RADIATION AND MASS TRANSFER EFFECTS ON MHD FREE CONVECTION FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE

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ABSTRACT

An analytical study is performed to study the effects of thermal radiation on unsteady free convection flow past an exponentially accelerated infinite vertical plate with mass transfer in the presence of magnetic field. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. The plate temperature is raised linearly with time and the concentration level near the plate is raised to C'_w. The governing equations are solved in closed form by the Laplace transform technique. The influence of various parameters, entering in the problem, on the velocity field and skin friction is discussed with the help of graphs.

Keywords: thermal radiation, free convection, MHD, exponential, accelerated, vertical plate.

INTRODUCTION

Natural convection induced by the simultaneous action of buoyancy forces from thermal and mass diffusion is of considerable interest in many industrial applications such as geophysics, oceanography, drying processes and solidification of binary alloy. The effect of the magnetic field on free convection flows is important in liquid metals, electrolytes and ionized gases. The thermal physics of MHD problems with mass transfer is of interest in power engineering and metallurgy. When free convection flows occur at high temperature, radiation effects on the flow become significant. Many processes in engineering areas occur at high temperatures, and knowledge of radiative heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines, and the various propulsion devices for aircraft, missiles, and space vehicles are examples of such engineering areas.

Gupta [1] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [2] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen kumar [3]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [4]. Basant kumar Jha [5] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Recently Muthucumaraswamy *et al.* [6] studied mass transfer effects on exponentially accelerated isothermal vertical plate.

Soundalgekar and Takhar [7] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar [8]. Raptis and Perdikis [9] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al.* [10] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique. Muthucumaraswamy and Janakiraman [11] studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion.

It is proposed to study the effects of thermal radiation and mass transfer on unsteady free convection flow past an exponentially accelerated infinite vertical plate with variable temperature in the presence of magnetic field. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

MATHEMATICAL ANALYSIS

The unsteady flow of an incompressible and electrically conducting viscous fluid past an infinite vertical plate with variable temperature has been considered. A magnetic field of uniform strength B_o is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. The flow is assumed to be in x'-direction which is taken along the vertical plate in the up ward direction. The y'-axis is taken to be normal to the plate. Initially the plate and the fluid are at the same temperature T'_∞ in the stationary condition with concentration level C'_∞ at all points. At time $t'>0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the plate temperature is raised linearly with time t and the level of concentration near the plate is raised to C'_w. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. The viscous dissipation is assumed to be negligible. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\beta\left(T'-T'_\infty\right) + g\beta^*\left(C'-C'_\infty\right) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

With the following initial and boundary conditions

$$t' \le 0\,, u' = 0\,, T' = T'_\infty\,,\ C' = C'_\infty \quad \text{for all } y'$$

$$t' > 0\,, u' = u_0\exp(a't')\,, T' = T'_\infty + \left(T'_w - T'_\infty\right)At'\,,$$

$$C' = C'_w \ \text{at}\ y' = 0$$

$$u' = 0\,, T' \to T'_\infty\,, C' \to C'_\infty\,, \text{ as } y' \to \infty \quad (4)$$

Where $A = \frac{u_0^2}{\nu}$. The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T_\infty'^4 - T'^4\right) \quad (5)$$

It is assumed that the temperature differences with in the flow are sufficiently small that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, thus

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} + 16a^*\sigma T_\infty'^3\left(T'_\infty - T'\right) \quad (7)$$

On introducing the following non-dimensional quantities:

$$u = \frac{u'}{u_0}, t = \frac{t'u_0^2}{\nu}, y = \frac{y'u_0}{\nu}, \theta = \frac{T'-T'_\infty}{T'_w - T'_\infty}, G_r = \frac{g\beta\nu\left(T'_w - T'_\infty\right)}{u_0^3}, C = \frac{C'-C'_\infty}{C'_w - C'_\infty}\ G_m = \frac{g\beta^*\nu\left(C'_w - C'_\infty\right)}{u_0^3},$$

$$P_r = \frac{\mu C_p}{\kappa}, S_c = \frac{\nu}{D}, M = \frac{\sigma B_0^2\nu}{\rho u_0^2}, R = \frac{16a^*\nu^2\sigma T_\infty'^3}{\kappa u_0^2},\ a = \frac{a'\nu}{u_0^2} \quad (8)$$

In equations (1) to (4), leads to

$$\frac{\partial u}{\partial t} = G_r\theta + G_m C + \frac{\partial^2 u}{\partial y^2} - Mu \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} - \frac{R}{P_r}\theta \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \quad (11)$$

The initial and boundary conditions in dimensionless form are as follows:

$$t \le 0: \quad u = 0\,, \theta = 0 \quad C = 0 \quad \text{for all } y$$

$$t > 0: \quad u = \exp(at)\,, \theta = t \quad C = 1 \quad \text{at } y = 0$$

$$u = 0\,, \theta \to 0\ C \to 0 \quad \text{as } y \to \infty \quad (12)$$

All the physical parameters are defined in the nomenclature. The dimensionless governing equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace transform technique and the solutions are derived as follows.

$$C = erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}\right) \quad (13)$$

$$\theta = \left(\frac{t}{2} + \frac{yP_r}{4\sqrt{R}}\right)\left[\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}} + \sqrt{\frac{Rt}{P_r}}\right)\right.$$

$$\left. + \left(\frac{t}{2} - \frac{yP_r}{4\sqrt{R}}\right)\left[\exp\left(-y\sqrt{R}\right)erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}} - \sqrt{\frac{Rt}{P_r}}\right)\right]\right] \quad (14)$$

$$u = \frac{\exp(at)}{2}\left[\exp\left(-y\sqrt{a+M}\right)erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(a+M)t}\right)\right.$$

$$\left. + \exp\left(y\sqrt{a+M}\right)erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(a+M)t}\right)\right]$$

$$+\left[\exp\left(-y\sqrt{M}\right)erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Mt}\right)\right]\left[\frac{ctG_r}{2d} - \frac{cyG_r}{4d\sqrt{M}} - \frac{1}{2}\left(\frac{G_r}{d} + \frac{G_m}{M}\right)\right]$$

$$+\left[\exp\left(y\sqrt{M}\right)erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Mt}\right)\right]\left[\frac{ctG_r}{2d} + \frac{cyG_r}{4d\sqrt{M}} - \frac{1}{2}\left(\frac{G_r}{d} + \frac{G_m}{M}\right)\right]$$

$$+\frac{G_r\exp(-ct)}{2d}\left[\exp\left(-y\sqrt{M-c}\right)erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(M-c)t}\right)\right.$$

$$\left. + \exp\left(y\sqrt{M-c}\right)erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(M-c)t}\right)\right]$$

$$+\frac{G_m\exp(bt)}{2M}\left[\exp\left(-y\sqrt{b+M}\right)erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(b+M)t}\right)\right.$$

$$\left. + \exp\left(y\sqrt{b+M}\right)erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(b+M)t}\right)\right]$$

$$+\left[\exp\left(-y\sqrt{R}\right)erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}} - \sqrt{\frac{Rt}{P_r}}\right)\right]\left[\frac{G_r}{2d} - \frac{ctG_r}{2d} + \frac{cyG_rP_r}{4d\sqrt{R}}\right]$$

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$$+\left[\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}}+\sqrt{\frac{Rt}{P_r}}\right)\right]\left[\frac{G_r}{2d}-\frac{ctG_r}{2d}-\frac{cyG_rP_r}{4d\sqrt{R}}\right]$$

$$-\frac{G_r\exp(-ct)}{2d}\left[\exp\left(-y\sqrt{R-cP_r}\right)erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}}-\sqrt{\frac{(R-cP_r)t}{P_r}}\right)\right.$$

$$\left.+\exp\left(y\sqrt{R-cP_r}\right)erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}}+\sqrt{\frac{(R-cP_r)t}{P_r}}\right)\right]$$

$$-\frac{G_m\exp(bt)}{2M}\left[\exp\left(-y\sqrt{bS_c}\right)erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}-\sqrt{bt}\right)\right. \tag{15}$$

$$\left.+\exp\left(y\sqrt{bS_c}\right)erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}+\sqrt{bt}\right)\right]+\frac{G_m}{M}erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}\right)$$

Where $b=\frac{M}{S_c-1}, c=\frac{R-M}{P_r-1}, d=c\left(R-M\right)$

SKIN-FRICTION

We now study skin-friction from velocity field. It is given in non-dimensional form as

$$\tau=\left.\frac{-du}{dy}\right]_{y=0} \tag{16}$$

Then from equations (15) and (16), we have

$$\tau=\exp(at)\left[\sqrt{a+M}.erf\left(\sqrt{(a+M)t}\right)+\frac{\exp\left(-(a+M)t\right)}{\sqrt{\pi t}}\right]$$

$$+\left[\frac{ctG_r}{d}-\frac{G_r}{d}-\frac{G_m}{M}\right]\left[\sqrt{M}.erf\left(\sqrt{Mt}\right)+\frac{\exp(-Mt)}{\sqrt{\pi t}}\right]$$

$$+\frac{cG_r}{2d\sqrt{M}}erf\left(\sqrt{Mt}\right)+\frac{G_r\exp(-ct)}{d}\left[\sqrt{M-c}.erf\left(\sqrt{(M-c)t}\right)+\frac{\exp\left(-(M-c)t\right)}{\sqrt{\pi t}}\right]$$

$$+\frac{G_m\exp(bt)}{M}\left[\sqrt{b+M}.erf\left(\sqrt{(b+M)t}\right)+\frac{\exp\left(-(b+M)t\right)}{\sqrt{\pi t}}\right]-\frac{cP_rG_r}{2d\sqrt{R}}erf\left(\sqrt{\frac{Rt}{P_r}}\right)$$

$$+\left[\frac{G_r}{d}-\frac{ctG_r}{d}\right]\left[\sqrt{R}erf\left(\sqrt{\frac{Rt}{P_r}}\right)+\sqrt{\frac{P_r}{\pi t}}\exp\left(\frac{-Rt}{P_r}\right)\right]$$

$$-\left[\frac{G_r\exp(-ct)}{d}\right]\left[\sqrt{R-cP_r}.erf\left(\sqrt{\frac{(R-cP_r)t}{P_r}}\right)+\sqrt{\frac{P_r}{\pi t}}\exp\left(\frac{-(R-cP_r)t}{P_r}\right)\right]$$

$$-\left[\frac{G_m\exp(bt)}{M}\right]\left[\sqrt{bS_c}.erf\left(\sqrt{bt}\right)+\sqrt{\frac{S_c}{\pi t}}\exp(-bt)\right]+\frac{G_m}{M}\sqrt{\frac{S_c}{\pi t}} \tag{17}$$

GRAPHS

Figure 1. Velocity Profiles when R=4,Pr=0.71,a=0.5,S=2.01,t=0.2

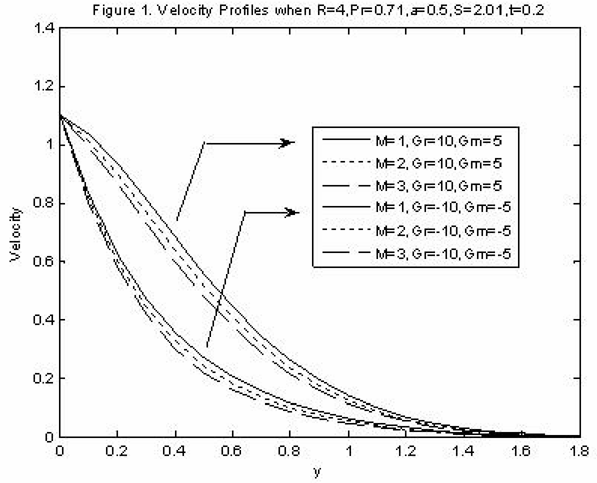


Figure 2. Velocity Profiles when R=4,Pr=0.71,a=0.5,S=2.01,t=0.4

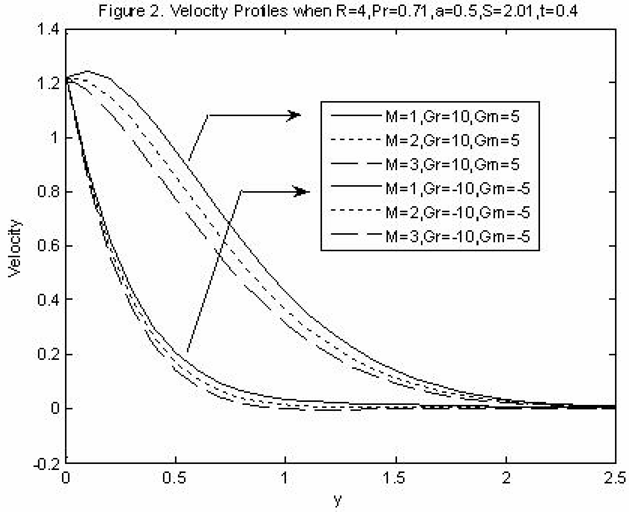
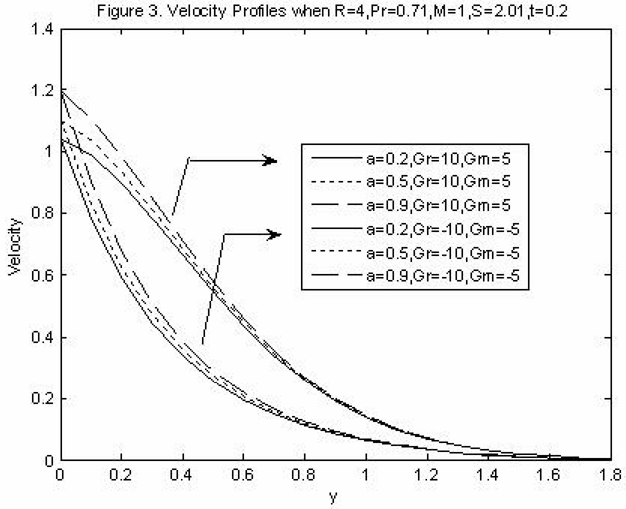


Figure 3. Velocity Profiles when R=4,Pr=0.71,M=1,S=2.01,t=0.2



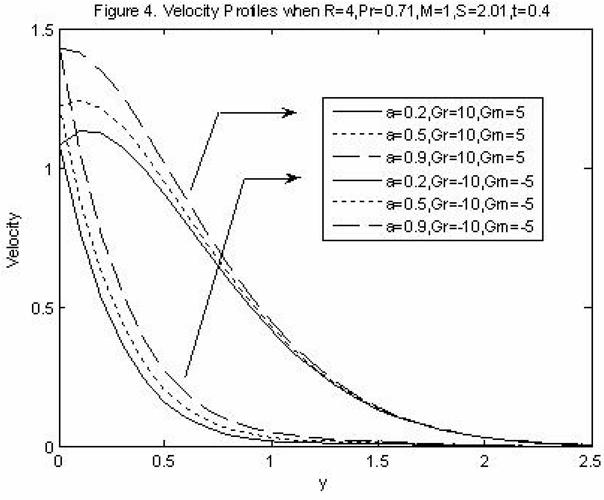


Figure 4. Velocity Profiles when R=4,Pr=0.71,M=1,S=2.01,t=0.4

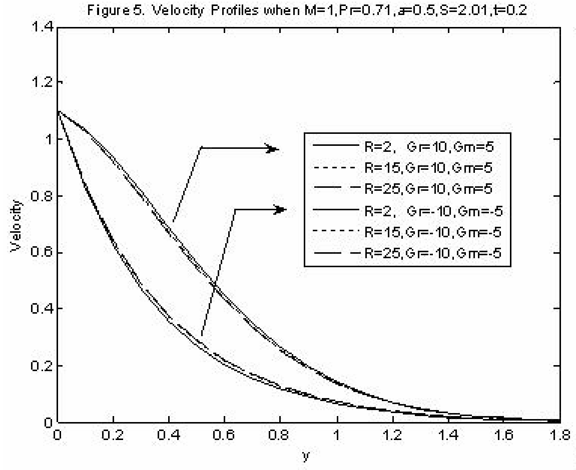


Figure 5. Velocity Profiles when M=1,Pr=0.71,a=0.5,S=2.01,t=0.2

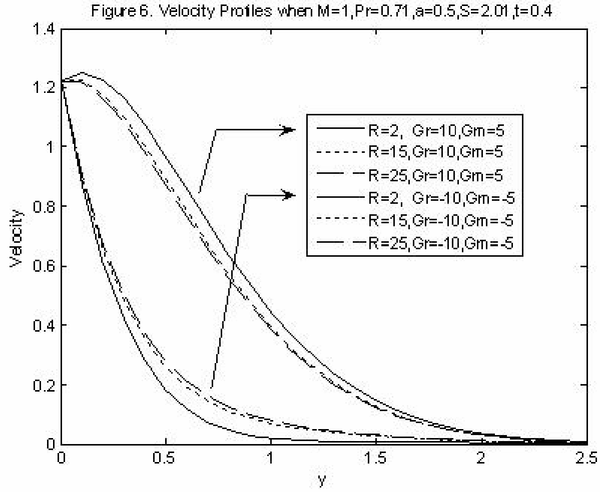


Figure 6. Velocity Profiles when M=1,Pr=0.71,a=0.5,S=2.01,t=0.4

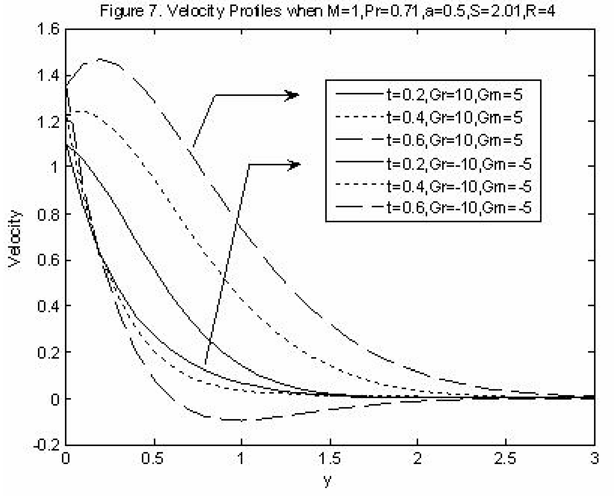


Figure 7. Velocity Profiles when M=1,Pr=0.71,a=0.5,S=2.01,R=4

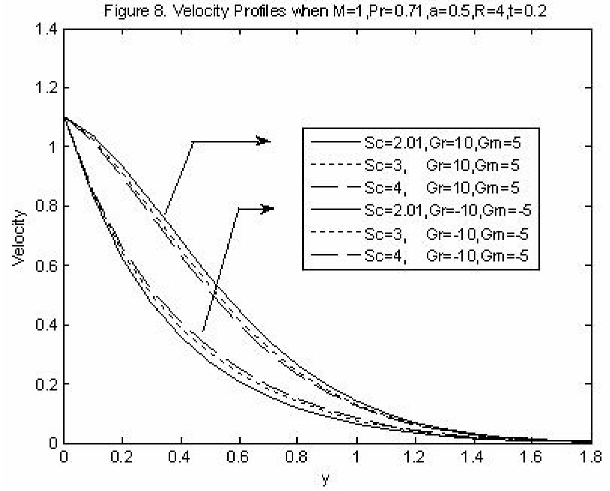


Figure 8. Velocity Profiles when M=1,Pr=0.71,a=0.5,R=4,t=0.2

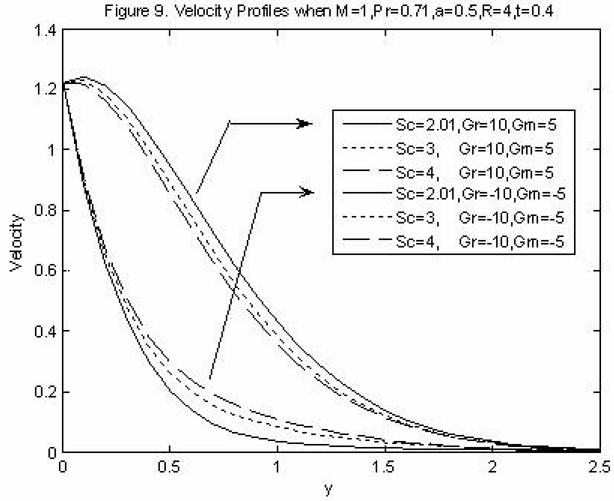


Figure 9. Velocity Profiles when M=1,Pr=0.71,a=0.5,R=4,t=0.4

Figure 10. Velocity Profiles when M=1,Sc=2.01,a=0.5,R=4,t=0.2

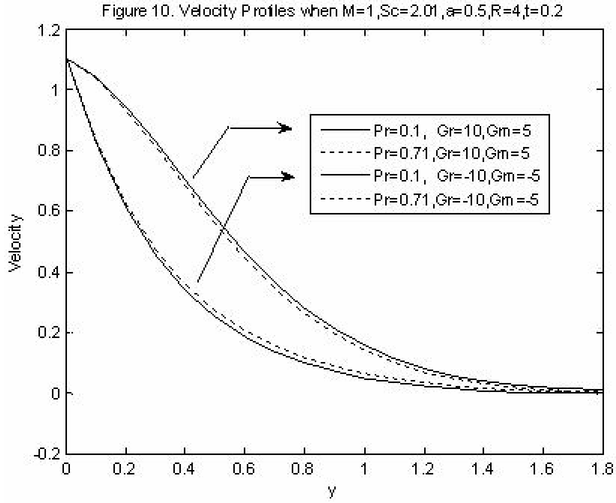


Figure 11. Velocity Profiles when M=1,Sc=2.01,a=0.5,R=4,t=0.4

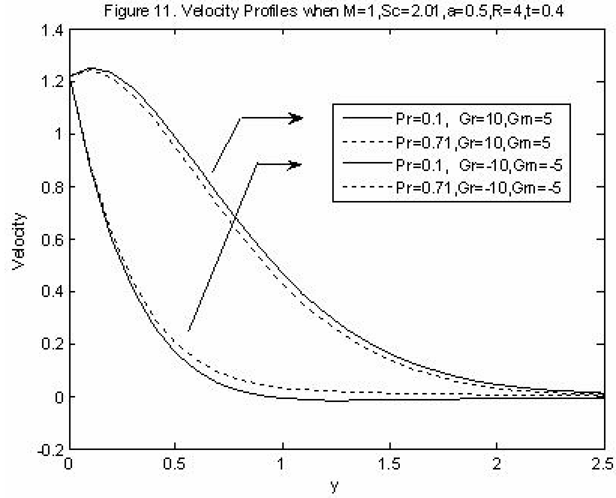


Figure 12. Velocity Profiles when M=1,Pr=0.71,a=0.5,R=4,Sc=2.01,t=0.2

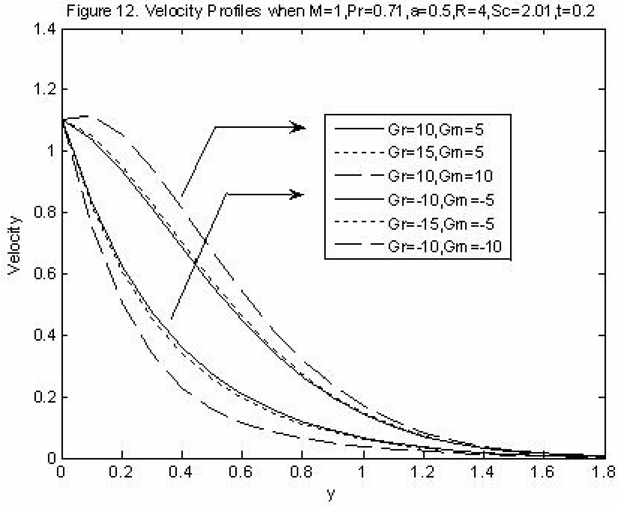


Figure 13. Velocity Profiles when M=1,Pr=0.71,a=0.5,R=4,Sc=2.01,t=0.4

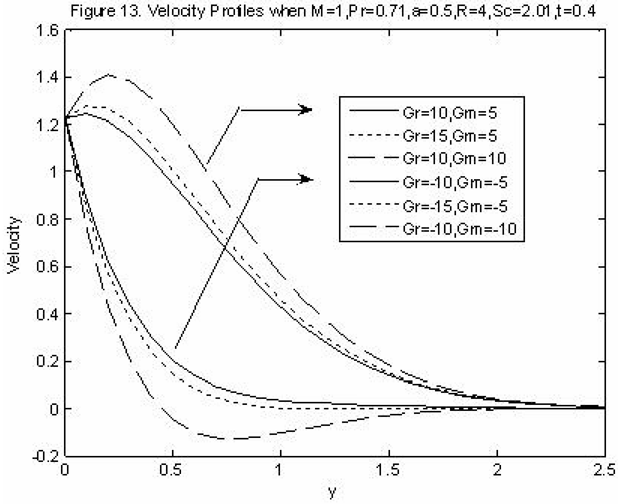


Figure 14. Skin friction

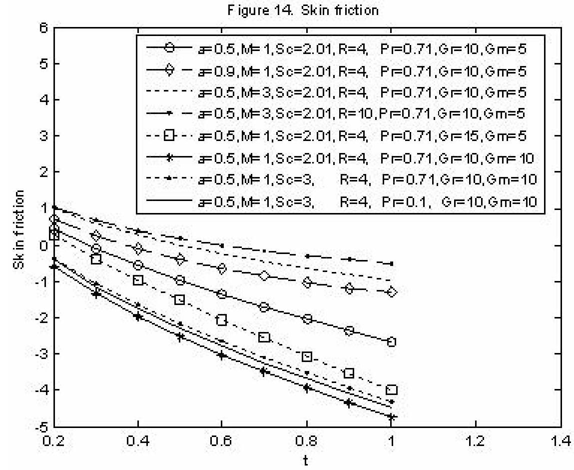
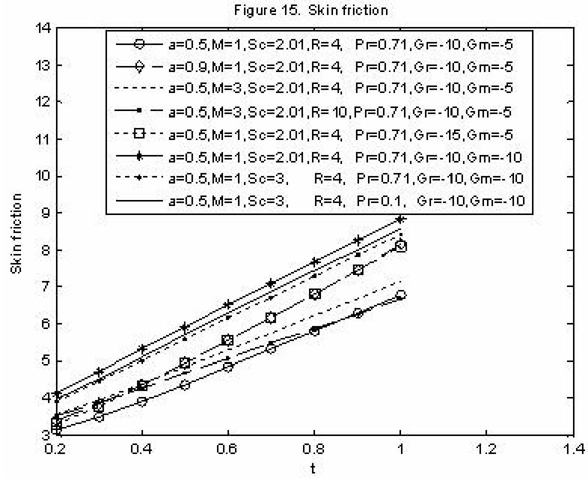


Figure 15. Skin friction



RESULTS AND DISCUSSIONS

The numerical values of the velocity and Skin friction are computed for different physical parameters like magnetic field parameter (M), Radiation parameter(R), Schmidt number(Sc), Thermal Grashof number(Gr), Mass Grashof number(Gm), time(t), Prandtl number(Pr) and Accelerating parameter (a). The purpose of the calculations given here is to study the effects of the parameters M, R, Sc, Gr, Gm, Pr, t and a upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

The velocity profiles for different parameters M, a, R, t, Sc, Pr, Gr and Gm are presented in Figures, 1-13 for the cases of heating (Gr<0, Gm<0) and cooling (Gr>0, Gm>0) of the plate. The heating and cooling take place by setting up free-convection current due to temperature and concentration gradient. From Figures, 1-4 we conclude that the velocity decreases with an increase in M (Magnetic parameter) and increases with increase of a in cases of both cooling and heating of the plate. From Figures, 5-6 and 8-11 we observe that in the case of cooling the velocity decreases with increase in R (Radiation parameter), Sc (Schmidt number) and Pr (Prandtl number). The reverse effect is observed in the case of heating of the plate. From Figures, 12 and 13 it is found that the velocity increases with an increase in Gr (Thermal Grashof number) and Gm (Mass Grashof number) in the case of cooling of the plate and the reverse phenomena is observed in the case of heating of the plate. From Figure-7 it is observed that in the case of cooling the velocity increases with an increase in t, but in the case of heating the velocity increases with increase in t near the plate and the reverse effect is observed away from the plate.

The Skin friction is presented in Figures 14 and 15. From these figures we conclude that the Skin friction increases with an increase in M and a for both cooling and heating of the plate. It is also observed that Skin friction increases with increase in R, Sc and Pr and decreases with increase in Gr and Gm for cooling of the plate. But the reverse effect is observed in the case of heating of the plate. Again it is found that the Skin friction decreases in case of cooling of the plate and increases in case of heating of the plate with an increase in t.

APPENDIX

Nomenclature

a^*	Absorption coefficient
A	Constant
B_0	External magnetic field
C'	Species concentration in the fluid
C'_w	Concentration of the plate
C'_∞	Concentration in the fluid far away from the plate
C	Dimensionless concentration
C_p	Specific heat at constant pressure
D	Chemical Molecular diffusivity
g	Acceleration due to gravity
G_r	Thermal Grashof number
G_m	Mass Grashof number
κ	Thermal conductivity of the fluid
M	Magnetic field parameter
P_r	Prandtl number
q_r	Radiative heat flux in the y direction
R	Radiation parameter
S_c	Schmidt number
T'	Temperature of the fluid near the plate
T'_w	Temperature of the plate
T'_∞	Temperature of the fluid far away from the plate
t'	Time
t	Dimensionless time
u'	Velocity of the fluid in the x' -direction
u_0	Velocity of the plate

u	Dimensionless velocity
y'	Coordinate axis normal to the plate
y	Dimensionless coordinate axis normal to the plate

Greek symbols

α	Thermal diffusivity
β	Volumetric coefficient of thermal expansion
β^*	Volumetric coefficient of expansion with concentration
μ	Coefficient of viscosity
ν	Kinematic viscosity
ρ	Density of the fluid
σ	Electric conductivity
τ	Dimensionless skin friction
θ	Dimensionless temperature
erf	Error function
$erfc$	Complementary error function

Subscripts

w	Conditions on the wall
∞	Free stream conditions

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