



## SIMULATION STUDY OF THREE-PHASE INDUCTION MOTOR WITH VARIATIONS IN MOMENT OF INERTIA

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### ABSTRACT

Transient performance of any electrical machine is greatly affected by sudden changes in its supply system, operating speed, shaft load including any variations in moment of inertia due to gear arrangement applications. D, q- axis modeling which is universally acceptable to determine such analysis may be adopted using stator reference frame/rotor reference frame/synchronously rotating reference frame. In this paper, rotor reference frame is used for the simulation study of three phase induction motor. MATLAB/SIMULINK based modeling is adopted to compare the transient performance of three-phase induction motor including main flux saturation with and without the moment of inertia (MOI) of the system attached to the motor. Simulated results have been compared and verified with experimental results on a test machine set-up. A close agreement between the simulated and experimental results proves the validity of proposed modeling.

**Keywords:** modeling, induction motor, rotor reference frame, simulation, transient analysis, moment of inertia.

### INTRODUCTION

In order to investigate the problems like large currents, voltage dips, oscillatory torques and harmonics in power systems during severe transient operations and startup, the d, q-axis model has been found to be well tested and proven [1-3] describes the basic concept of transient modeling of the machine. Dynamic behavior of the machine may be analyzed using rotor reference frame. [4-5] recommends specific tests to estimate the machine parameters to proceed with transient modeling. In He [6] and Levi [7] the effect of considering the main flux saturation is investigated. It has been shown that the main magnetizing field contributes significantly to the disparity between the induction machines computer simulation results and the experimentally derived results [6]. So, the effect of saturation in induction machines can be included through variation of main flux inductance while assuming the leakage inductances to be constant. Further, rotor reference frame is recommended for analysis involving saturation effect [8].

MATLAB / SIMULINK, which has been found to be a very useful tool for modeling electrical machine and it is used to predict the dynamic behavior of the machines [9-11].

In this paper, MATLAB / SIMULINK based model using rotor reference frame and including saturation

effect is proposed for simulation purpose. Simulated results as obtained have been compared with experimental results on a test machine with and without including the MOI of the external system attached. During simulation sufficient time span is included to predict the complete behavior of the machine.

### PROBLEM FORMULATION

Figure-1 shows the representation of three phase induction motor with q, d axis superimposed. In order to maintain clarity, phase's b and c are not shown. qdo modeling in common reference frame results into the following equations:

$$\begin{bmatrix} F_{qdo} \end{bmatrix} = \begin{bmatrix} T_{qdo} \end{bmatrix} \begin{bmatrix} F_{abc} \end{bmatrix} \quad (1)$$

Where

$$\begin{bmatrix} T_{qdo} \end{bmatrix} = 2/3 \begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ \sin\theta & \sin(\theta-2\pi/3) & \sin(\theta+2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \quad (2)$$

The voltage balance equation for the d, q coils in arbitrary reference frames are as follows [2]:

$$\begin{bmatrix} V_{qs}^c \\ V_{ds}^c \\ V_{qr}^c \\ V_{dr}^c \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_c & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s + L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega - \omega_r) L_m & R_r + L_r p & (\omega_c - \omega_r) L_r \\ -(\omega_c - \omega_r) L_m & L_m p & -(\omega_c - \omega_r) L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs}^c \\ i_{ds}^c \\ i_{qr}^c \\ i_{dr}^c \end{bmatrix} \quad (3)$$

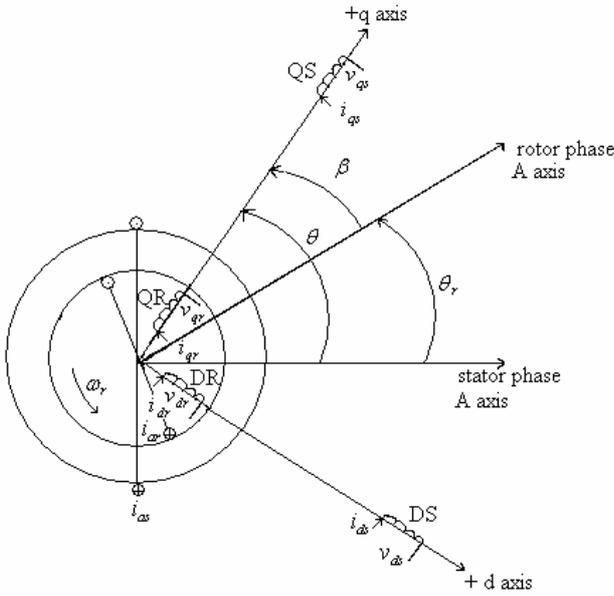


Figure-1. Q, d - axis superimposed into a three-phase induction motor.

**Rotor reference frame**

It is always recommended to transform equation (3) to d-q axis fixed either on stator, the rotor or rotating in synchronism with the supply voltages. In order to study the effects of rotor side disturbances (such as sudden change in shaft load, moment of inertia etc.) of the machine, rotor reference frame as explained below may be adopted successfully. For rotor reference frame

$$\omega_c = \omega_r \text{ Stator supply angular frequency (radian /sec). (4)}$$

From Figure-1

$$\theta = \theta_r + \beta$$

And

$$\beta = 0$$

So, the instantaneous angular position is

$$\theta = \theta_r \tag{5}$$

Equations (3) and (4) gives:

$$\begin{bmatrix} V_{qs}^r \\ V_{ds}^r \\ V_{qr}^r \\ V_{dr}^r \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_r L_s & L_m p & \omega_r L_m \\ -\omega_r L_s & R_s + L_s p & -\omega_r L_s p & L_m p \\ L_m p & 0 & R_r + L_r p & 0 \\ 0 & L_m p & 0 & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs}^r \\ i_{ds}^r \\ i_{qr}^r \\ i_{dr}^r \end{bmatrix} \tag{6}$$

The electromagnetic torque is,

$$T_e = \frac{3P}{2} L_m (i_{qs}^r i_{dr}^r - i_{ds}^r i_{qr}^r) \tag{7}$$

(N.m)

If the bus bar voltages are

$$\begin{aligned} V_{as} &= V_m \cos(\omega_s t) \\ V_{bs} &= V_m \cos(\omega_s t - 2\pi/3) \\ V_{cs} &= V_m \cos(\omega_s t + 2\pi/3) \end{aligned} \tag{8}$$

The transformation from abc to dqo variables is found by substituting (5) into equation (2) and is given as

$$[T_{abc}^r] = \frac{2}{3} \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{9}$$

The mechanical motion described by

$$p\omega_r = (T_e - T_L) / J \tag{10}$$

**Model developed to include saturation**

During the development of dynamic models of induction motors, most of the researchers neglected the effect of magnetic saturation and assumed inductances to be constant. Whereas, in this paper, an attempt has been made to account the effect of magnetizing saturation in dynamic model of the motor.

Modeling results in to the following non-linear relation

between magnetizing reactance,  $X_m$  and magnetizing

Current,  $i_m$ ,

$$x_m = -0.000i_m^3 + 0.0084i_m^2 - 0.6768i_m + 27.2815 \Omega \tag{11}$$

The magnetizing current,  $i_m$  is defined as:

$$i_m = \sqrt{i_{dt}^2 + i_{qt}^2} \tag{12}$$

And

$$i_{dt}^r = i_{ds}^r + i_{dr}^r \tag{13}$$

$$i_{qt}^r = i_{qs}^r + i_{qr}^r \tag{14}$$

**MODELING OF THREE-PHASE INDUCTION MOTOR USING MATLAB SIMULINK**

Stator d-q axis currents and rotor d-q axis currents may be computed by solving the differential equation (6). Equations (7), (11) and (12) may be used to determine electromagnetic torque, magnetizing reactance and magnetizing current of the machine. The third order nonlinear polynomial equation (11) is used to include the saturation in magnetic circuit.

Electromagnetic torque, mechanical sub-block and simulink model as proposed are shown in Figures 2, 3 and 4, respectively.

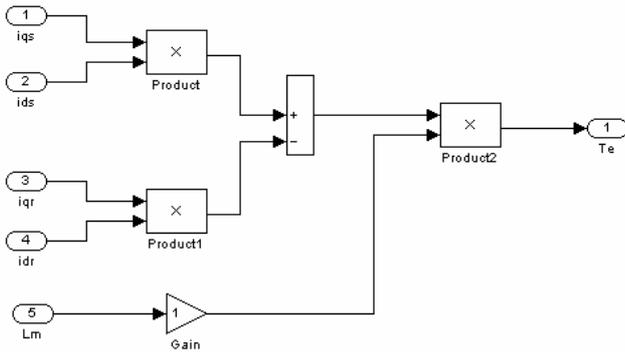


Figure-2. Electromagnetic torque.

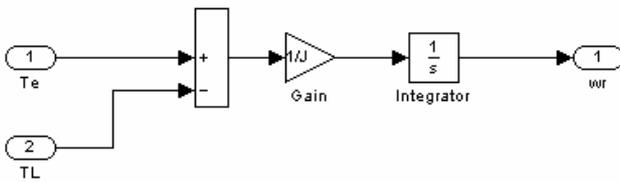


Figure-3. Mechanical sub-block.

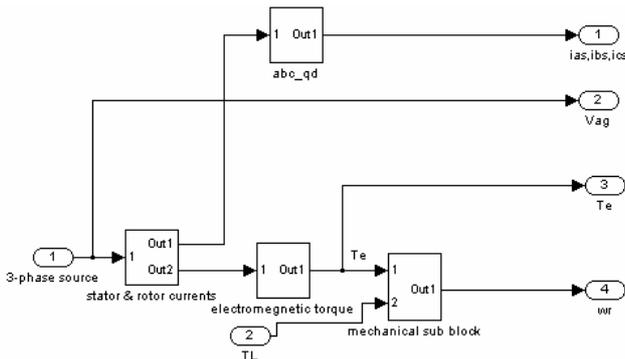


Figure-4. Matlab simulink model of total proposed system.

**RESULTS AND DISCUSSIONS**

Figure-5 and Figure-6 shows the comparison of simulated results with experimental results on a test machine (Appendix-1). Comparison has been carried out for simulated results with ( $J = 0.913 \text{ kgm}^2$ , Figure-6 and without ( $J = 0.113 \text{ kgm}^2$ , Figure-5 including the effect of MOI (Appendix-2) of external system coupled with the motor. A close agreement between the simulated and experimental results (Appendix 3) with MOI as  $0.913 \text{ kgm}^2$  confirms the validity of model adopted. On the other hand as shown in Figure-5, simulated results deviates a lot in case MOI of external system is neglected. Figure-7 and 8 shows the effect of moment of inertia on simulated results using proposed model. Initially, the three-phase induction motor runs at no-load and after 3 seconds, it runs on full load.

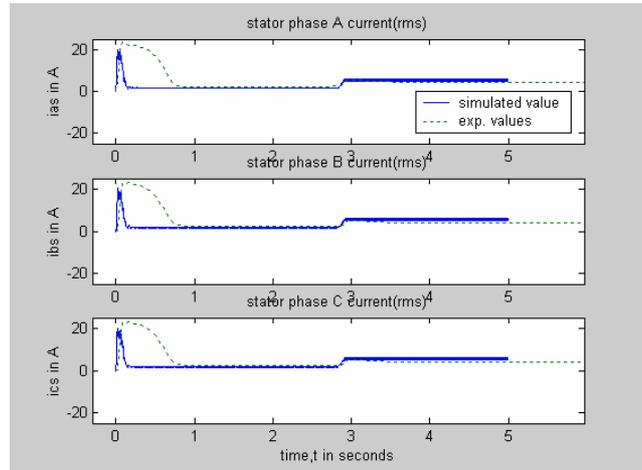


Figure-5. Stator phase currents in phase a, b and c when moment of inertia,  $J = 0.113 \text{ kgm}^2$ .

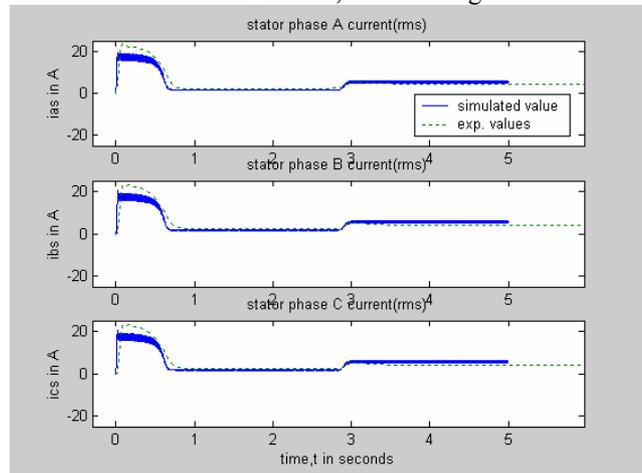


Figure-6. Stator phase currents in phase a, b and c when moment of inertia,  $J = 0.913 \text{ kgm}^2$ .

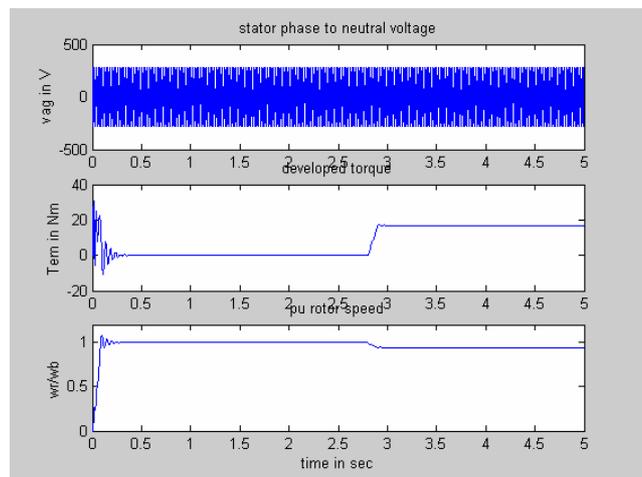
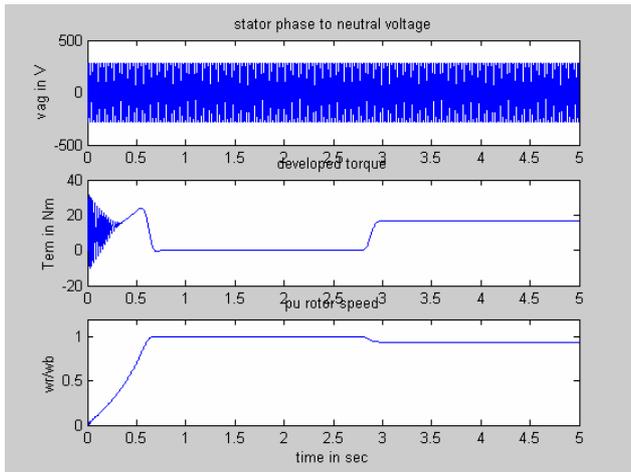


Figure-7. Stator phase to neutral voltage, electromagnetic torque and rotor speed when moment of inertia,  $J = 0.113 \text{ kgm}^2$ .



**Figure-8.** Stator phase to neutral voltage, electromagnetic torque and rotor speed when moment of inertia,  $J = 0.913 \text{ kgm}^2$ .

From the above figures, it is observed that:

- As observed from Figures 5 and 6, MOI greatly affects the inrush current drawn by the machine;
- Moment of inertia has significant effects during starting in contrast of steady state operations;
- Low values of MOI results into a low settling time, whereas it is increasing with increase in MOI; and
- Speed build up is found to be smooth with large value of MOI.

## CONCLUSIONS

In this paper an attempt has been made to study the effect of moment of inertia on the transient performance of an induction motor. Further, the effect of saturation has been accounted, which is generally omitted by most of the researchers. Simulated results as obtained have been compared with experimental results on a test machine with and without including the MOI of the external system attached. A close agreement between simulated and experimental results frames the validity of the model adopted.

## Nomenclature

$V_{as}, V_{bs}, V_{cs}$	Bus bar voltages for phase a, b and c, respectively
$V_m$	Maximum Voltage
$V^c qds, V^r qds$	Stator q and d axes voltages in arbitrary and rotor reference frame
$V^c qdr, V^r qdr$	Rotor q and d axes voltages in arbitrary and rotor reference frame
$i^c qds, i^r qds$	Stator q and d axes currents in arbitrary and rotor reference frame
$i^c qdr, i^r qdr$	Rotor q and d axes voltages in

$R_s, L_s$	arbitrary and rotor reference frame Stator Phase Resistance and Stator Self inductance
$L_m$	Mutual inductance
$R_r, L_r$	Rotor Phase Resistance and Rotor self inductance
$\omega_c, \omega_r, \omega_s$	Angular speed (rad/sec.) in arbitrary, rotor and synchronously rotating reference frame
$\theta, \theta_r$	Angular position in arbitrary and rotor reference frame
$T_e, T_L$	Electrical Torque and Load Torque
$p$	Operator for differentiation

## Subscripts

q	= Quadrature axis
d	= Direct axis
s	= Stator quantities
r	= Rotor quantities

## Superscripts

c	= Quantity referred to arbitrary frame
r	= Quantity referred to rotor reference frame

## Appendix 1

3-hp, 3-phase, 50 Hz, 230 volts, delta connected Induction Motor,

Stator Resistance  $R_s = 3.35 \text{ ohms}$

Rotor Resistance  $R_r = 1.76 \text{ ohms}$

MOI of induction motor,  $J = 0.113 \text{ kgm}^2$

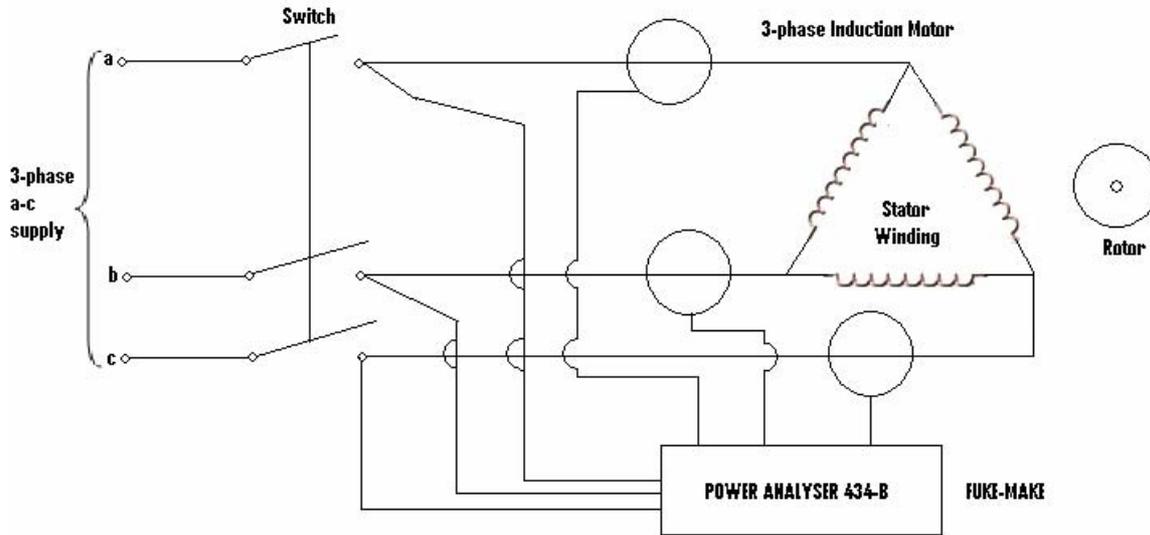
MOI of total system,  $J = 0.913 \text{ kgm}^2$

Stator and Rotor Inductance  $L_s = L_r = 15.43 \text{ mh}$

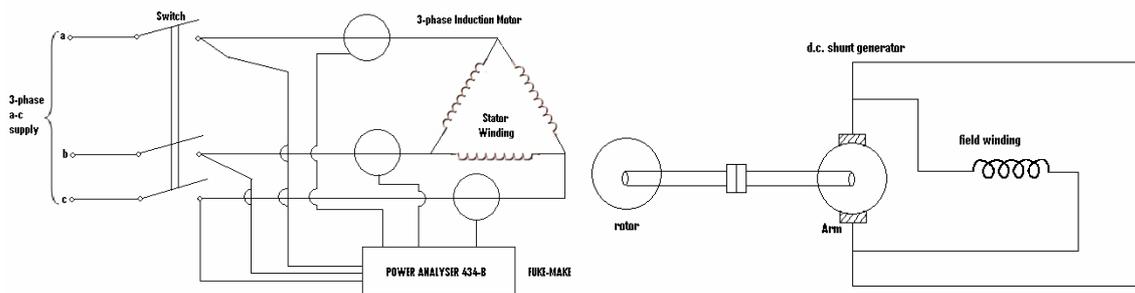
## Appendix 2

### Experimental set-up to determine the moment of inertia

Circuits as shown in Figure-9(a) and 9(b) may be used to perform the retardation tests on given motor with and without any coupling with auxiliary machine. After taking the appropriate observations, following expressions may be used to determine the moment of inertia. The power, P consumed in overcoming the rotational losses is given by,



**Figure-9(a).** Experimental set-up to determine moment of inertia without external system.



**Figure-9(b).** Experimental set-up to determine moment of inertia with external system.

$$P = J \times \frac{4\pi^2}{3600} \times N \times \frac{dN}{dt}$$

Where N is operating speed of motor in rpm.  $dN / dt$  at normal rated speed can be found graphically using the oscillogram of speed vs. time.

### Appendix 3

#### Experimental set-up for verification of simulated results

A dc shunt generator is coupled with the induction motor is used to load the induction motor. The rating and parameters of the dc shunt machine are:

kw: 1.75

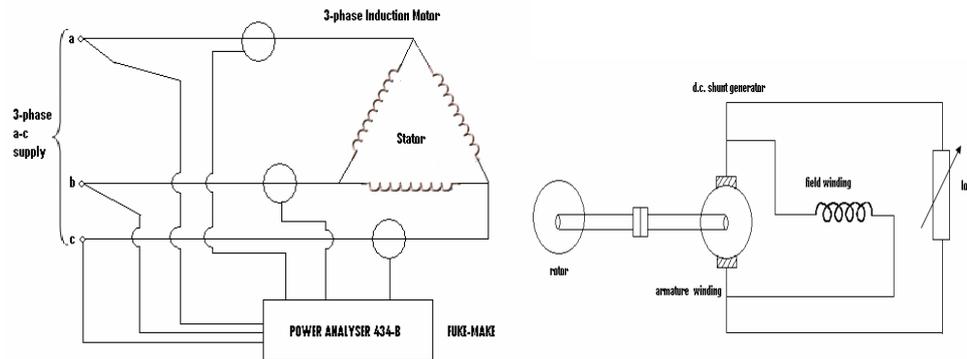
Amperes: 8

Rpm: 1420

Volts: 220.

Armature resistance,  $R_a = 16.97 \Omega$ .

Field resistance,  $R_f = 394 \Omega$ .



**Figure-10.** Experimental set-up for verification of simulated results.

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