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APPLICATION OF BRINKMAN MODEL TO THE UNSTEADY FLOW OF A BINGHAM FLUID IN CONTACT WITH A NEWTONIAN FLUID

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ABSTRACT

Unsteady flow of a Bingham fluid in contact with a Newtonian fluid between two permeable beds of different permeabilities is studied. We used the Brinkman model for this problem. Expressions for the interface velocity, velocity distributions in the porous and non-porous regions and mass flow rate are obtained. These expressions are evaluated numerically for different values of the parameters.

Keywords: bingham fluid, newtonian fluid, brinkman model.

INTRODUCTION

The problems of fluid flow through channels or tubes consisting of two different fluids with different viscosities are of growing interest because of its diverse applications particularly with respect to biofluids.

In physiological flows, the boundary is typically coated with a fluid different property from those of the fluid flows. As a first step, towards understanding the effect of a fluid coating on the flow, it is of interest to extend the single fluid analysis to two fluid analyses by including a peripheral layer of different viscosity. Several researchers studied the physiological applications of two fluid analyses. Srivastava *et al.*, [1] studied the peristaltic motion of two fluids in non- uniform axisymmetric tubes. They have shown the applicability of their model to the flow in small intestine and ductus efferentus of the reproductive tract.

The fluid mechanical description of the esophageal peristaltic transport with the help of two-fluid model has been explained by Brasseur [2]. Srivastava and Srivastava [3] have investigated the problem of peristaltic transport of blood in a uniform and non-uniform geometries by considering blood as a two layered fluid model consisting of a central layer of suspension of all erythrocytes, etc assumed to be a Casson fluid, which is a yield stress fluid and a peripheral layer of plasma as a Newtonian fluid. Comparini and Mannucci [4] studied the flow of a Bingham fluid in contact with a Newtonian fluid, playing the role of a lubricant.

Flows of non-Newtonian fluids with permeable boundaries have been the subject of research for many years because of their important role in engineering and medicine. Vajravelu *et al.*, [5] investigated hydromagnetic unsteady flow of two immiscible fluids between two permeable beds. Beavers and Joseph [6] studied the flow of a viscous flow in a channel bounded below by a naturally permeable wall. However, in several applications involving one dimensional flow of immiscible fluids through a porous channel, the permeability of the bonding medium need not be small. Further, the viscous stresses in the porous medium may be of comparable magnitude, thus requiring the consideration of Brinkman model. In view of this Sacheti [7] discussed the flow of immiscible fluids through a porous channel using Brinkman model. Further Bugliarello and Sevilla [8] reported that blood through small vessels consists of two layers; one is a peripheral layer of plasma and the other, a core layer containing a suspension of erythrocytes. Hence it is interesting to study the flow of a Bingham fluid in contact with a Newtonian fluid in a porous channel using Brinkman model.

In this paper the unsteady flow of two immiscible fluids in a porous channel is studied. The core contains Bingham fluid whereas the peripheral layer contains a Newtonian fluid. The effects of the permeability and the viscosity ratio on the flow characteristics are discussed.

MATHEMATICAL FORMULATION OF THE PROBLEM

We consider the flow of a Bingham fluid in contact with a Newtonian fluid between two permeable beds. The flow between permeable beds consists of three layers. The core layer consists of a Bingham fluid which is surrounded by a Newtonian fluid forming two layers. The peripheral layer is bounded above by a permeable bed. The flow in the permeable bed is governed by Brinkman model. For simplicity we consider half of the channel. X-axis is taken in the mid way in the plug flow region.
A line perpendicular to it is taken as y-axis.

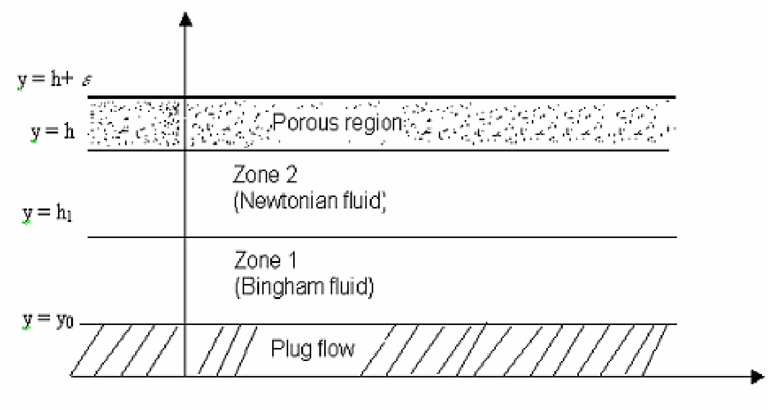

Figure-1. Physical model.

The following assumptions are made in the analysis of the problem:

- The porous beds are homogeneous and isotropic;
- The flow in the x-direction is driven by an exponentially time dependent pressure gradient;
- The flow is unsteady and fully developed so that all physical characteristics except pressure are functions of y and t only; and
- The velocity field, the pressure distribution and the yield stress vary exponentially with time.

In view of these assumptions, the basic equations reduced to

$$\frac{\partial u_i}{\partial t} = \frac{-1}{\rho_i}\frac{\partial p_i}{\partial x} + \frac{\mu}{\rho_1}\frac{\partial \sigma^i_{xy}}{\partial y} \quad \text{Where i =1, 2}$$

Where $\sigma^i_{xy} = \mu_i \frac{\partial u_i}{\partial y} - \tau_0$, i = 1

$$= \mu_i \frac{\partial u_i}{\partial y} \quad , \text{i =2} \tag{1}$$

Where i = 1 corresponds to flow in zone1,
i = 2 corresponds to flow in zone 2
The flow in the porous region is governed by

$$\frac{\partial u_3}{\partial t} = -\frac{1}{\rho_2}\frac{\partial p_3}{\partial x} + \frac{\mu_2}{\rho_2}\frac{\partial^2 u_3}{\partial y^2} - \frac{\mu_2}{\rho_2 k}u_3 \tag{2}$$

The boundary conditions are given by

$$\mu_1 \frac{du_1}{dy} = \tau_0 \quad \text{at} \quad y = o \tag{3}$$

$$u_1 = u_0 \text{ at } \quad y = h_1 \tag{4}$$

$$u_2 = u_0 \text{ at } \quad y = h_1 \tag{5}$$

$$u_2 = u_3 \text{ at } \quad y = h \tag{6}$$

$$\mu_2 \frac{du_2}{dy} = \mu_2 \frac{du_3}{dy} \quad \text{at } y = h \tag{7}$$

$$u_3 = 0 \text{ at } \quad y = 1 + \varepsilon \tag{8}$$

It is convenient to introduce the following non dimensional quantities.

$$u_i^* = \frac{u_i}{u_{av}},\ p_i^* = \frac{p}{\rho u_{av}^2},\ x^* = \frac{x}{h},\ y^* = \frac{y}{h},\ t^* = \frac{t u_{av}}{h},$$

$$\tau_0^* = \frac{\tau_0}{\rho_1 u_{av}^2}$$

After non dimensionalisation the basic equations (1) and (2) and the boundary conditions (3)-(8) can be expressed as

$$\frac{\partial u_i}{\partial t} = -\frac{\partial p_i}{\partial x} + \frac{1}{R_i}\frac{\partial^2 u_i}{\partial y^2} \quad , \text{i = 1,2} \tag{9}$$

$$\frac{\partial u_3}{\partial t} = \frac{\partial p_3}{\partial x} + \frac{1}{R_2}\frac{\partial^2 u_3}{\partial y^2} - \frac{\sigma^2}{R_2}u_3 \tag{10}$$

$$\frac{\partial u_1}{\partial y} = \sigma_0 R_1 \quad \text{at } y = 0 \tag{11}$$

$$u_1 = u_0 \quad \text{at} \quad y = \alpha \tag{12}$$

$$u_2 = u_0 \quad \text{at} \quad y = \alpha \tag{13}$$

$$u_2 = u_3 \quad \text{at } y = 1 \tag{14}$$

$$\frac{du_2}{dy} = \frac{du_3}{dy} \quad \text{at } y = 1 \tag{15}$$

$$u_3 = 0 \text{ at } \quad y = 1 + \varepsilon \tag{16}$$

Where $\alpha = \frac{h_1}{h}$, $R_i = \frac{u_{av} h \rho_i}{\mu_i}$

SOLUTION OF THE PROBLEM

In view of assumption (d), it follows that

$$\frac{\partial p_i}{\partial x} = -\text{pi}\, e^{ct},\quad u_i = S_i e^{ct} \quad \sigma_0 = \tau_0 e^{ct}, \text{(i = 1, 2, 3)} \tag{17}$$

Using (17), the governing equations (9) and (10) and the boundary conditions (11)-(15) can be written as

$$\frac{d^2 S_i}{dy^2} - cR_i S_i = -p_i R_i \quad \text{i = 1, 2} \tag{18}$$

$$\frac{d^2 S_3}{dy^2} - (cR_2 + \sigma^2)S_3 = -p_2 R_2 \tag{19}$$

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$\frac{ds_1}{dy} = \tau_0 R_1$ At y =0 (20)

$S_1 = S_0$ at $y = \alpha$ (21)

$S_2 = S_0$ at $y = \alpha$ (22)

$S_2 = S_3$ at y =1 (23)

$\frac{dS_2}{dy} = \frac{dS_3}{dy}$ At y = 1 (24)

$S_3 = 0$ at $y = 1 + \varepsilon$ (25)

Solving (18) and (19) subject to the boundary conditions (20)-(25) we get the velocities in the porous and nonporous regions as given below

ZONE-1

$S_1 = A_1 \cosh a_1 y + B_1 \sinh a_1 y + \frac{p_1}{c}$ (26)

Where $A_1 = \frac{1}{Cosh(a_1\alpha)}[(S_0 - \frac{p_1}{c}) - \frac{\tau_0 R_1}{a_1} Sinh(a_1\alpha)]$

$B_1 = \frac{\tau_0 R_1}{a_1}$ $a_1 = \sqrt{cR_1}$

By putting $y = y_0$ in (26), we get the plug flow velocity as

$S_p = A_1 \cosh a_1 y_0 + B_1 \sinh a_1 y_0 + \frac{p_1}{c}$ (27)

ZONE-2

$S_2 = A_2 \cosh a_2 y + B_2 \sinh a_2 y + \frac{p_2}{c}$ (28)

Where $A2 = d_{16} S_0 + d_{17}$, $B_2 = d_{14} S_0 + d_{15}$, $a_2 = \sqrt{cR_2}$

POROUS REGION

$S_3 = A_3 \cosh a_3 y + B_3 \sinh a_3 y + \frac{p_3 R_2}{a_3^2}$ (29)

Where $a_3 = \sqrt{cR_2 + \sigma^2}$, $A3 = \frac{d_8(d_{12}S_0 + d_{13}) - d_9 d_{11}}{d_8 d_{10} - d_7 d_{11}}$;

$B_3 = \frac{d_9 d_{10} - d_7(d_{12}S_0 + d_{13})}{d_8 d_{10} - d_7 d_{11}}$ $d_1 = \sinh a_2(\alpha - 1)$, $d_2 = -\cosh a_3 Cosh(a\alpha)$

$d_3 = -Sinh a_3 Cosh(a_2\alpha)$,

$d_4 = \frac{p_3}{c}\cosh a_2 - (\frac{p_3 R_2}{a_3^2} - \frac{p_3}{c})\cosh(a_2\alpha)$,

$d_5 = -a_2 Cosh a_2(\alpha - 1)$,

$d_6 = \frac{p_3 a_2}{c} Sinh a_2$, $d_7 = Cosh a_3(1+\varepsilon)$, $d_8 = Sinh a_3(1+\varepsilon)$,

$d_9 = -\frac{p_3 R_2}{a_3^2}$, $d_{10} = a_3 d_1 d_3 - d_2 d_5$,

$d_{11} = a_3 d_1 d_2 - d_3 d_5$, $d_{12} = -(a_2 d_1 Sinh a_2 - d_5 Cosh a_2)$

$d_{13} = d_4 d_5 + d_1 d_6$, $d_{14} = \frac{d_2 d_8 d_{12} - d_3 d_7 d_{12}}{d_1(d_8 d_{10} - d_7 d_{11})} + \frac{\cosh a_2}{d_1}$,

$d_{15} = \frac{d_2 d_8 d_{13} - d_2 d_9 d_{11} - d_3 d_7 d_{13} + d_3 d_9 d_{10}}{d_1(d_8 d_{10} - d_7 d_{11})} + \frac{d_4}{d_1}$,

$d_{16} = \frac{1 - d_{14}\sinh a_2\alpha}{\cosh a_2\alpha}$, $d_{17} = \frac{-(d_{15} Sinh(a_2\alpha) + \frac{p_3}{c}}{Cosh(a_2\alpha)}$

DETERMINATION OF INTERFACE VELOCITY

The continuity of shear stress gives the condition (in non dimensional form)

$\frac{ds_1}{dy} - \tau_0 R_1 = \mu \frac{ds_2}{dy}$, at $y = \alpha$ (30)

Using (30), the interface velocity is obtained as

$$S_0 = \frac{[\mu d_{17} a_2 \sinh(a_2\alpha) + \mu d_{15} a_2 Cosh(a_2\alpha) + \frac{p_1}{c} a_1 Tanh(a_1\alpha) + \tau_0 R_1 Sinh(a_1\alpha) Tanh(a_1\alpha) - \tau_0 R_1 Cosh(a_1\alpha) + \tau_0 R_1)]}{a_1 Tanh(a_1\alpha) - \mu d_{16} a_2 \sinh(a_2\alpha) - \mu d_{14} a_2 \cosh(a_2\alpha)} \quad (31)$$

MASS FLOW

The dimensional mass flow rate per unit width of the channel is

$Q = Q_0$ ECT where $Q_0 = F_1 + F_2$ (32)

$F_1 = \int_0^\alpha S_1 dy = \int_0^\alpha (A_1 \cosh a_1 y + B_1 \sinh a_1 y + \frac{p_1}{c}) dy =$

$= \frac{A_1}{a_1} \sinh a_1\alpha + \frac{B_1}{a_1}(\cosh a_1\alpha - 1) + \frac{p_1}{c}\alpha$

$F_2 = \int_\alpha^1 S_2 dy = \int_\alpha^1 (A_2 \cosh a_2 y + B_2 \sinh a_2 y + \frac{p_2}{c}) dy =$

$\frac{A_2}{a_2}[\sinh a_2 - \sinh a_2\alpha] + \frac{B_2}{a_2}[\cosh a_2 - \cosh a_2\alpha] + \frac{p_2}{c}(1-\alpha)$

SHEAR STRESS

The dimensionless shear stress in the channel is given by

$\tau_{xy} = \tau_0 \; 0 < y \le y_0$

$= a_1 A_1 \sinh a y + a_1 B_1 \cosh a y - \tau_0$, $y_0 < y < h_1$

$= a_2 A_2 \sinh a_2 y + a_2 B_2 \cosh a_2 y$, $h_1 < y < 1$ (33)

DISCUSSIONS

The variation of interface velocity with the permeability parameter σ is calculated from the equation (31) for different viscosity ratios and is shown in Figure-3 for fixed values of $R_1 = 3$, $R_2 = 7$, $\alpha = 0.7$, $\varepsilon = 0.5$, $\tau_0 = 0.5$, $\mu = 3$. We observe that the interface velocity decreases with increasing permeability parameterσ. For a given, S_0 decreases with increasing viscosity ratio μ ***i.e.*** the interface velocity decreases with the increase in the peripheral layer viscosity.

The variation of velocity with y is calculated numerically for different values of the viscosity ratios and is shown in Figure-2 for fixed values of $R_1 = 3$, $R_2 = 7$, $\tau = 0.2$, $\mu = 0.3$, $\varepsilon = 0.3$. It is observed that the velocity is constant in the plug flow region of the core layer. This is due to the presence of yield stress in the Bingham fluid. Also the velocity increases to a maximum value in the non plug flow region of the core layer for $\mu < 1$. After attaining the maximum value the velocity decreases in the peripheral layer and porous regions with the increment in y.

In the non plug flow region the velocity profile is a parabola for $\mu = 1$. When $\mu > 1$, the velocity curves are parabolas in the non plug flow region of the core layer and in the peripheral layer. When the ratio of viscosities of the peripheral and core layers decreases, the velocity increases in all the three regions.

The variation of the interface velocity for different values of porosity ε is shown in the Figure-3. It is observed that the interface velocity decreases with an increase in ε.

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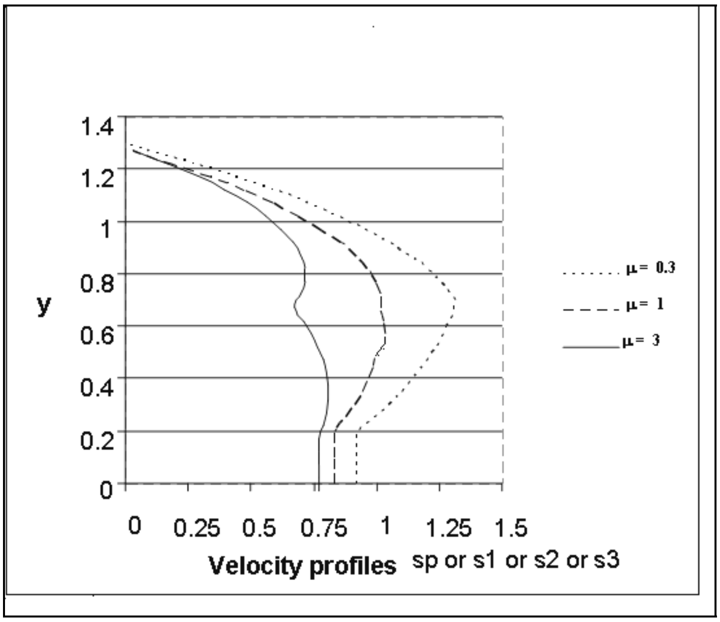

Figure-2. The velocity profiles in different regions for varying y.

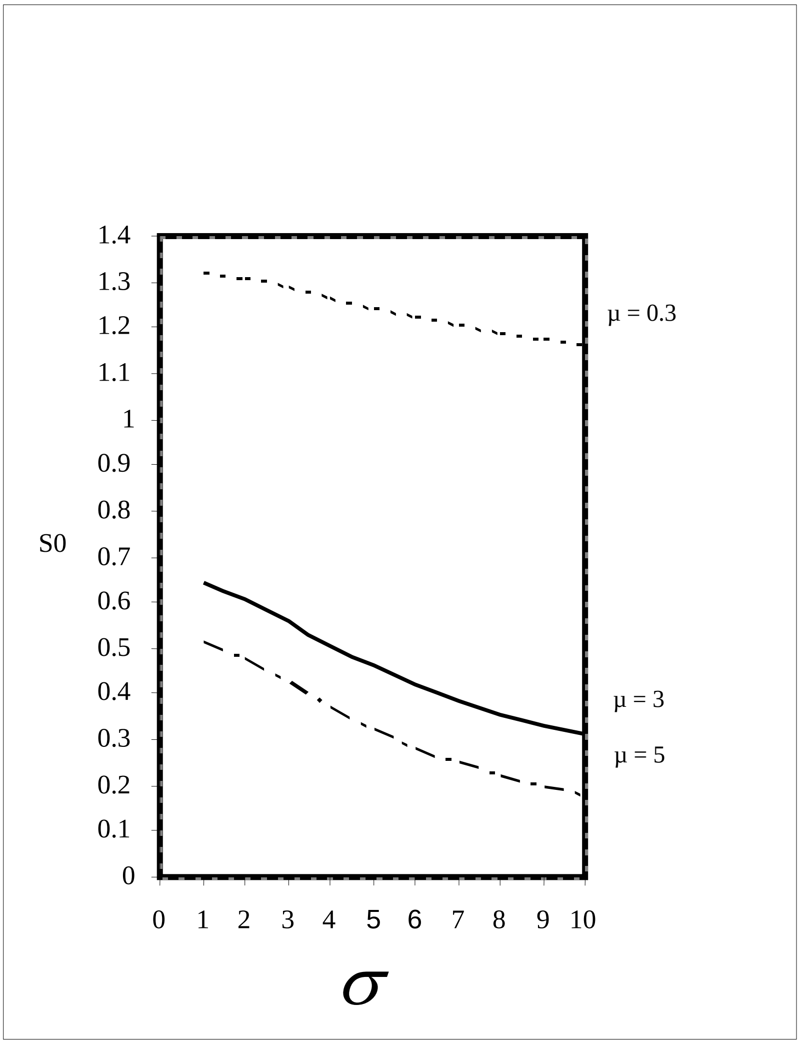

Figure-3. The Interface velocity (S0) for varying σ for different μ.

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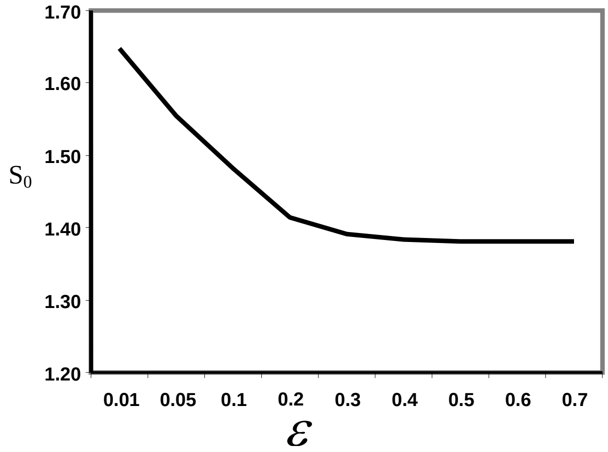

Figure-4. The interface velocity (S0) for varying ε .