



A GENETIC ALGORITHM BASED APPROACH FOR OPTIMAL ALLOCATION OF DISTRIBUTED GENERATIONS IN POWER SYSTEMS FOR VOLTAGE SENSITIVE LOADS

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ABSTRACT

Connecting green power sources to power grid are gradually becoming popular. Proper placement and sizing of these energy sources is important in order to obtaining their maximum potential benefits. There have been studies to allocate the DG, in which loads are generally modeled as constant power or constant current types of loads. Since most of the distribution system loads are uncontrolled and dependent on the voltage and frequency of the system. Accordingly in this study a methodology has been presented that uses genetic algorithm to optimally place the DGs in terms of locations and sizes so as to minimize the per unit locational charges for active power at buses by incorporating different voltage dependent static load models. The presence of DG sources at distribution level has changed the characteristics of distribution network from passive to active. Nodal pricing and per unit locational charges often used in the pricing of short-term operation in transmission are good candidates to consider in distribution. Several simulation studies have been conducted on radial feeder as well as networked systems with single DG and multiple DG separately, subjected to no voltage violation at any of the buses.

Keywords: genetic algorithms, distributed generation, locational charges, distribution network, optimal location, optimal size.

1. INTRODUCTION

As the green power sources such as; hydro, wind, solar, geothermal, biomass, ocean energy and fuel cells, all over the world are becoming popular, naturally more studies is being carried out for connecting these energy sources to the grid. Distributed generation (DG) is a generating plant serving a customer on site or providing support to a distribution network, connected to grid at distribution level voltages. DGs are considered as small power generators that complement central power stations by providing incremental capacity to power system [1-2].

Influence of DG may affect the flow of power and voltage scenario at load sites. These impacts may be categorized as positive or negative depending on the distribution system operating characteristics and DG characteristics. Positive impacts are generally referred as "system support benefits", that can be explained in terms of reducing losses and on-peak operating costs, improving voltage profiles, eliminating construction of new transmission lines, improving system reliability, integrity and efficiency. For extracting the maximum potential benefits, the DG must be reliable and dispatchable in terms of proper siting and sizing [1-6].

Most of the literatures have considered power losses of the system as objective function to be minimized [3]-[6] for locating the site for DG installation. A "2/3 rule" is described in [8] to place the DG on a radial feeder with uniformly distributed loads. This method cannot be extended to a feeder with other types of loads, or to networked systems. An analytical method has been described to optimally place the DG in radial as well as networked systems [9]. A genetic tabu search algorithms has been described for optimal DG allocation in distribution networks [10]. Methods presented in [9-10]

determines only the location of DG, whereas, losses depends on location and size both. A method is suggested in [11] for optimal placement and sizing of DG source using analytical approach considering different static load models, but its application is restricted to radial feeders only. An analytical method presented in [12] for optimal placement and size of DG is very accurate and effective, but it can not be extended for multi DG placement, while it is more beneficial to place relatively small amounts of dispersed installation rather than a single DG installation [14].

In the competitive market, injection of distributed generation (DG) is becoming more popular. Now the distribution network has become more active rather than passive and adopted many of characteristics of transmission. The nodal pricing and per unit locational charges are often used in short term operations in transmission. Hence, pricing mechanism that has been used in transmission such as nodal pricing and per unit locational charges are good candidates to consider in distribution. Nodal pricing is an economically efficient pricing mechanism for short-term operation of transmission systems that has been implemented in various forms by electricity markets in New York, New England, New Zealand, Argentina and Chile. Clearly, this is a pricing mechanism that has a great deal of experience and confidence [16]. If the losses and the line loading reduce due to the presence of DG, the locational charge will reduce. However, if it increases losses and line loading per unit locational charges will increase [15, 16].

In the studies cited above, the loads are generally modeled as constant power or constant current types of loads. Since most of distribution system loads are voltage dependent [19], hence determination of optimum location



and size of DG considering the constant power or constant current load model should be reviewed. Accordingly, in our study for allocating DG, voltage dependent model of load has been entrusted for further analysis.

This paper presents an approach applying genetic algorithms (GA) for optimal placement as well as optimal sizing of DGs for the reduced average per unit locational charges for active power at buses by giving a proper consideration to voltage dependent model of the loads. An analysis that reflects the effects of the load modeling to the optimal placement and size of DG is also presented. The bus at which per unit locational charges for active power is maximum may be considered for DG placement, and this approach will certainly bring down the per unit locational charges at that bus, but in some cases the per unit locational charges at other buses may increase. Hence we have considered average of per unit locational charges for active power at buses as the objective function to be minimized for DG placement. Several simulation studies have been conducted on radial and networked system with single DG and multiple DGs separately. It is observed that the location of DG is almost fixed, while the size of DGs is under the influence of the value of load exponents or type of loads. If the DG size is fixed, the objective function reduces to minimum with the increment of load exponents. Any further increment of load exponent leads to an increasing objective function. It is found that placing relatively small amount of multiple DG rather than a large single DG reduces the objective function many folds in case of radial feeder, while its effect in networked system is also considerable.

1.1 Static load models

In conventional load flow analysis, active and reactive loads at buses are assumed constant value, regardless of the magnitude of voltages at the buses. In actual power system operation, different categories and types of loads such as residential, industrial and commercial loads might be present. The nature of these types of loads is such that their active and reactive powers are dependent on voltage and frequency of the system. The static load models used in power systems for active and reactive powers are expressed in a polynomial or an exponential form. The characteristic of the exponential load models can be expressed as [19];

$$P = P_0 \left(\frac{V}{V_0} \right)^{na}$$

$$Q = Q_0 \left(\frac{V}{V_0} \right)^{nr}$$

Where na and nr stand for load exponents, P_0 and Q_0 stand for real and reactive powers at the nominal voltage. V And V_0 stand for load voltage and load nominal voltage, respectively. Special values of the load exponents can cause specific load types such as 0: constant power, 1:

constant current, 2: constant impedance. The values of these coefficients are determined for different load types in transmission and distribution systems. Usually data, determined from experience, could be used for the estimation. Common values of the exponents of static load models for different load components are given in Table-1.

Table-1. Common values for the exponents for different static load models.

Load component	na	nr
Battery charging	2.59	4.06
Fluorescent lamps	2.07	3.21
Constant impedance	2	2
Fluorescent lighting	1	3
Air conditioner	0.5	2.5
Constant current	1	1
Resistance space heater	2	0
Pumps, fans and other motors	0.08	1.6
Incandescent lamps	1.54	0
Compact fluorescent lamps	1	0.35
Small industrial motors	0.1	0.6
Large industrial motors	0.05	0.5
Constant power	0	0

2. FORMULATION OF PROBLEM

By definition, marginal loss coefficients (MLCs) measure the change in total active power losses L due to a marginal change in consumption/generation of active power P_i and reactive power Q_i at each node i in the network, i.e.,

$$\rho_{P_i} = \frac{\partial L}{\partial P_i}$$

$$\rho_{Q_i} = \frac{\partial L}{\partial Q_i}$$

Where ρ_{P_i} and ρ_{Q_i} represent the active and reactive power related MLCs. A method presented in [17] has been used to obtain the real and reactive marginal loss coefficients, ρ_{P_i} and ρ_{Q_i} . Per unit locational charges are calculated at buses using power flows by giving a proper consideration to voltage dependent static load models described in [19] for the given per unit cost of active power at the reference bus. The spot price at each bus is location specific and differs by locational charges. At reference bus locational charges are considered to be zero. At any bus i , the per unit active and reactive locational charges can be obtained as follows:



$$P_{ai} = \lambda \cdot \rho_{pi}$$

$$P_{ri} = \lambda \cdot \rho_{Qi}$$

Where λ is the per unit cost for active power at reference bus.

The formulation of the problem can be summarized as-

$$\text{Minimize } f = \frac{1}{n-1} \sum_{i=1}^{n-1} P_{ai}; n \text{ is the number of buses.}$$

Subject to voltage at all buses should be within specified limits i.e.

$$V_{\min} \leq V \leq V_{\max}$$

3. APPLICATION NETWORKS

The proposed method is tested on radial as well as networked systems. The description of networks is given as follows:

Radial distribution network

The network under study is a 13-bus radial distribution network. The network data are given in Appendix B. The loads of the system are assumed to be voltage sensitive. The unit cost of active power at bus 1 (reference bus) are assumed to be 24 USD/MWh [17].

IEEE 6-bus networked system

An IEEE 6-bus networked system has been shown in Figure-1. The parameters of the system are given in Appendix A. The loads of the system are assumed to be voltage sensitive. The unit cost of active power at bus 1 (reference bus) are assumed to be 24 USD/MWh [17].

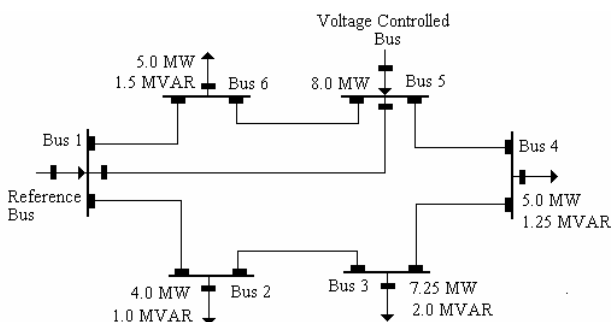


Figure-1. An IEEE 6-bus networked system.

4. GENETIC ALGORITHMS FOR OPTIMAL PLACEMENT AND SIZING OF DISTRIBUTED GENERATORS

In a DG enhanced feeder, the optimization is not straightforward as in the case of a conventional feeder, due to the presence of additional generators. The per unit locational charge at buses depend on DG location and DG size both. We produce a simple genetic algorithm (GA) based method to solve for optimal position and optimal size of DG simultaneously. Genetic algorithm effectively implements the 'survival of the fittest' strategy using the principles of Darwinian natural selection and biological inspired operations. Genetic algorithm typically starts with

randomly generated initial population of individuals and iteratively transforms the initial population into a new generation of the population using two genetic operators' mutation and crossover. The individuals are probabilistically selected to participate in the genetic operation based on their fitness measures. The iterative transformation of the population is executed inside the main generational loop of the run of genetic algorithm. After the termination criterion is satisfied, the single best individual in the population produced during the run is harvested and designated as the result of the run. If the run is successful, the result may be the solution to the problem.

In our application, the initial population vector is randomly generated. All the buses except the slack bus and all the DG size up to maximum DG size are randomly generated. Each element of initial population vector (DG location and corresponding DG size) is placed on the network one by one and DGs are considered as negative real load at the corresponding bus. Now, for the placement of each element the voltage limit at each bus is determined. If voltage is within specified limits for all the buses, the placed element is the survived element of survived population vector. The performance index of elements of survived population vector is determined for optimization. The survived population of initial population vector is further used to generate next generation population using two genetic operators' crossover and mutation. The above-mentioned steps are repeated to obtain the best results.

5. THE PROPOSED ALGORITHM

In order to determine the best location and the best size of the DG unit for distribution network, algorithm has been suggested considering the appropriate voltage levels. The major steps of the proposed algorithm are:

- Step1. Create an initial population by randomly generating DG location and DG Size.
- Step2. Apply each element of initial population on the network i.e. generated DG size is considered as negative real load at the corresponding generated location.
- Step3. Check for the voltage violation at all the buses for each element applied on network.
- Step4. If voltage is within specified limits for all the buses, the element is survived element of survived population
- Step5. Modify the loads of the system as per the voltage variations for further analysis.
- Step6. Calculate the average of per unit locational charges for active power at the buses for the elements of survived population vector. The element for which the average of per unit locational charges for active power is minimum is the optimal element.
- Step7. If the convergence criterion is satisfied, stop and display the results, otherwise go to step 8.



Step8. Apply the elements of short listed survived population vector from the initial population vector to generate next generation population using two genetic operators' crossover and mutation, and repeat step 3 to step7.

The proposed method can be applied for the placement of single DG and can easily be extended for any number of DG placements.

6. SIMULATION RESULTS

The proposed method is applied on several standard test systems for both radial and networked systems, considering single DG and multiple DGs separately. In case of multiple DG placements, we have considered the placement of four DGs at a time. Since most of the loads of distribution system are uncontrolled and voltage sensitive. Accordingly, the voltage dependent static load models [19] has been implemented to ascertain the actual loads of the system in accordance with the voltage variation for further analysis in each case. The types of loads determine the extent of voltage dependence of loads, and load exponents *na* and *nr* decide the types of loads. In our study, we have analyzed the impact of varieties of voltage dependent loads on optimal siting and sizing of DGs by varying *na* and *nr* between 0 and 10 in each case. The simulation results of each case are analyzed separately.

6.1 Single DG placement radial distribution network

The system under test is a 13-bus radial distribution network with three different loading conditions; increasingly distributed, uniformly distributed, and centrally distributed load profile. Each loading condition is analyzed separately.

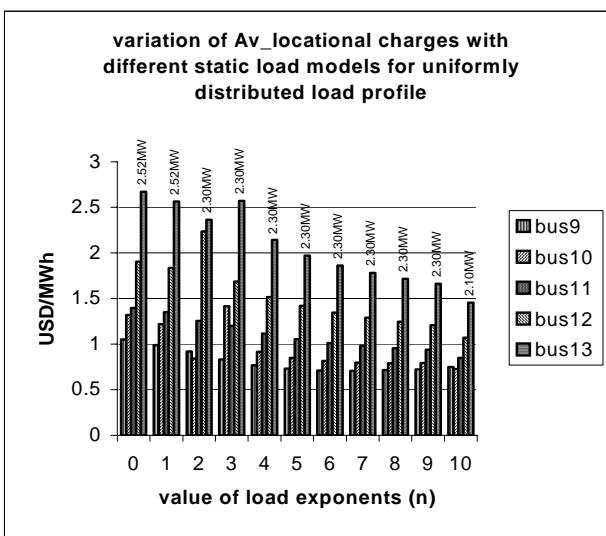


Figure-2. Variation of average locational charges with different static load models for uniformly distributed load profile.

Under uniformly distributed loading condition optimal location and size of DG is determined for different static load models and the results of simulation are shown in Figure-2. A few buses are shown in test results, because presence of DG at other buses violates the voltage limits. It may be observed that bus 9 is the universal optimal location irrespective of the value of load exponents, while optimal size is changing with load exponents. The optimal size of DG in MW for each category of load model is mentioned against respective category in Figure-2. The DG size of 2.3 MW is found to be optimal in majority of cases. The system was simulated for a fixed DG size of 2.3 MW at bus 9 for different static load models and results are presented in Figure-5. It may be seen that for increasing load index n, the objective function reduces to minimum value. Any further increase in load index n yields with increased objective function.

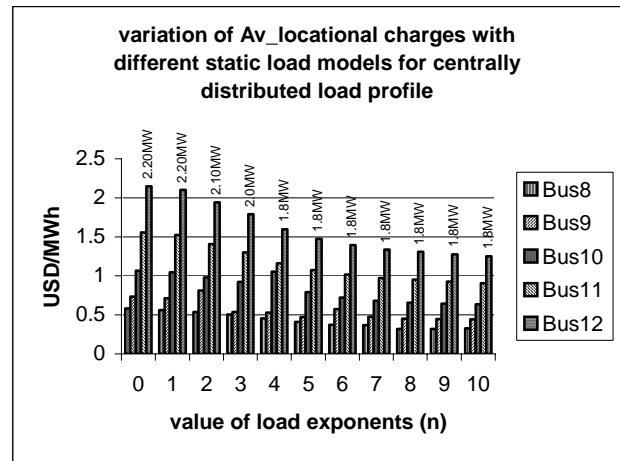


Figure-3. Variation of average locational charges with different static load models for centrally distributed load profile.

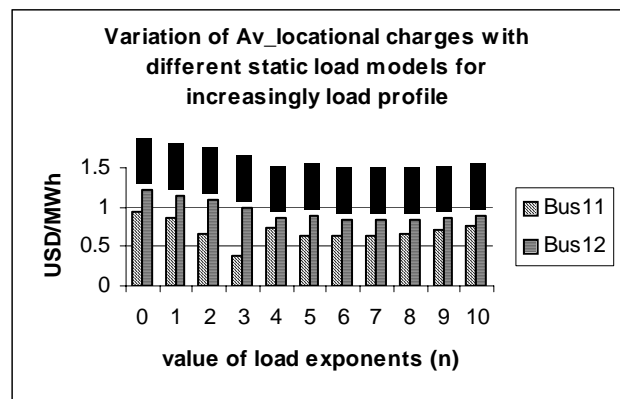


Figure-4. Variation of average locational charges with different static load models for increasingly distributed load profile.

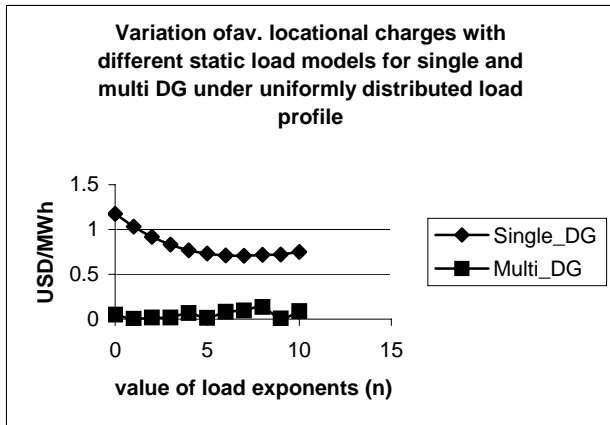


Figure-5. Variation of average locational charges with different static load models for single and multi DGs under uniformly distributed load profile.

The simulation results of centrally distributed loading condition are shown in Figure-3. The bus 8 is the optimal location irrespective of the value of load exponents, while optimal size of DG is changing with load exponents. The DG size of 1.8 MW is the optimum size in majority of cases. The system is simulated with a fixed DG size of 1.8 MW at bus 8 for the different load models, and results are presented in Figure-6. The observations are similar to uniformly distributed condition.

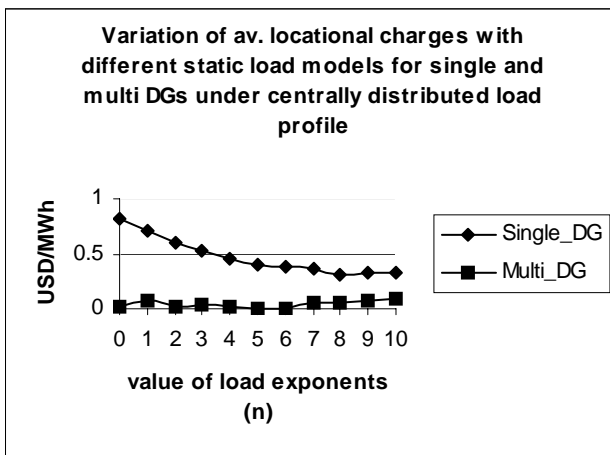


Figure-6. Variation of average locational charges with different static load models for single and multi DGs under centrally distributed load profile.

In case of increasingly distributed loading condition, the results are shown in Figure-4. In the results only two buses that are found suitable for DG placement, because presence of DG at other buses do not satisfy voltage constraints. The bus 11 is the optimal location irrespective of the value of load exponents, while DG size is changing with load exponents. The DG size of 2.2 MW is found to be optimal in majority of cases. The system is simulated with a constant DG size of 2.2 MW at bus 11

and results are shown in Figure-7. The observations are similar to the two previous cases reported earlier.

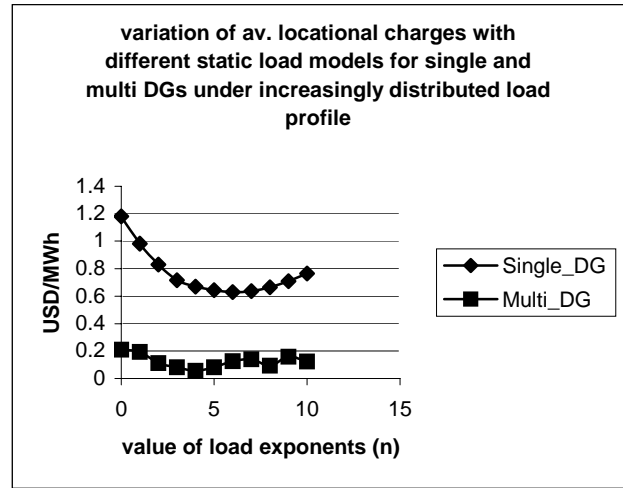


Figure-7. Variation of average locational charges with different static load models for single and multi DGs under increasingly distributed load profile.

IEEE 6-bus networked system

Applying proposed method has showed the results of simulation for this network in Figure-8.

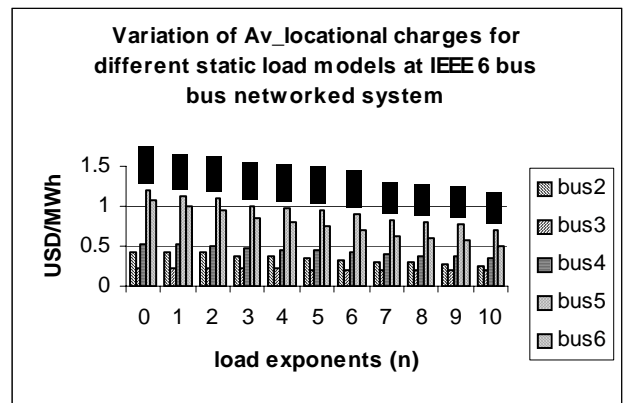


Figure-8. Variation of average locational charges for different static load models at IEEE 6 bus networked system.

The Bus 3 is the optimal location irrespective of the value of load exponents, while the DG size is varying with load exponents. The optimal size for each category is mentioned against respective categories. The system is tested with a fixed DG size of 11 MW at bus 3 for different static load models are shown in Figure-9.

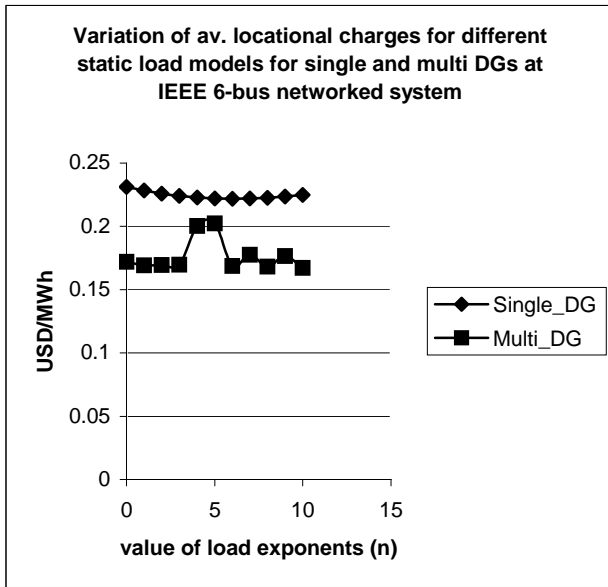


Figure-9. Variation of average locational charges with different static load models for single and multi DGs under an IEEE 6 bus networked system.

It is observed that as the load exponent increases, the objective function starts to decrease and reaches to minimum value. If load exponents still increases, then objective function begins to increase. The impact of load exponent on objective function is marginal compared to radial distribution system due to less variation of voltages in the networked system.

6.2 Multiple DG placements

The placement of multiple DGs in radial and networked system has been considered separately. These are the same system that is simulated under single DG placement. In our application we have considered maximum four DG to be placed in the distribution system at a time. The genetic algorithm has been applied to optimally allocate the DGs in the system in terms of locations and sizes by giving a proper consideration to different voltage dependent static load models. The results of simulation that determine the optimal locations and sizes of DGs for radial distribution network with different loading conditions, each taken separately have been shown in tabular form. Tables 3 and 4 depict the results for uniformly and centrally distributed loading conditions respectively, while the results of increasingly distributed loading condition is shown in Table-2. The optimal locations and sizes of DG for the networked system are shown in Table-5.

Table-2. Optimal locations and sizes of DGs for increasingly distributed load profile under multiple DG placements.

Load exponents	Total DG Injection (MW)	Optimal loc. 1	Optimal size 1 (MW)	Optimal loc. 2	Optimal size 2 (MW)	Optimal loc. 3	Optimal size 3 (MW)	Optimal loc. 4	Optimal size 4 (MW)
0	3.02	4	0.67	9	0.78	10	0.73	13	0.84
1	3.23	13	0.77	10	0.85	7	0.76	6	0.85
2	3.22	11	0.85	8	0.73	13	0.80	6	0.84
3	2.0	13	0.62	12	0.76	11	0.54	9	0.08
4	2.88	11	0.56	5	0.73	10	0.79	12	0.80
5	2.56	12	0.58	11	0.60	6	0.75	9	0.63
6	3.32	7	0.83	4	0.84	12	0.82	10	0.83
7	2.46	7	0.61	12	0.57	11	0.47	9	0.81
8	2.78	13	1.46	12	0.47	10	0.85	-	-
9	3.08	13	0.74	10	0.85	8	0.67	7	0.82
10	3.09	12	0.85	10	0.80	6	0.77	9	0.67

**Table-3.** Optimal locations and sizes of DGs for uniformly distributed load profile under multiple DG placement.

Load exp- onents	Total DG Injection (MW)			
	Optimal loc. 1	Optimal size 1 (MW)	Optimal loc. 2	Optimal size 2 (MW)
0	2.49	11	0.62	9
1	2.29	13	1.07	12
2	2.35	13	0.70	10
3	2.33	11	0.62	9
4	2.01	13	0.90	12
5	2.01	12	0.68	10
6	2.73	8	0.73	4
7	2.84	12	0.62	3
8	2.74	6	0.63	8
9	3.01	13	0.71	10
10	2.8	13	1.30	12
			0.60	10
			0.90	9
			-	-
			-	-

Table-4. Optimal locations and sizes of DGs for centrally distributed load profile under multiple DG placements.

Load exp- onents	Total DG Injection (MW)			
	Optimal loc. 1	Optimal size 1 (MW)	Optimal loc. 2	Optimal size 2 (MW)
0	1.86	13	0.87	12
1	1.94	11	0.9	9
2	1.89	13	0.45	10
3	1.7	13	0.80	12
4	1.82	11	0.40	9
5	2.5	13	0.56	12
6	1.55	13	0.26	10
7	2.05	7	0.57	11
8	1.97	5	0.20	9
9	2.25	11	0.39	6
10	2.23	11	0.53	7
			1.07	3
			0.63	0.63
			-	-
			-	-

A comparison that reflect the changes in objective function for single and multiple DGs placement are shown in Figures 5, 6 and 7 for radial distribution system, while Figure-9 depicts the effect in networked system. It is observed that small capacity multiple DG placements are more beneficial rather than a large capacity single DG placement irrespective of different load models in both types of systems. In case of radial distribution system, multiple DG placements reduce objective function in many folds in comparison of a single DG, while its effect is considerable too in case of networked system. In some cases it is also seen that two DG placement or three DG placement is more efficient than four DG placement. The multiple DG placements invite more DG penetration in the network that improves the objective of reducing average locational charges at buses with satisfying the voltage constraints.



Table-5. Optimal locations and sizes of DGs for IEEE 6-bus networked system under multiple DG placement.

Load exp- onents	Total DG Injection (MW)	Optimal loc. 1	Optimal size 1 (MW)	Optimal loc. 2	Optimal size 2 (MW)	Optimal loc. 3	Optimal size 3 (MW)	Optimal loc. 4	Optimal size 4 (MW)
0	13.5	4	3.04	3	6.71	2	3.75	-	-
1	13.8	4	3.02	3	7.05	2	3.75	-	-
2	13.3	4	2.76	3	6.75	2	3.75	-	-
3	13.2	4	3.03	3	6.46	2	3.75	-	-
4	10.3	4	2.18	3	8.10	-	-	-	-
5	9.7	4	1.94	3	4.79	2	2.96	-	-
6	13.0	4	2.94	3	6.32	2	3.75	-	-
7	10.1	4	1.16	3	5.54	2	5.76	-	-
8	11.6	4	2.30	3	5.88	2	3.44	-	-
9	10.3	4	2.77	3	5.26	2	2.28	-	-
10	11.8	4	2.51	3	5.58	2	3.75	-	-

7. CONCLUSIONS

This study presents an approach by the use of genetic algorithm for optimal placement and size of DG in radial as well as networked system with single and multiple DGs separately so as to minimize the average locational charges at buses by giving a proper consideration of voltage dependent static load models. It is observed that optimal location is almost fixed irrespective of different load models, while DG size is under the influence of load models in both types of network. If a fixed size DG is located at optimal position for different static load models, it is observed that as the load exponent increases, the objective function starts to decrease and reaches to minimum value. If load exponents still increases, then objective function begins to increase in both types of network. The impact of load exponent on objective function is marginal in networked system compared to radial distribution system due to less variation of voltages in the networked system. In case of radial distribution system, multiple DG placement reduces objective function in many folds in comparison of a single

DG, while its effect is considerable too in case of networked system. In some cases it also seen that two DG placement or three DG placement is more efficient than four DG placements.

The multiple DG placements invite more DG penetration in the network that improves the objective of reducing average location charges at buses with satisfying the voltage constraints

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Appendix A

Table-A(1). Bus data (parameters of the system in Figure-1[9]).

Bus No.	Voltage (p.u.)	Bus Power (MVA)
1	1.0 + j 0.0	Slack bus
2	-	-4.0-j1.00
3	-	-7.25-j2.00
4	-	-5.00-j1.25
5	$ V_5 $	8.00
6	-	-5.00-j1.50

Table-A(2). Line data.

From	To	Z_{serial} (p. u.)	Z_{shunt} (p. u.)
1	2	0.2238 + j 0.5090	j0.0012
2	3	0.2238 + j0.5090	j0.0012
3	4	0.2238 + j0.5090	j0.0012
4	5	0.2238 + j0.5090	j0.0012
5	6	0.2238 + j0.5090	j0.0012
6	1	0.2276 + j0.2961	j0.0025
1	5	0.2603 + 0.7382	j0.0008

Appendix B

Table-B(1). Parameters of 13-bus radial feeder [11].

Line parameters	$R = 0.538 \Omega/\text{km}$, $X_L = 0.4626$ Line length between two buses:2.5 km Bus Voltage: 12.5 kV		
Load profile	Uniformly distributed	Centrally distributed	Increasingly distributed
Bus No.	Load (MW)	Load (MW)	Load (MW)
1	0.3	0.06	0.09
2	0.3	0.12	0.12
3	0.3	0.18	0.15
4	0.3	0.24	0.18
5	0.3	0.3	0.21
6	0.3	0.36	0.24
7	0.3	0.42	0.27
8	0.3	0.36	0.3
9	0.3	0.3	0.33
10	0.3	0.24	0.36
11	0.3	0.18	0.39
12	0.3	0.12	0.42
13	0.3	0.06	0.45
Total Load	3.9	2.94	3.51