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INVERSE METHOD TO DETERMINE ELASTIC CONSTANTS USING A SIMPLY SUPPORTED CIRCULAR PLATE AND STRAIN-GAUGE

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ABSTRACT

We proposed an inverse method, using a simply supported circular plate under lateral concentrated load, for the simultaneous determination of two elastic constants E and υ from strain data of two-element strain-gage rectangular rosette. Several series and different position of strain-gauges provided full field information about strain field of the plate. An overdeterministic approach using least-square method is implemented to fit the experimentally determined strain field to the theoretical solution. Accuracy of the proposed method is verified experimentally.

INTRODUCTION

Determining the elastic constants of material by analyzing their strain field is a known technique. Its non destructive and economics character, accuracy of the result provided, simplicity, and ease of implementation make it attractive for research as well as industrial environments.

A simply supported circular plate with lateral concentrated load is an experimental configuration that is easy to realize. With a well established theoretical strain field [1-3], a classical coefficient inverse approach can be implemented to determine elastic constants of material from the experimentally strain field.

Full field strain information is obtained from two-element strain-gage rectangular rosette, which is bonded point to point to the specimen in different and series radius position. Error during strain measurement caused by non linearity of the strain field and caused by transfer sensitivity of the strain-gauge can be easily corrected by well established strain measurement theory [1, 4].

In the previous paper the feasibility of using a simply supported circular plate was investigated [5]. However, only non linearity of the strain field was considered with only one radius position of the strain-gage.

In this paper, an inverse approach is proposed to determine two elastic constants from the strain field information provided by two-element strain-gage rectangular rosette. Correction of error during measurement, both caused by non linearity of strain field and transfer sensitivity of the strain-gauge is considered. The strain information is obtained at every point in different radius, which allows the use of an over deterministic analysis by the least-square method. The young modulus and poison ratio are determined simultaneously from the strain measured. To verify the proposed method, we implemented an experiment to determine the elastic constants of aluminum.

BACKGROUND

Strain field in simply supported circular plate

Consider simply supported circular plate shown in Figure-1. Concentrated load P is applied on its center of the plates and R is radius of the support. The governing differential equation of plate (in bending) in polar coordinate is given as [1, 3]:

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)=\frac{p(r,\theta)}{D} \tag{1}$$

$$p(r,\theta)=\begin{cases}0 & r\neq 0\\ P/A & r=0\end{cases},\qquad D=\frac{Eh^3}{12\left(1-v^2\right)} \tag{2}$$

Where h is the thickness of the plate, p(r,θ) is the function of stress, P is the acting external force and A is surface area

The boundary condition associated with equation (1) and equation (2) for simply supported circular plate is [1, 3]:

$$w(R)=0\ \ and\ \left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)_{r\,=\,R}=0 \tag{3}$$

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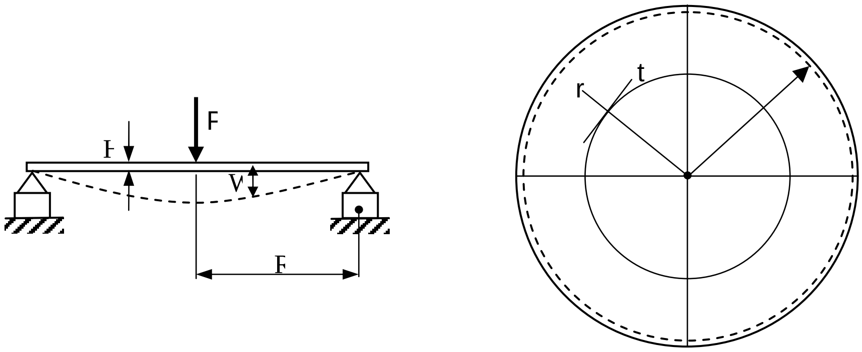

Figure-1. Simply supported circular plate with applied concentrated load P on its center.

Solving equations (1), (2) and (3), obtains the function of deflection in the following form:

$$w(r) = \frac{P}{8\pi D}\left[\frac{3+\nu}{1+\nu}\left(R^2 - r^2\right) - 2r^2 \ln\left(\frac{R}{r}\right)\right] \quad (4)$$

Strain field of the plate in tangential direction (ε_t) and in radial direction (ε_r) is given by the equation of [1, 3]:

$$\varepsilon_r = -z\frac{d^2 w}{dr^2} \qquad \varepsilon_t = -\frac{z}{r}\frac{dw}{dr} \quad (5)$$

The corresponding analytical form of strain field on the surface of the plate (z = h/2) are:

$$\varepsilon_t = \frac{Ph}{8\pi D}\left(\frac{1}{1+\nu} + \ln\frac{R}{r}\right) \quad (6)$$

$$\varepsilon_r = \frac{Ph}{8\pi D}\left(\frac{1}{1+\nu} + \ln\frac{R}{r} - 1\right) \quad (7)$$

Graph of ε_t and ε_r are plotted in Figure-2. The support radius R = 105 mm and plate thickness h = 8.5 mm, the applied load was 200 N and the elastic constants for aluminum
(E = 70 GPa and υ = 0.30) were used for the assumption.

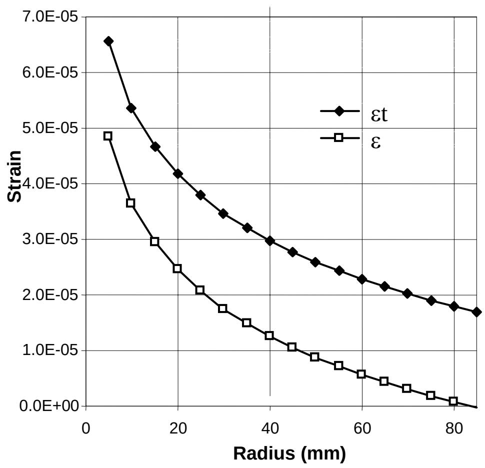

Figure-2. Graph of ε_t and ε_r as a function of radius r.

Two-element strain-gage rectangular rosette

Strain-gage is probably the most common method of strain measurement on experimental analysis utilized by many industrial and Scientifics applications [1]. In this method, strain-gage is deform together with the specimen, strain to be measured is the strain at the center of the gage, but the obtained measurement is the average strain of all gage's wire. Equation (6) and (7) show non-linearity of the strain field in the form of logarithmic function of radius r, error will be detected during strain measurement. This error should be corrected by a correction factor. Consider two-element strain-gage rectangular rosette in Figure-3, theoretical strain at the center of the gage (ε_t and ε_r) are given by equation (6) and (7), while the measured strain is $\varepsilon_{t,m}$ and $\varepsilon_{r,m}$. Due to non linearity of the strain field, the average strain in all wire are :

$$\varepsilon_{t,av} = \sum_{n=0}^{N}\frac{1}{a}\int_0^a \varepsilon_X\left(r - \frac{b}{2} + n\xi\right)dx \quad (8)$$

$$\varepsilon_{r,av} = \sum_{n=0}^{N}\frac{1}{a}\int_0^a \varepsilon_y\left(-\frac{b}{2} + n\xi\right)dy \quad (9)$$

Where a = gauge length, b = gauge width, ζ distance between wire and N = number of wires.
The measured strain should be corrected by correction factor C_t and C_r.

$$C_t = \frac{\varepsilon_t}{\varepsilon_{t,av}} \qquad C_r = \frac{\varepsilon_r}{\varepsilon_{r,av}} \quad (10)$$

$$\hat{\varepsilon}_t = C_t\varepsilon_{t,m} \qquad \hat{\varepsilon}_r = C_r\varepsilon_{r,m} \quad (11)$$

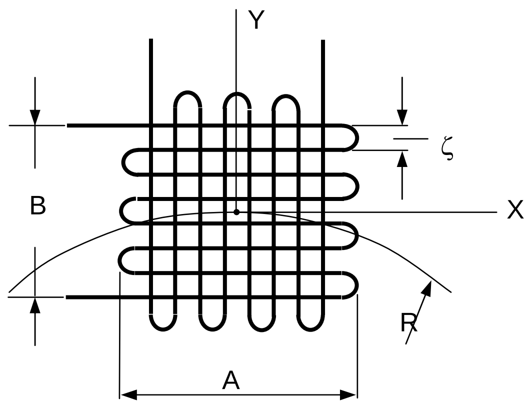

Figure-3. Two-element strain-gauge rosette 90°.

This formula is obtained the using correction factor C_r and C_t that are closed to unity. For simply supported circular plate with support radius R = 220mm, gauge position r = 45mm, gauge length a = 29mm, distance between wire ζ = 0.346mm, N = 14 and υ = 0.3 these value are C_r = 0.99 and C_t = 1.04 [5]

Additional second correction is required to compensate error caused by the influence of transfer sensitivity k_t on gage factor S_g. These corrections are in the following form [1, 4]:

$$\varepsilon_t = \frac{1-\upsilon_0 k_t}{1-k_t^2}\upsilon\left(\hat{\varepsilon}_t - k_t\hat{\varepsilon}_r\right) \qquad (12)$$

$$\varepsilon_r = \frac{1-\upsilon_0 k_t}{1-k_t^2}\left(\hat{\varepsilon}_r - k_t\hat{\varepsilon}_t\right) \qquad (13)$$

Over deterministic Inverse Approach

Sensitivity of the strain field

Equations (6) and (7) show that the elastic constants E and υ are coupled non-linearly. Therefore E and υ can be determined simultaneously using both equation. This approach requires accurate and sufficient data of the strain in radial and tangential direction from some two-element strain-gages rectangular rosettes. These gages should be placed in proper places and in different radius position.

The first consideration is sensitivity of ε_t and ε_r for combination of elastic constants E and υ as shown in graph of Figure-4. These graphs used the assumption of R = 105mm, P = 300 N, h = 8.5mm E = 70 GPa and υ = 0.30. It can be inferred from this graph that strain ε_t and ε_r will loose their sensitivity for r greater than 60 mm.

The second consideration is the sensitivity of ε_t and ε_r due to error caused by inaccurate position of the strain-gage; this is shown in graph of Figure-4b. In this graph, the strain-gage are placed at radius of r = r + 1 mm. It is shown, that ε_t and ε_r will loose their sensitivity for radius of r greater than 55 mm. This means that the error during the strain measurement in this radius will not be detected.

All graph show that strain ε_t and ε_r will increase excessively for r close to the plate's center. If the load is Increase, this zone may reads to the plastic zone, so that huge error will be detected on the strain measurement. It is clear that for simply supported circular plate being considered, the strain-gage should be placed at the radius of less than 55 mm but not closed to the plate's center

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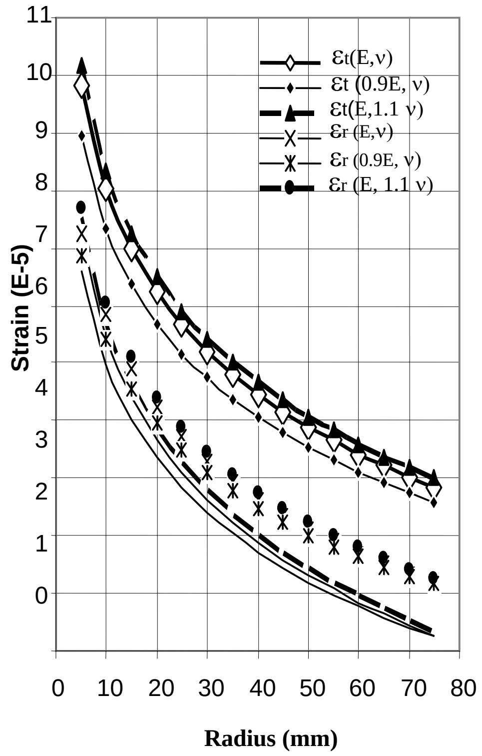

Figure-4.a. Sensitivity of strain field to elastics constant e and ν.

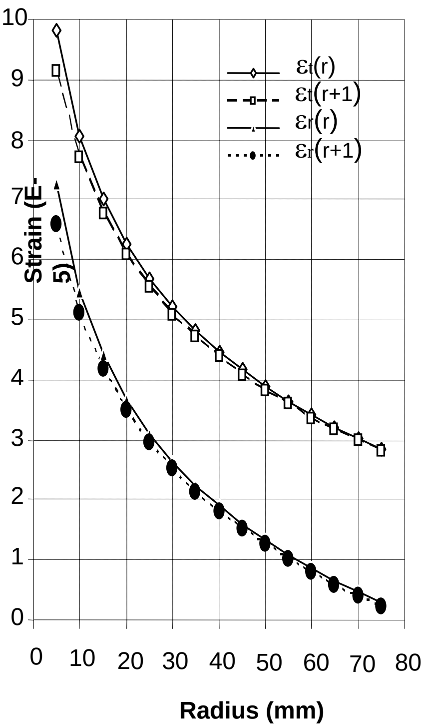

Figure-4.b. Sensitivity of strain field to r = r + 1.

Least-square analysis

The least-square method has been used in a regression analysis. The basic assumption underlying this approach is that there are always differences between experimental result and theoretical value [6]. Base on least-square algorithm by Sanford [5] and noting equation 6 and 7, the relation between experimental results and theoretical value can be expressed as:

$$f_{t,k}(E,\nu)=\frac{CE_k}{1-\upsilon_k^2}\left(\frac{1}{1+\upsilon_k}+\ln\frac{R}{r_k}-1\right)-\varepsilon_{t,k}=0 \qquad (14)$$

$$f_{r,k}(E,\upsilon)=\frac{CE_k}{1-\upsilon_k^2}\left(\frac{1}{1+\upsilon_k}+\ln\frac{R}{r_k}-1\right)-\varepsilon_{r,k}=0 \qquad (15)$$

Where $C=1.5\frac{P}{8\pi h^2}$, h = plate's thickness and k = 1, 2, M, are the points corresponding with the location of the strain-gages.

Taylor series expansion of equation 14 and 15 yield:

$$\left(f_{t,k}\right)_{i+1}=\left(f_{t,k}\right)_i+\left(\frac{\partial f_{t,k}}{\partial E}\right)_i\Delta E+\left(\frac{\partial f_{t,k}}{\partial\upsilon}\right)_i\Delta\upsilon \qquad (16)$$

$$\left(f_{r,k}\right)_{i+1}=\left(f_{r,k}\right)_i+\left(\frac{\partial f_{r,k}}{\partial E}\right)_i\Delta E+\left(\frac{\partial f_{r,k}}{\partial\upsilon}\right)_i\Delta\upsilon \qquad (17)$$

Where i am refer to the i_{th} step of iteration. It is evident from equation 16 and 17 that correction should be made given $f_{t,k}$ (E, υ) = 0 and $f_{r,k}$ (E,υ) = 0. This fact leads to the iterative equation:

$$\left(\frac{\partial f_{t,k}}{\partial E}\right)_i\Delta E+\left(\frac{\partial f_{t,k}}{\partial\upsilon}\right)_i\Delta\upsilon=-\left(f_{t,k}\right)_i \qquad (18)$$

$$\left(\frac{\partial f_{r,k}}{\partial E}\right)_i\Delta E+\left(\frac{\partial f_{r,k}}{\partial\upsilon}\right)_i\Delta\upsilon=-\left(f_{r,k}\right)_i \qquad (19)$$

This yield:

$$\begin{bmatrix}\frac{\partial f_{t,k}}{\partial E} & -\frac{\partial f_{t,k}}{\partial\upsilon}\\ \frac{\partial f_{r,k}}{\partial E} & -\frac{f_{r,k}}{\partial\upsilon}\end{bmatrix}_i\begin{Bmatrix}\Delta E\\ \Delta\upsilon\end{Bmatrix}_i=\begin{Bmatrix}f_{t,k}\\ f_{r,k}\end{Bmatrix}_i \qquad (20)$$

The value of E and υ can be determined simultaneously by solving equation 6 and 7, and optimized by equation 20.

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EXPERIMENTAL SET UP

An experimental for strain measurement with two-element strain-gage rectangular rosette was conducted to verify the proposed method. The specimen was a circular plate made from aluminum alloy shown in Figure-5b. It's diameter was 215 mm and the thickness was 8.5 mm. Strain gages were placed at the radius of 25mm to 50 mm with interval of 5 mm and placed on both side of the plate, i e. at the compression side and at the tension side. Strain-gage was two-element rectangular rosette with dimension of length a=11mm, b=5mm, and the number of wire N=7, the transfer sensitivity was K_t= 0.001 and gage factor S_g=2.14

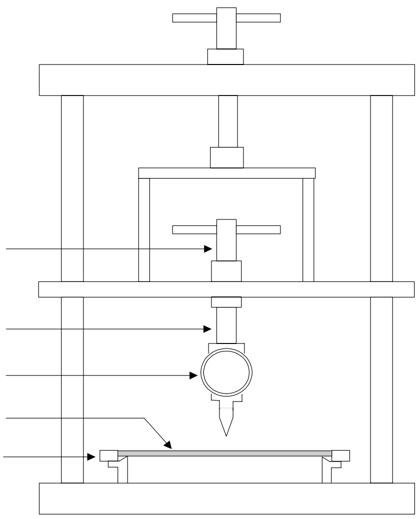

Figure-5a. Experimental set up.

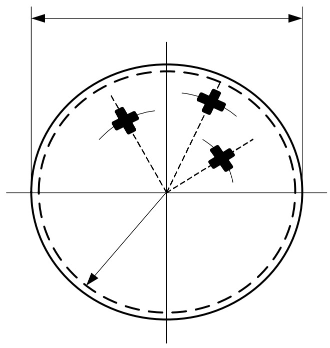

Figure-5b. Specimen.

A general purpose tension and compression machine was utilized for the experiment as shown in Figure-5a. The specimen was supported at radius of R = 105 mm. To ensure the proper position for the measurement, a pair of fixture was used to hold the specimen during setting and it will be removed before loading. The loading was achieved by turning the lead screw clockwise were produced linear movement of the load bar. This movement was transferred through a spherical indenter with radius of 1 mm and assumed to be a point. The load was measured with an accuracy of 0.1 N by an electrical load cell. All strain data produced by strain-gauges were recorded simultaneously by a precision data logger.

RESULTS AND DISCUSSIONS

The Load of 200 N and 300 N are applied to the specimen. All strain data recorded by precision data logger would be corrected. First correction is made by applying factor of correction C_t and C_r in equation 10 and the secondly corrected by a sensitivity factor K_t in equation 12 and 13; material constants E and ν were calculated by equation 6 and 7, the results are optimized using the least-square algorithm in equation 20. A set of data before and after correction is displayed in Table-1 along with the data after least-square.

Table-1 shows that the values of optimized data are closed to the value of corrected data. These indicate that the data during strain measurement were accurate.

It was seen that in higher radius the differences between measured strain and corrected strain became lower. At R = 50 mm measured and corrected strain are same. This is match with the over deterministic approach that ε_t and ε_r will loose their sensitivity for radius greater than 55 mm.

The elastic constants were evaluated are summarized in Table-2.

Lead Screw

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Table-1. Experimental data.

Load	**Strain**		**Radius** (strain gauge position) **mm**					
			25	**30**	**35**	**40**	**45**	**50**
200 N	Strain measured	ε_t	38.15	35.77	33.10	30.73	28.74	26.58
	(x 10^{-6})	ε_r	21.05	18.05	15.41	12.94	10.78	8.88
	Strain corrected	ε_r	38.27	35.81	33.11	30.70	28.70	26.58
	(x 10^{-6})	ε_t	20.83	17.92	15.37	12.90	10.72	8.88
	Optimized data	ε_t	38.80	35.65	32.87	30.52	28.45	26.60
	(x 10^{-6})	ε_r	21.20	17.99	15.28	12.93	10.86	9.00
300 N	Strain measured	ε_t	57.51	52.88	48.82	46.74	42.90	38.96
	(x 10^{-6})	ε_r	31.65	26.90	22.60	19.67	16.29	13.92
	Strain corrected	ε_r	57.69	52.94	48.82	46.70	42.84	38.96
	(x 10^{-6})	ε_t	31.32	26.66	22.58	19.66	16.74	14.14
	Optimized data	ε_t	58.05	53.26	49.21	45.70	42.61	39.84
	(x 10^{-6})	ε_r	31.78	26.99	22.94	19.43	16.34	13.57

Table-2. Experimental result.

Load	**E** (GPa)	υ
200 N	70.05 ± 0.015	0.30 ± 0.005
300 N	69.9 ± 0.03	0.290 ± 0.005
Hand book [7]	70-79	0.33

The comparison with the hand book data for aluminum alloy (E = 70-79 GPa and υ = 0.33) show that the value of E is satisfactory but the value of υ is slightly smaller for load of 300 N, which confirm the validity of the proposed method.

CONCLUSIONS

An inverse method has been proposed to determine the elastic constants E andυ. The method is base on the theoretical strain field of simply supported circular plate under lateral concentrated load and strain-gage. The overdeterministic approach using the least-squares method was implemented to fit the experimentally strain data to the theoretical solution. A computer simulation was conducted to investigate the effect of variation of E and υ to the sensitivity of the strain field. It was found that the result were sensitivity of the strain field of plate being considered is less for radius greater than 55 mm.

An experimental with two-element strain-gage rectangular rosette was conducted to verify the proposed method experimentally. With two strain field ε_t and ε_r, two elastic constants E and υ were determined simultaneously with enough accuracy.

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