant.

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(i) Nominal performance

The closed-loop system achieves nominal performance if the performance objective is satisfied for the nominal plant model, *G*nom. In this problem, that is equivalent to: Nominal Performance $\|W_P\,(I + G\text{nom}\,K)^{-1}\|_\infty < 1$

(ii) Robust stability

The closed-loop system achieves robust stability, if the closed loop system is internally stable for all of the possible plant models P .In this problem that is equivalent to a simple norm test on a particular nominal closed-loop transfer function.

Robust stability condition

$\|W_{del}KG\text{nom}(I + KG\text{nom})^{-1}\|_\infty < 1$

(iii) Robust performance

The closed-loop system achieves robust performance if the closed-loop system is internally stable for all P, and in addition to that, the performance objective, $\|W_P\,(I + GK)^{-1}\|_\infty < 1$, is satisfied for every **P**. The property of robust performance is equivalent to a structured singular value test (a generalization of the two H_∞ norm tests in the previous conditions) on a particular, nominal closed- loop transfer function.

4. D-K ITERATION ALGORITHM

The objective is to design a controller which minimizes the upper bound to μ for the closed loop system $\|DF_l(P(s),K(s)D^{-1}\|_\infty$.The major problem in doing this is that the D-Scale that results from μ calculation is in the form frequency by frequency data and the D-scale required above must be a dynamic system. This requires an approximation to the upper bound D-Scale in the iteration. Now we will look at this issue more closely. It can be summarized as follows:

Initialize procedure with $K_o(s)$: H_∞ controller for P(s)

- Calculate the resulting closed loop: Fl (P(s), K(s))
- Calculate D scale for μ upper bound ‖D Fl (P(s), K(s) D-1‖∞. Approximate the frequency data D (w), by D € RH∞, with $\tilde{D}$ (jw) ≈ D (w)
- Design H∞ Controller for $\tilde{D}$(s) P(s) $\tilde{D}$-1(s).

We have used the notation D(w) to emphasize that the D-Scale arises from frequency by frequency μ analyses of $G(jw)=F_l(P(jw),K(jw))$ and therefore is a function of ω. Note that it is NOT the frequency response of some transfer function and therefore we do NOT use the notation D(j ω). The μ analysis of the closed loop system is unaffected by the D-Scales. However the H_∞ design problem is strongly affected by scaling

The procedure aims at finding at D such that the upper bound for the closed loop system is a closed approximation to μ for the closed loop system. At each frequency, a scaling matrix, D(w) , can be found such that $\sigma_{max}(D(w)G(jw)D(w)^{-1})$ is a closed upper bound to μ(G(jw)).

Another aspect of this is to consider that as the iteration approaches the optimal μ value, the resulting controllers often have more and more response at high frequencies.

The above discussion used an H_∞ controller to initialize the iteration. Actually any stabilizing controller can be used. In higher order, lightly damped, interconnection structures, the H-inf design of $K_o(s)$ may be badly conditioned. In such case the software may fail to generate a controller, or may give controller which doesn't stabilize the system. A different controller (the H_2 controller is often the good choice) can be used to get a stable closed loop system, and thereby obtain D scales. Application of these D scales often results in a better conditioned H-inf design problem and the iteration can proceed.

The robust performance difference between the H_∞ controller, $K_0(s)$, and K(s), can be dramatic even after a single D-K iteration. The H_∞ problem is sensitive to the relative scaling between v and w. The D-scale provides the significance better choice of relative scaling for closed loop robust performance. Even the application of a constant D scale can have dramatic benefits [7].

5. SIMULATION RESULTS

The stability and performance measure of the plant G(s) = $\dfrac{0.0506e^{-6s}}{S}$ are analyzed and are given in the following figures as mentioned below:

Figure-3 Frequency roll of the plant with and without controller;
Figure-4 Magnitude plot of the weighing functions;
Figures 5 to 6 represent the performance measure [D-K iteration results]; and
Figure-7 Stability analysis of the controller.

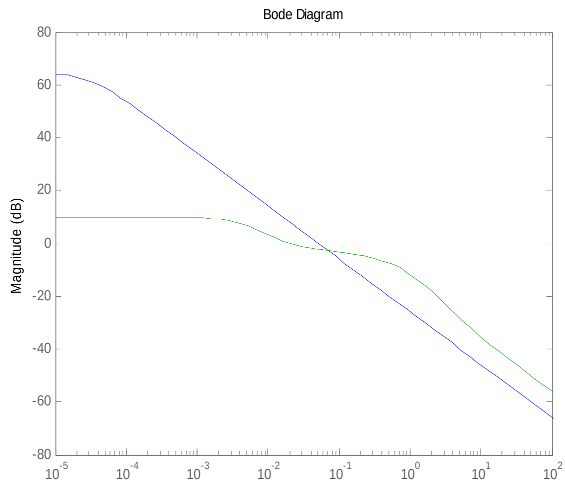

Figure-3. Bode plot of the plant and plant with controller.

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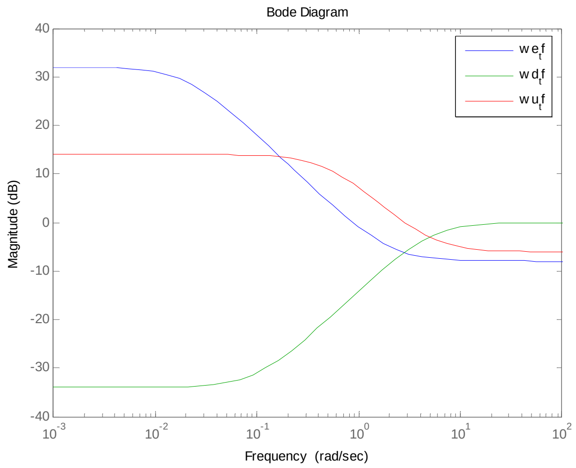

Figure-4. Bode plot of the weighting function.

5.1 Results of D-K iteration (robust performance analysis)-µ synthesis

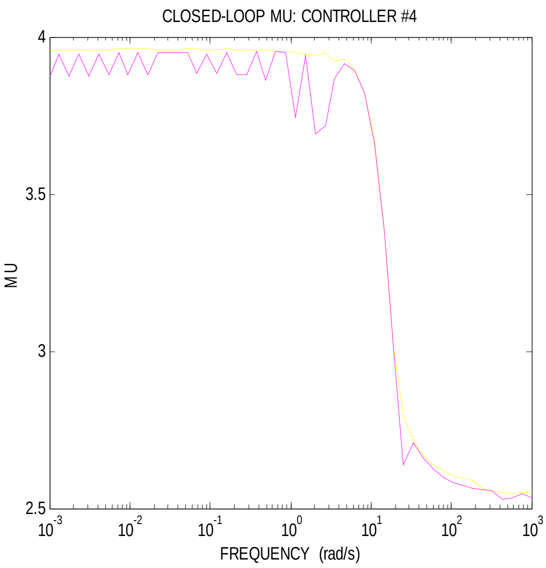

Figure-5. Mu bound without D scaling. Peak value of the Mu plot is not less than one.

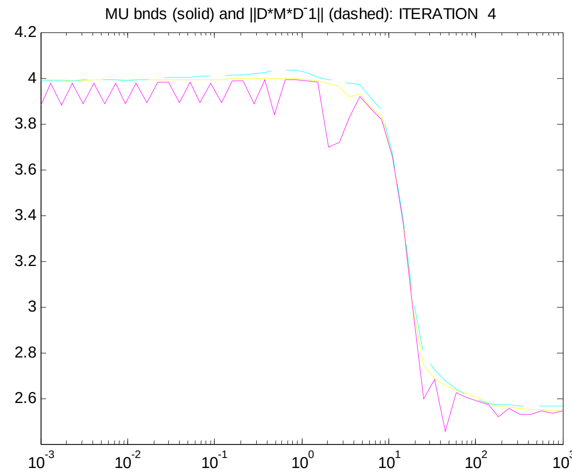

Figure-6. Mu bound with D scaling. Peak value of the Mu plot is not less than one.

From the Figures 5 and 6 we can assure that the performance specification for the system is not yet met, because the peak value of the plot should be less than one before and after the D-K iteration also.

5.2 Stability Analysis

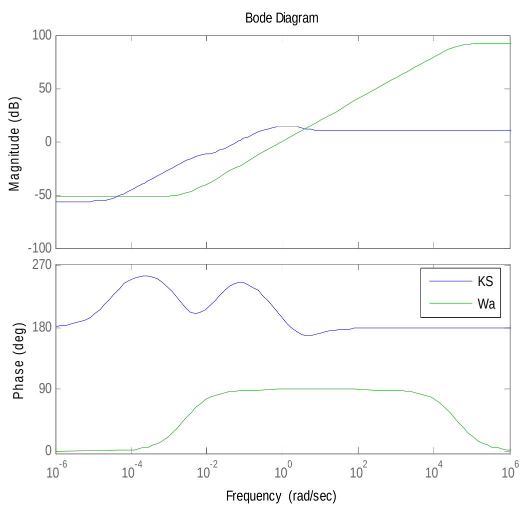

Figure-7. The robust stability criteria does not hold in the frequency range 0.00008 rad/s to 1.2 rad/s.

Therefore, the controller is not robustly stable with respect to the additive uncertainty set.

6. Future Work

The weighting functions need to adjust to satisfy the robust stability and performance conditions. Loop shaping can also be made for the system to improve its performance.

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7. CONCLUSIONS

Robust controller has been designed for the unstable process with uncertainty. The analysis of robust performance was done using μ synthesis (D-K scaling). The weighting functions needs to be adjusted to minimize the peak of the μ plot to be less than one, to maintain the robust performance of the system. After finding the uncertainty Del (Δ), the weighting function for the uncertainty has to be selected as a covering function of the magnitude plot of the Del (Δ), to assure the robust stability. The controller bandwidth has been increased with the perturbed system to maintain the controller stability.

When analyzing the robust stability criteria with respect to bode magnitude plot, it does not hold in the frequency range between 0.00008 rad/sec to 1.2 rad/sec (Figure-7). Therefore the controller is not robustly stable with respect to the additive uncertainty set.

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