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STATIC DISPLACEMENTS DUE TO SINGLE COUPLES IN A LAYERED HALF-SPACE

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ABSTRACT

Closed form analytical expressions for the displacement field due to single couples in a layer overlying a half-space using the Fourier Transform Method in term of Galerkin vector are obtained. In engineering, elastic layer represents an elastic plate while in geophysics it represents a lithosphere. This type of coupling of an elastic layer overlying an elastic half-space corresponds to the realistic earth model-lithosphere lying over an asthenoshere. These expressions are true for all Lame's parameters to remain non-zero and non-infinite. The final results are simple and quite convenient.

Keywords: single couple, displacement, layered half-space.

1. INTRODUCTION

Mindlin and Cheng (1950) obtained the solution, in term of Galerkin vector, for various single couples in a uniform half-space. The corresponding displacements were obtained by Cheng (1961). However, some of the results given by Cheng (1961) are in error. Moreover, a multiplying constant has been suppresses by him. Singh and Singh (1989) obtained the results afresh with the correct multiplying constant. Though, at present half-space model is considered as adequate for most applications, the welded elastic half-spaces model are useful for considering the effect of internal boundaries. Heaton and Heaton (1989) obtained analytical expressions for the displacements for various single couples placed in an elastic half-space in welded contact with another elastic half-space assuming the half-spaces to be poissonian. Kumari *et al.* (1992) obtained the result for single couple's sources which are valid for arbitrary values of the Poisson's ratio of the two media.

The dislocation problem in layered media has always received a great attention in the geophysical literature, but most solutions are generally limited to point sources probably due to the major mathematical difficulties that occur when extended dislocation sources are considered. Ben-Menahem and Singh (1968) extended the results of Stekette (1958) and Maruyama (1964) to a layered half-space. They investigated the effect of a crustal layer on the displacement fields. Since then, great progress has been achieved using this theory and its geophysical applications. Model involving dislocation theory and a layer over a half-space have been studied by many researchers.

In this paper, starting from the solution for the single force derived in the previous paper Sahrawat and Singh (2008), we obtained the displacements field for six-single couples in a layer overlying a half space using Fourier Transform method in term of Galerkin vector by the process of differentiation and superposition. The result will be without any restriction on the Poisson's ratio and has included the restoring force of gravity and allowed the Lame's parameters to remain non-zero and non-infinite. Fourier analysis is used as a tool to simplify the mathematical problem into one that can be more easily solved and more rapidly computed.

2. DISPLACEMENT FIELD

Consider a medium (z<0) in which a homogenous, isotropic, perfectly elastic layer in welded contact with another homogenous, isotropic, perfectly elastic half-space. We obtain the displacements field for six single couples designated as (12), (21), (13), (31), (23) and (32). Let $u_{(n)i}^{mk}$ $(k \neq m)$ denote the displacement components u_i (i = x, y, z) in medium n (n = 1, 2) resulting from the single couple of unit strength acting at (0, 0, -h) whose forces are parallel to the x_m- direction and whose arms is in the x_k–direction in a layer overlying a half-space.

2.1 Single couple (12) in the xy-plane

The displacement components $u_{(n)i}^{(12)}$ resulting from single couple (12) of unit moment with forces parallel to the x-direction and arm in the y-direction in medium n (n = 1, 2) in a layer overlying a half-space are obtained using the relation

$$u_{(n)i}^{(12)} = -\frac{\partial}{\partial y} u_{(n)i}^{x} \qquad (2.1.1)$$

Where $u_{(n)i}^{x}$ are the displacement components due to single force in x-direction (Sahrawat and Singh, 2008). Therefore, equation (2.1.1) gives the displacement components due to single couple (12):

$$u_{(1)x}^{12} = i2\pi(\alpha_1 - 2)\beta_1^2 k_y B_1 e^{-i\beta_1 x}, \qquad (2.1.2)$$

$$u_{(1)y}^{12} = -i4\pi^2 \alpha_1 \beta_1 k_y^2 B_1 e^{-i\beta_1 x}, \qquad (2.1.3)$$

$$u_{(2)x}^{12} = -i2\pi(\alpha_2 - 2)\beta_1^2 k_y \left[A_2 e^{i\beta_1 x} - B_2 e^{-i\beta_1 x}\right], \qquad (2.1.4)$$

$$u_{(2)y}^{12} = -i4\pi^2 \alpha_2 \beta_1 k_y^2 \left[A_2 e^{i\beta_1 x} + B_2 e^{-i\beta_1 x}\right], \qquad (2.1.5)$$

The expressions for $u_{(1)z}^{12}$ and $u_{(2)z}^{12}$ can be obtained from the corresponding expressions for $u_{(1)y}^{12}$ and $u_{(2)y}^{12}$ respectively on replacing k_y^2 by $k_y\, k_z$.

2.2 Single couple (21) in the xy-plane

The displacement component $u^{(21)}_{(n)i}$ resulting from single couple (21) of unit moment in the xy-plane are obtained using the relation

$$u^{(21)}_{(n)i} = -\frac{\partial}{\partial x} u^{y}_{(n)i} \tag{2.2.1}$$

Where $u^{y}_{(n)i}$ are the displacement components due to single force in y-direction (Sahrawat and Singh, 2008)? Therefore, equation (2.2.1) gives the displacement components due to single couple (21):

$$u^{21}_{(1)x} = -i4\pi^2\alpha_1\beta_2 k_x^2 D_1 e^{-i\beta_2 y}, \tag{2.2.2}$$

$$u^{12}_{(1)y} = i2\pi(\alpha_1 - 2)\beta_2^2 k_x D_1 e^{-i\beta_2 y}, \tag{2.2.3}$$

$$u^{21}_{(2)x} = -i4\pi^2\alpha_2\beta_2 k_x^2\left[C_2 e^{i\beta_2 y} + D_2 e^{-i\beta_2 y}\right], \tag{2.2.4}$$

$$u^{12}_{(2)y} = -i2\pi(\alpha_2 - 2)\beta_2^2 k_x\left[C_2 e^{i\beta_2 y} - D_2 e^{-i\beta_2 y}\right], \tag{2.2.5}$$

The expressions for $u^{21}_{(1)z}$ and $u^{21}_{(2)z}$ can be obtained from the corresponding expressions for $u^{21}_{(1)x}$ and $u^{21}_{(2)x}$ respectively replacing k_x^2 by $ik_x k_z$.

2.3 Single couple (13) in the zx-plane

The displacement component $u^{(13)}_{(n)i}$ resulting from single couple (13) of unit moment in the zx-plane are obtained using the relation

$$u^{(13)}_{(n)i} = -\frac{\partial}{\partial h} u^{x}_{(n)i} \tag{2.3.1}$$

Therefore, equation (2.3.1) gives the displacement components due to single couple (13):

$$u^{13}_{(1)x} = i2\pi(\alpha_1 - 2)\beta_1^2 k_z B_1 e^{-i\beta_1 x}, \tag{2.3.2}$$

$$u^{13}_{(1)y} = -i4\pi^2\alpha_1\beta_1 k_y k_z B_1 e^{-i\beta_1 x} \tag{2.3.3}$$

$$u^{13}_{(2)x} = -i2\pi(\alpha_2 - 2)\beta_1^2 k_z\left[A_2 e^{i\beta_1 x} - B_2 e^{-i\beta_1 x}\right], \tag{2.3.4}$$

$$u^{13}_{(2)y} = -i4\pi^2\alpha_2\beta_1 k_y k_z\left[A_2 e^{i\beta_1 x} + B_2 e^{-i\beta_1 x}\right] \tag{2.3.5}$$

The expressions for $u^{13}_{(1)z}$ and $u^{13}_{(2)z}$ can be obtained from the corresponding expressions for $u^{13}_{(1)y}$ and $u^{13}_{(2)y}$ on replacing $k_y k_z$ by k_z^2.

2.4 Single couple (31) in the zx-plane

The displacement component $u^{(31)}_{(n)i}$ resulting from single couple (31) of unit moment in the zx-plane are obtained using the relation

$$u^{(31)}_{(n)i} = -\frac{\partial}{\partial x} u^{z}_{(n)i} \tag{2.4.1}$$

Where $u^{z}_{(n)i}$ are the displacement components due to single force in z-direction (Sahrawat and Singh, 2008). Therefore, equation (2.4.1) gives the displacement components due to single couple (31):

$$u^{31}_{(1)x} = -4\pi^2\alpha_1 k_x^2\left[E_1\beta e^{\beta z} + F_1\beta e^{-\beta z} + J_1(1+\beta z)e^{\beta z} - H_1(1-\beta z)e^{-\beta z}\right], \tag{2.4.2}$$

$$u^{31}_{(1)z} = i2\pi\alpha_1\beta k_x\left[E_1\beta e^{\beta z} - F_1\beta e^{-\beta z} + J_1(2+\beta z - \frac{2}{\alpha_1})e^{\beta z} + H_1(2-\beta z - \frac{2}{\alpha_1})e^{-\beta z}\right] \tag{2.4.3}$$

$$u^{31}_{(2)x} = -4\pi^2\alpha_2 k_x^2\left[E_2\beta e^{bz} + J_2(1+\beta z)e^{\beta z}\right], \tag{2.4.4}$$

$$u^{31}_{(2)z} = i2\pi\alpha_2\beta k_x\left[E_2\beta e^{\beta z} + J_2(2+\beta z - \frac{2}{\alpha_2})e^{\beta z}\right], \tag{2.4.5}$$

The expressions for $u^{31}_{(1)y}$ and $u^{31}_{(2)y}$ can be obtained from the corresponding expressions for $u^{31}_{(1)x}$ and $u^{31}_{(2)x}$ respectively on replacing k_x^2 by $k_x k_y$.

2.5 Single couple (23) in the yz-plane

The displacement component $u^{(23)}_{(n)i}$ resulting from single couple (23) of unit moment in the yz-plane are obtained using the relation

$$u^{(23)}_{(n)i} = -\frac{\partial}{\partial h} u^{y}_{(n)i} \tag{2.5.1}$$

Therefore, equation (2.1.1) gives the displacement components due to single couple (23):

$$u^{23}_{(1)x} = 4\pi^2\alpha_1\beta_2 k_x k_z D_1 e^{-i\beta_2 y}, \tag{2.5.2}$$

$$u^{23}_{(1)y} = -2\pi(\alpha_1 - 2)\beta_2^2 k_z D_1 e^{-i\beta_2 y}, \tag{2.5.3}$$

$$u^{23}_{(2)x} = 4\pi^2\alpha_2\beta_2 k_x k_z\left[C_2 e^{i\beta_2 y} + D_2 e^{-i\beta_2 y}\right] \tag{2.5.4}$$

$$u^{23}_{(2)y} = 2\pi(\alpha_2 - 2)\beta_2^2 k_z\left[C_2 e^{i\beta_2 y} - D_2 e^{-i\beta_2 y}\right] \tag{2.5.5}$$

The expressions for $u^{23}_{(1)z}$ and $u^{23}_{(2)z}$ can be obtained from the corresponding expressions for $u^{23}_{(1)x}$ and $u^{23}_{(2)x}$ respectively on replacing $k_x k_z$ by $i k_z^2$

2.6 Single couple (32) in the yz-plane

The displacement component $u^{(32)}_{(n)i}$ resulting from single couple (32) of unit moment in the yz-plane are obtained using the relation

$$u^{(32)}_{(n)i} = -\frac{\partial}{\partial y} u^{z}_{(n)i}$$

(2.6.1)

Therefore, equation (2.1.1) gives the displacement components due to single couple (32):

$$u^{32}_{(1)x} = -4\pi^2\alpha_1 k_x k_y\left[E_1\beta e^{\beta z} + F_1\beta e^{-\beta z} + J_1(1+\beta z)e^{\beta z} - H_1(1-\beta z)e^{-\beta z}\right], \tag{2.6.2}$$

$$u^{32}_{(1)z} = i2\pi\alpha_1\beta k_y\left[E_1\beta e^{\beta z} - F_1\beta e^{-\beta z} + J_1(2+\beta z - \frac{2}{\alpha_1})e^{\beta z} + H_1(2-\beta z - \frac{2}{\alpha_1})e^{-\beta z}\right] \tag{2.6.3}$$

$$u^{32}_{(2)x} = -4\pi^2\alpha_2 k_x k_y\left[E_2\beta e^{bz} + J_2(1+\beta z)e^{\beta z}\right] \tag{2.6.4}$$

$$u^{32}_{(2)z} = i2\pi\alpha_2\beta k_y\left[E_2\beta e^{\beta z} + J_2(2+\beta z - \frac{2}{\alpha_2})e^{\beta z}\right] \tag{2.6.5}$$

The expressions for $u^{32}_{(1)y}$ and $u^{32}_{(2)y}$ can be obtained from the corresponding expressions for $u^{32}_{(1)x}$ And $u^{32}_{(2)x}$ respectively on replacing $k_x k_y$ by k_y^2

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The value of the constants is given by Sahrawat and Singh (2008).

3. CONCLUSIONS

Heaton and Heaton (1989) obtained analytical expressions for the displacements for various single couples in an elastic half-space in welded contact with another elastic half-space assuming the half-spaces to be poissonian. Kumari *et al.* (1992) obtained the result for various dipolar sources which are valid for arbitrary values of the Poisson's ratio of the two media. We have obtained the result be without any restriction on the Poisson's ratio and has included the restoring force of gravity and allowed the Lame's parameters to remain non-zero and non-infinite. Fourier analysis is used as a tool to simplify the mathematical problem into one that can be more easily solved and more rapidly computed. The present formulation is simple and quite convenient.

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FIGURES

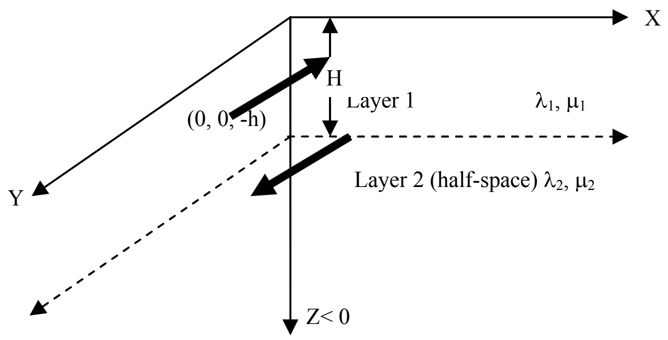

Figure-1a. Single couple 12.

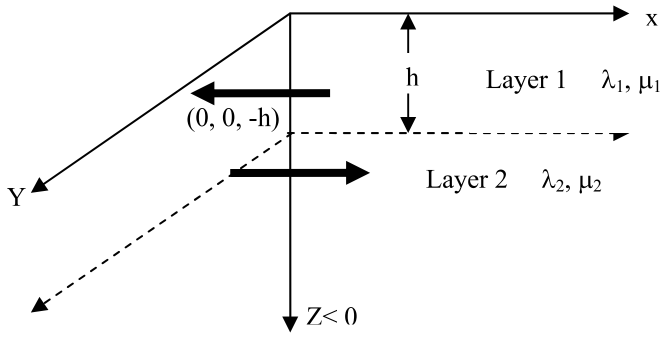

Figure-1b. Single couple 21.

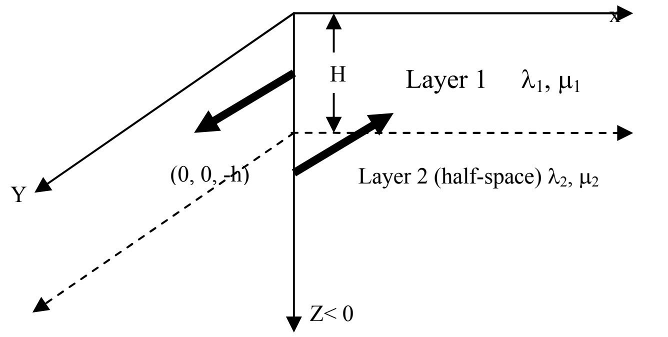

Figure-1c. Single couple 13.

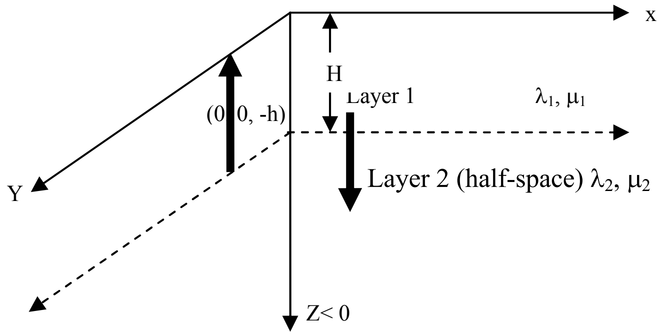

Figure-1d. Single couple 31

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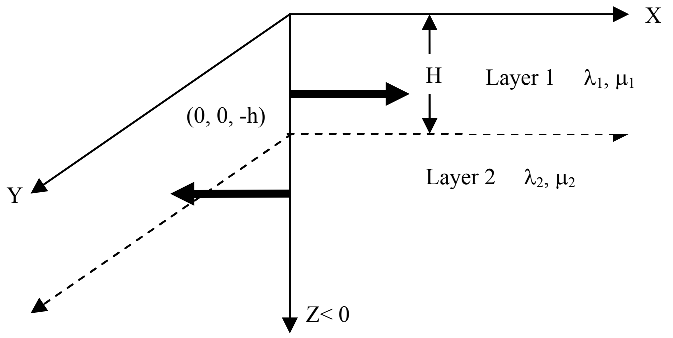

Figure-1e. Single couple 23.

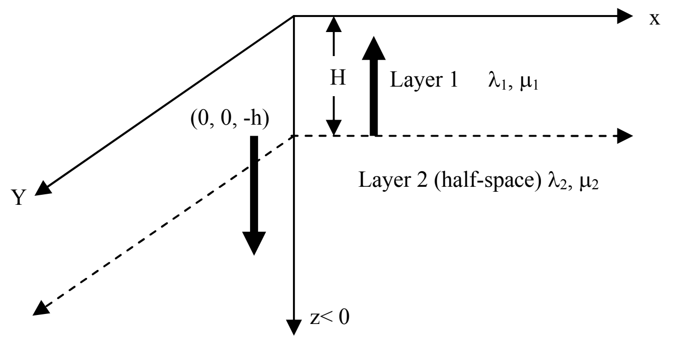

Figure-1f. Single couple 32.