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LMS AND RLS ALGORITHMS FOR SMART ANTENNAS IN A W-CDMA MOBILE COMMUNICATION ENVIRONMENT

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ABSTRACT

Wireless mobile communication systems will be more sophisticated and wide spread in future. This growth demands not only for capacity but also high quality of service and better coverage without increase in radio frequency spectrum allocated for mobile applications. Wireless systems used fixed antenna systems in the past, but space division multiple access systems use smart antennas. These smart antennas dynamically adapt to changing traffic requirements. Smart antennas are usually employed at the base station and radiate narrow beams to serve different users. The complex weight computations based on different criteria are incorporated in the signal processor in the form of software algorithms. This article focuses on adaptive beam forming approach based on smart antennas and adaptive algorithms used to compute the complex weights like Least Mean Square (LMS) and Recursive Least Squares (RLS) algorithms.

Keywords: beam forming, mobile communication, space division multiple access, adaptive arrays, weight vector.

INTRODUCTION

Future wireless mobile system demand for better coverage, high quality of service and more capacity. The frequency reuse concept increases capacity. However, increasing the number of cells to accommodate growing subscriber needs is not effective and not an economical option. Within the cells in a cellular network a further increase in capacity is achieved by efficiently sharing the frequency channels using multiple access techniques. Space Division Multiple Access (SDMA) has emerged as a key technology for future mobile communication. SDMA exploits the spatial domain of the mobile radio channel to bring about increase in network capacity in the existing wireless systems. SDMA based systems uses smart antennas that adapt to changing traffic requirements. Smart antennas, usually employed at the base station, radiate narrow beams to serve different users. As long as the users are well separated spatially the same frequency can be reused if the users are in the same cell. This additional intra cell channel reused based on spatial separation is the key in achieving an increase in the capacity of the system.

The Smart antenna systems can generally be classified as either switched beam or adaptive array systems. In a switched beam system multiple fixed beams in predetermined directions are used to serve the users. In this approach the base station switches between several beams that give the best performance as the mobile user moves through the cell. Adaptive beam forming uses antenna arrays backed by strong signal process capability to automatically change the beam pattern in accordance with the changing signal environment. It not only directs maximum radiation in the direction of the desired mobile user but also introduces nulls at interfering directions while tracking the desired mobile user at the same time.

The adaptation achieved by multiplying the incoming signal with complex weights and then summing them together to obtain the desired radiation pattern. These weights are computed adaptively to adapt to the changes in the signal environment. The complex weight computation based on different criteria and incorporated in the signal processor in the form of software algorithms.

Adaptive algorithms form the heart of the array processing network. Several algorithms have been developed based on different criteria to compute the complex weights. They have their own advantages and disadvantages as far as the convergence speed, complexity and other aspects are concerned. There is still a room for an improvement in this regard to improve the performance of the whole adaptive system by improving present algorithms. This article discusses the LMS and RLS algorithms that are used to compute the complex weights and their performance is compared. The switched beam system is as shown in Figure-1.

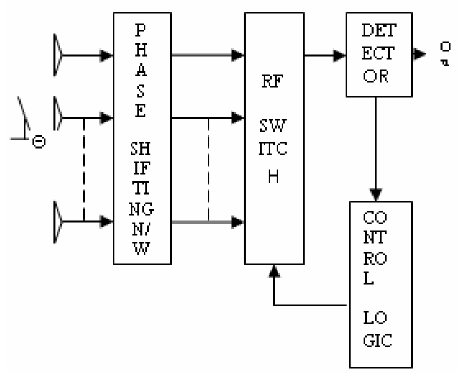

Figure-1. Block diagram of switched beam system.

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A typical switched beam system for the base station would consist of multiple arrays with each array covering a certain section in the cell. It consists of phase shifting network, which forms multiple beams looking in certain directions. The control logic selects the right beam. The control logic is governed by an algorithm which scans all the beams and selects the one receiving the strongest signal based on the measurement made by the detector. The technique is simple in operation but is not suitable for high interference areas. Thus switched beam systems offer limited performance enhancement when compared to conventional antenna systems in wireless communication.

However, greater performance improvements can be achieved by implementing advanced signal processing techniques to process the information obtained by the antenna arrays. The adaptive array systems are smart because they are able to dynamically react to the changing RF environment. Adaptive array uses antenna arrays but it is controlled by signal processing. The basic block diagram of adaptive array system is as shown in Figure-2.

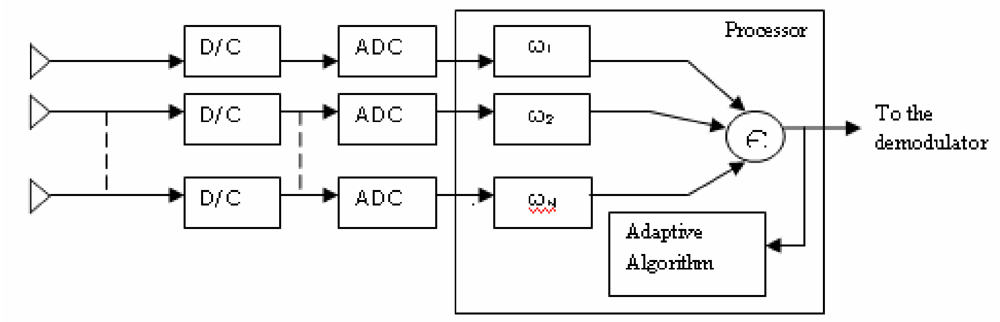

Figure-2. Block diagram of adaptive array system.

A smart antenna system can perform the following functions: First the direction of arrival of all the incoming signals including the interfering signals and multipath signals are estimated using the direction of arrival algorithms. Secondly, the desired user signal is identified and separated from the rest of the unwanted incoming signals. Finally a beam is steered in the direction of the desired signal and the user is tracked as he moves while placing nulls at the interfering signal direction by constantly updating the complex weights. In a beam forming network, typically the signals incident at the individual elements are combined intelligently to form single desired beam forming output. Before the incoming signals are weighted they are brought down to base band or intermediate frequencies IF's. The receivers provided at the output of each element perform the necessary frequency down conversion. As adaptive array systems use digital signal processors to weight the incoming signal, it is required to digitize the down converted signal, by using Analog to Digital Converters (ADCs) before they are processed by the DSP. The digital signal processor forms the heart of the system. The processor interprets the incoming data information, determines the complex weights and multiplies the weights to each element output to optimize the array pattern. The optimization is based on minimizing the contribution from the noise and interference while producing maximum beam gain at the desired direction. There are several algorithms based on different criteria for updating and computing the optimum weights. Based on information required, the adaptive algorithms are classified into two categories.

Reference signal based algorithms and Blind adaptive algorithm. Reference signal based algorithms are based on minimization of the mean square error between the received signal and reference signal. Therefore reference signal is required which has high correlation with the desired signal.

Adaptive beam forming

Adaptive beam forming is a technique in which an array of antennas exploited to achieve maximum reception in desired direction and rejecting signals of same frequency in the other direction. This is achieved by adjusting weights of each of the antennas used in the array. It uses the idea that the signal from different transmitters occupies the same frequency channel arriving from different directions. This spatial separation is exploited to separate desired signal from the interfering signals.

Beam forming is generally accomplished by phasing the feed to each element of an array so that signals received or transmitted from all elements in phase in a particular direction. The phases and amplitudes are adjusted to optimize the received signal. The array factor for N element equally spaced linear array is given by

$$AF(\Phi) = \sum_{n=0}^{N-1} A_n . e^{(jn(\frac{2\pi d}{\lambda}\cos\Phi + \alpha))} \quad (1)$$

The inter element phase shift α is given by

$$\alpha = \frac{-2\pi d}{\lambda_0}\cos\Phi_0 \quad (2)$$

$\Phi_{0 \text{ is desired}}$ beam direction
Λ0 is the wavelength

Adaptive beam forming problem setup

The adaptive beam forming problem configuration is shown below in Figure-3.

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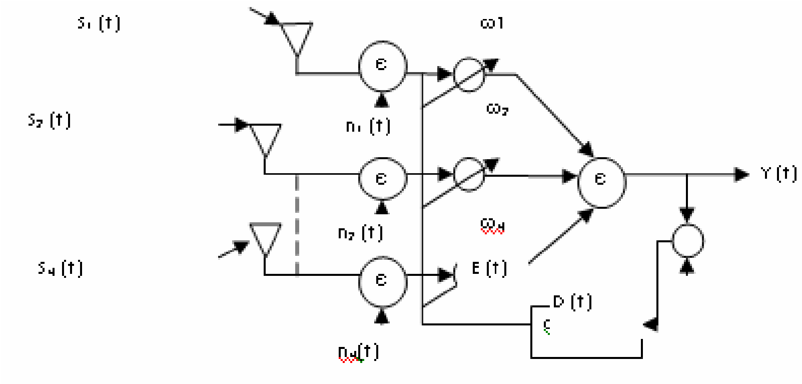

Figure-3. An adaptive array system.

The output of the array y (t) is the weighted sum of the received signals. $S_i(t)$ at the array elements and the noise n(t) at the receivers connected to each element. The weights iteratively computed based on the array output y (t), a reference signal d (t) that approximates the desired signal and previous weights. The reference signal is approximated to the desired signal using a training sequence or a spreading code, which is known at the receiver.
The array output is given by

$$y(t) = \omega^H x(t) \tag{3}$$

Where ω^H denotes the complex conjugate transpose of the weight vector ω.

In order to compute the optimum weights the array response vector or steering vector from the sampled data of the array output has to be known. The array response vector is a function of the incident angle as well as the frequency. The baseband received signal at the N^{th} antenna is sum of phase shifted and attenuated versions of the original signal S_i (t).

$$x_N(t) = \sum_{i=1}^{N} a_N(\theta_i) S_i(t) e^{-j2\pi f_c \tau_N \theta_i} \tag{4}$$

The $S_i(t)$ consists of both the desired and the interfering signal.
$\tau_N \theta_i$ the delay, f_c is the carrier frequency

$$a_N(\theta_i) = \left[a_1(\theta_i)e^{-j2\pi f_c \tau_1 \theta_i}, a_2(\theta_i)e^{-j2\pi f_c \tau_2 \theta_i}, \ldots\ldots\ldots\ldots a_N(\theta_i)e^{-j2\pi f_c \tau_N \theta_i}\right]^T \tag{5}$$

Now

$$A(\theta) = \left[a(\theta_1).a(\theta_2)\ldots\ldots\ldots.a(\theta_d)\right] \tag{6}$$

$$S(t) = \left[S_1(t).S_2(t)\ldots\ldots\ldots..S_d(t)\right]^T \tag{7}$$

So that

$$x(t) = A(\theta)S(t) \tag{8}$$

With noise

$$x(t) = A(\theta)S(t) + n(t) \tag{9}$$

The beam former response can be expressed in the vector form as

$$r(\theta,\omega) = \omega^H a(\theta,\omega) \tag{10}$$

This equation includes the possible dependency of a (θ) on ω as well.
Rewriting x (t) by separating the desired signal from the interfering signals.

$$x(t) = S(t)a(\theta_0) + \sum_{i=1}^{N_u} U_i(t)a(\theta_i) + n(t) \tag{11}$$

Let s(t) denote the desired signal arriving at an angle of incidence θ_0 at the array and $u_i(t)$ denotes the number of undesired interfering signals arriving at angles of incidence θ_i .The direction of arrival are known a priori using a direction of arrival (DOA) algorithm. Where a (θ_i) is the array propagation vector of the i^{th} interfering signal. A (θ_0) is the array propagation vector of desired signal.

Having the above information adaptive algorithms are required to estimate s(t) from x(t) while minimizing the error between the estimates $\hat{s}$ (t) and the original signal s(t). Let d*(t) represent a signal that is closely correlated to original desired signal s (t). D*(t) is referred to as the reference signal the mean square error ϵ^2 (t) between the beam former output and the reference signal can now be computed as

$$\varepsilon^2(t) = \left[d^*(t) - \omega^H x(t)\right]^2 \tag{12}$$

Taking expectation on both the sides of the equation, we get

$$E\left[\varepsilon^2(t)\right] = E\left\{\left[d^*(t) - \omega^H x(t)\right]^2\right\} \tag{13}$$

$$E\left[\varepsilon^2(t)\right] = E\left\{\left[d^2(t)\right]\right\} - 2\omega^H E\left[d^*(t)x(t)\right] + E\left[\omega^H x(t)\right] \tag{14}$$

$$E\left[\varepsilon^2(t)\right] = E\left\{\left[d^2(t)\right]\right\} - 2\omega^H \gamma + \omega^H R\omega \tag{15}$$

Where $\gamma = E\{[d^*(t)x(t)]\}$ is cross correlation matrix between the desired signal and the received signal?
R= E[x (t) x^H(t)]: is the auto correlation matrix of the received signal also known as co variance matrix. The minimum MSE can be obtained by setting the gradient vector of the eq(14) with respect to ω equal to 0.

$$\Delta_\omega\left[E\{\varepsilon^*(t)\}\right] = -2\gamma + 2R\omega = 0 \tag{16}$$

Therefore the optimum solution for the weight vector ω* is given by

$$\Omega^* = R^{-1}r \tag{17}$$

The optimal weight vector ω* sometimes called the wiener weight vector.

Least mean square algorithm

The Least Mean Square (LMS) algorithm uses a gradient based method of steepest decent. LMS algorithm uses the estimate of the gradient vector from the available data. LMS algorithm is important because of its simplicity and ease of computation. The LMS incorporates an iterative procedure that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error.

LMS Algorithm and Adaptive Arrays

Consider a uniform linear array with N isotropic elements, which forms the integral part of the adaptive beam forming system as shown in Figure-4.

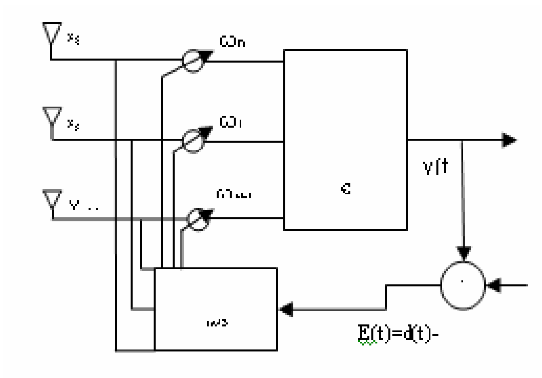

Figure-4. LMS adaptive beam forming network.

The output of the antenna array is given by

$$x(t) = S(t)a(\theta_0) + \sum_{n=1}^{N_u} U_i(t)a(\theta_i) + n(t) \quad (18)$$

As shown in fig(4) the output of the individual elements are linearly combined after weight modification such that the antenna pattern is optimized to have maximum gain in the desired direction and nulls in the undesired direction. The weights will be computed using LMS algorithm based on minimum mean square error criteria.

The weight update equation is

$$\omega(n+1) = \omega(n) + \mu x(n)\left[d^*(n) - x^H \omega(n)\right]$$
$$= \omega(n) + \mu x(n) e^*(n) \quad (19)$$

Where μ is the step size parameter and controls the convergence characteristics of the LMS algorithm.

The LMS algorithm is initiated with an arbitrary value of ω (0) for the weight vector at n=0. The successive corrections of the weight vector eventually leads to minimum value of the mean squared error.

The summary of LMS algorithm is

Output $y(n) = \omega^h x(n)$ (20)

Error $e(n) = d^*(n) - y(n)$ (21)

Weight $\omega(n+1) = \omega(n) + \mu x(n) e^*(n)$ (22)

The convergence of weight vector is given by

$$0 < \mu < \frac{1}{\lambda_{\max}} \quad (23)$$

Where λ_{max} is the largest eigen value of the correlation matrix R. If μ is chosen to be very small, the algorithm converges very slowly. A large value of μ may lead to a faster convergence but the stability around a minimum value will be lost.

The recursive least squares algorithm (RLS)

The convergence speed of the LMS algorithm depends on the Eigen values of the array correlation matrix. In an environment yielding an array correlation matrix with large eigen value spread the algorithm converges with a slow speed This problem is solved with the RLS algorithm by replacing The gradient step size μ with a gain matrix $\hat{R}^{-1}(n)$ at the nth iteration, producing the weight update equation

$$\omega(n) = \omega(n-1) - \hat{R}^{-1}(n) x(n) \varepsilon^*(\omega(n-1)) \quad (24)$$

Where $\hat{R}(n)$ is given by

$$\hat{R}(n) = \delta_0 \hat{R}(n-1) + x(n)^* x^H(n)$$

$$= \sum_{k=0}^{n} \delta_o^{\,n-k} x(k).x^H(k) \quad (25)$$

Where δo denoting a real scalar less than but close to 1.

The δo is used for exponential weight of past data and is referred to as the forgetting factor as the update equation tends to de-emphasize the old samples. The quantity $\frac{1}{1-\delta_0}$ is normally referred to us the algorithm memory.

Thus for δ o=0.99 the algorithm memory is close to 100 samples. The RLS algorithm updates the required inverse of using the previous inverse and the present samples as

$$\hat{R}^{-1}(n) = \frac{1}{\delta_0}\left[\hat{R}^{-1}(n-1) - \frac{\hat{R}^{-1}(n-1)x(n)x^H(n)\hat{R}^{-1}(n-1)}{\delta_0 + x^H(n)\hat{R}^{-1}(n-1)x(n)}\right] \quad (26)$$

The matrix is initialized as

$$\hat{R}^{-1}(0) = \frac{1}{\varepsilon_0} I, \varepsilon_0 \phi 0 \quad (27)$$

The RLS algorithm minimized the cumulative square error

$$J(n) = \sum_{k=0}^{n} \delta_0^{\,n-k} |\varepsilon(k)|^2 \quad (28)$$

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And its convergence is independent of the Eigen values distribution of the correlation matrix.

Simulation and performance evaluation

For simulation purpose the uniform linear array with N number of elements is considered. The spacing between the individual elements is considered to be half wavelength.

Case I: It is considered that the desired user is arriving at an angle 30 degrees and an interferer at an angle 60 degrees. The array factor for number of elements equal to 5, 8 and 10 is computed and Figure-5 shows the array factor plots and how the LMS algorithm places deep nulls in the direction of interfering signals and maximum in the direction of the desired signal.

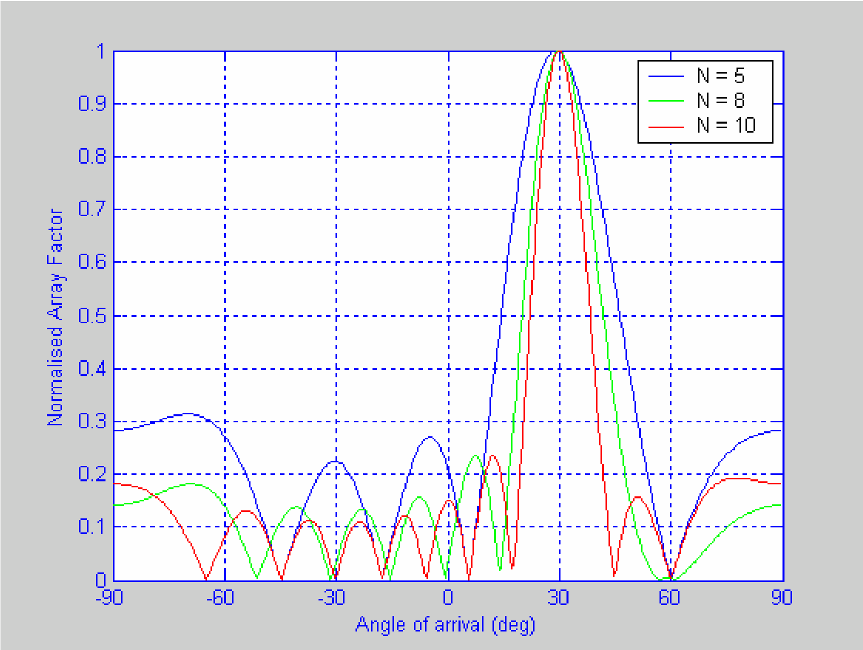

Figure-5. Array factor plots for LMS algorithm when the desired user with AOA 30 deg and interferer with AOA 60 deg. the spacing between elements = 0.5 λ.

Case II: It is considered that the desired user is arriving at an angle 0 degrees and an interferer at an angle 60 degrees. The array factor for number of elements equal to 5, 8 and 10 is computed and Figure-6 shows the array factor plots and how the LMS algorithm places deep nulls in the direction of interfering signals and maximum in the direction of the desired signal.

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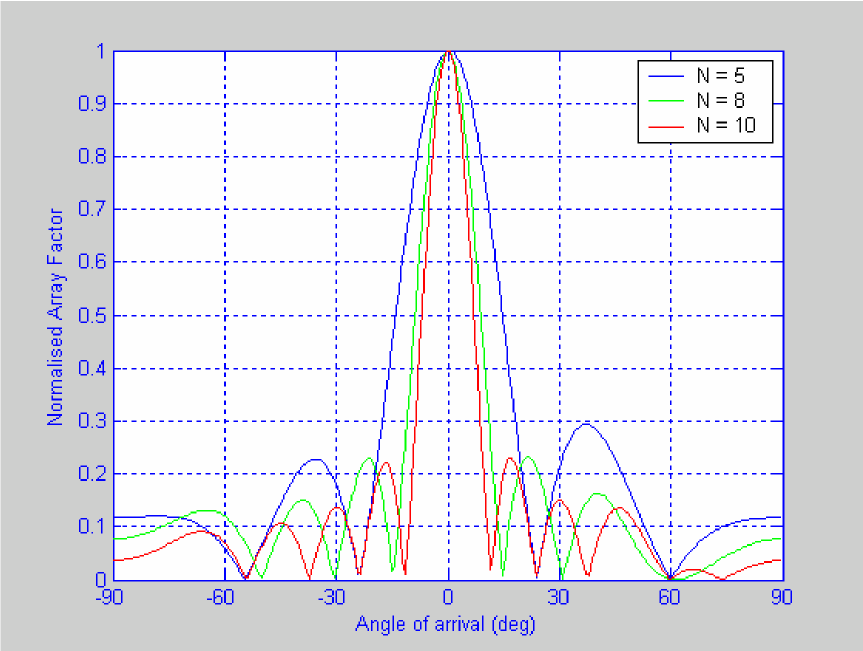

Figure-6. Array factor plots for LMS algorithm when the desired user with AOA 0 deg and interferer with AOA 60 deg the spacing between elements = 0.5 λ.

Case III: Finally it is considered that the desired user is arriving at an angle -45 degrees and an interferer at angle 45 degrees. The array factor for number of elements equal to 5, 8 and 10 is computed and Figure-7 shows the array factor plots and how the LMS algorithm places deep nulls in the direction of interfering signals and maximum in the direction of the desired signal.

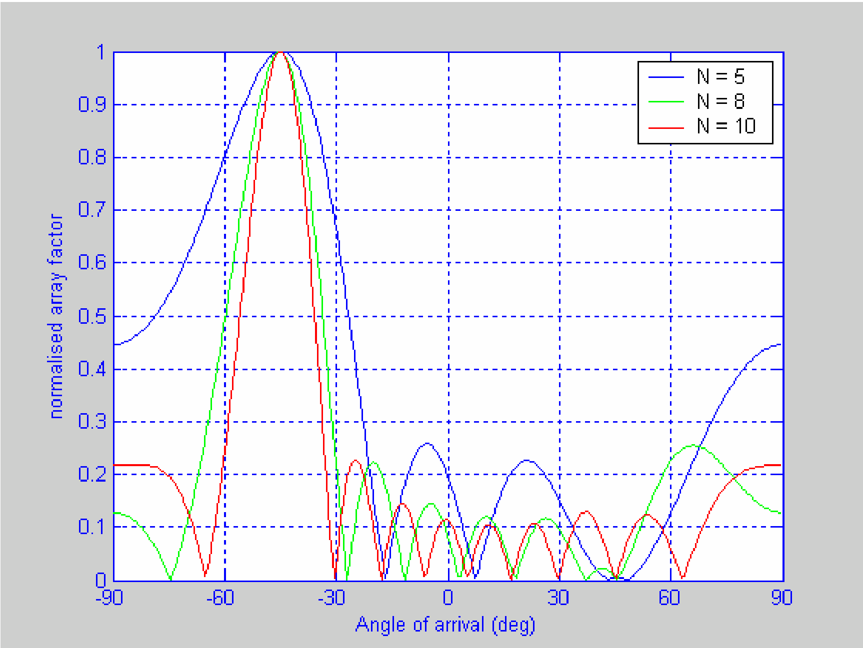

Figure-7. Array factor plots for LMS algorithm when the desired user with AOA -45 deg and interferer with AOA 45 deg the spacing between elements = 0.5 λ

The array factor for the number of elements equal to five and the spacing between elements equal to λ/2, λ/4 and λ/8 is computed and Figure-8 shows the array factor plots. Similarly, Figure-9 shows the array factor plots for the number of elements equal to eight and Figure-10 shows the array factor for the number of elements is equal to ten respectively. From Figures 8, 9 and 10 it is clear that

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the spacing between the elements equal to half wavelength is the optimum value of spacing as it gives accurate result.

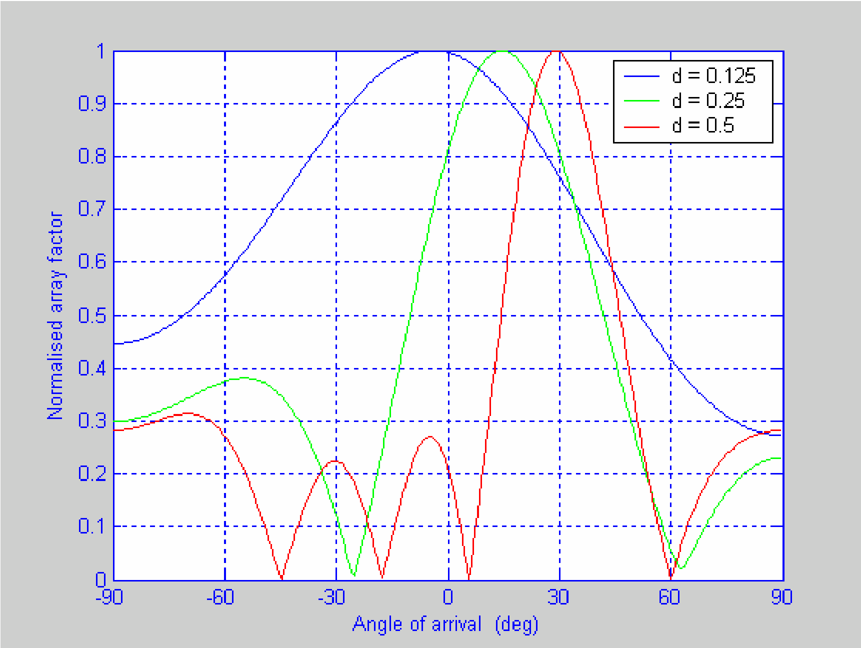

Figure-8. Array factor plot for N = 5.

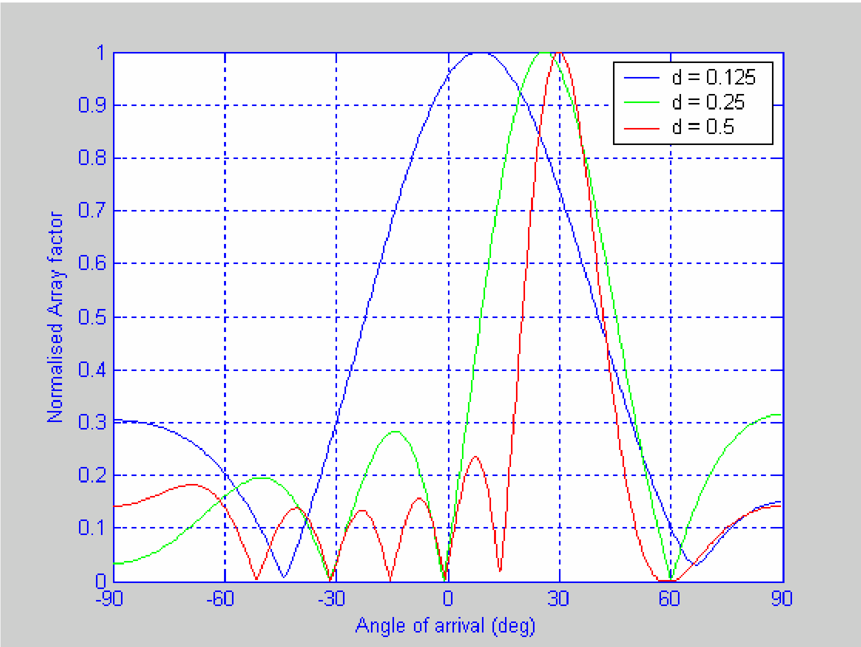

Figure-9. Array factor plot for N = 8.

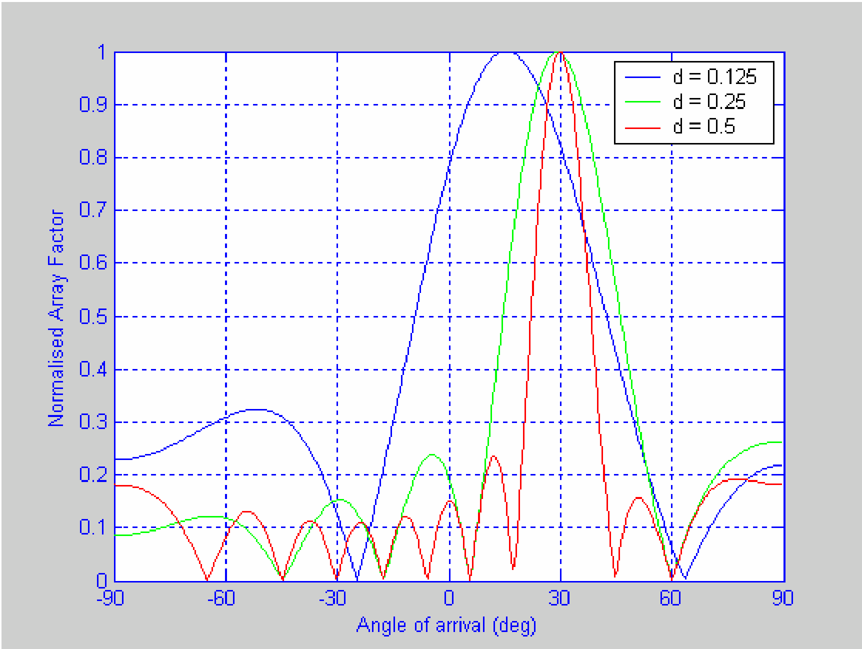

Figure-10. Array factor plot for N = 8.

Figures 11, 12 and 13 show the algorithm converges faster and stable for spacing between the elements equal to half wave length. The minimum mean square error is achieved after 50 iterations in case of N = 5 and after 30 iterations in case of N = 8 and after 10 iterations in case of N = 10. This shows that the algorithm converges faster when the number of elements in the array is increased.

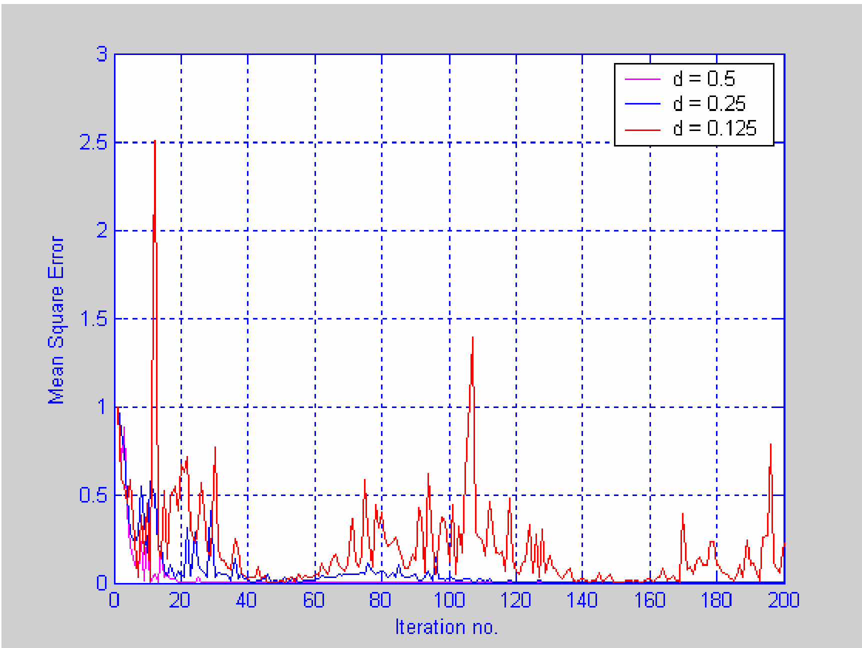

Figure-11. Mean square error plot for LMS algorithm for N = 5.

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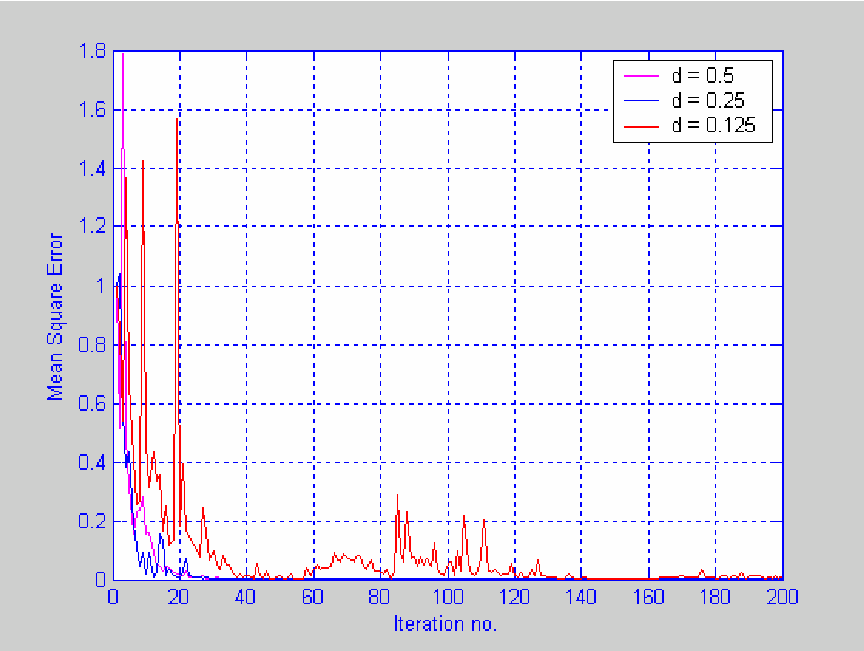

Figure-12. Mean Square Error plot for LMS algorithm for N = 8.

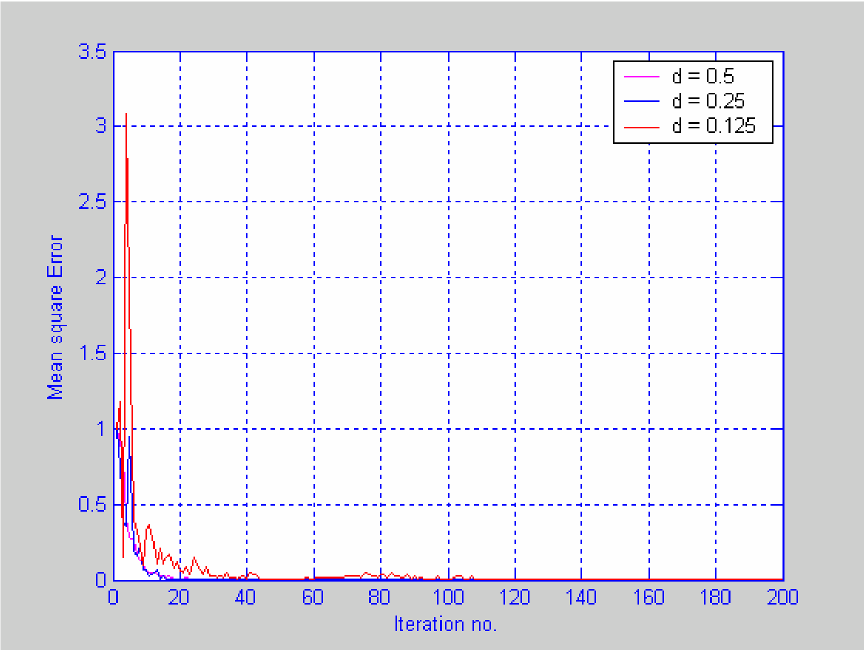

Figure-13. Mean square error plot for LMS algorithm for N = 10.

Simulation results of RLS algorithm

Figure-14 shows the array factor plots for RLS algorithm when the angle of arrival of the desired user is at 30 deg and interferer at -60 degrees for spacing between the elements equal to half wave length. The RLS algorithm places adaptively the maxima in the direction of desired user and nulls at the AOA of the interferer for various values of N. Figures, 15 and 16 show the array factor plots for spacing between the elements equal to quarter wave length and one eight wave length respectively. From these simulations it is evident that the optimum spacing between the elements is half wave length.

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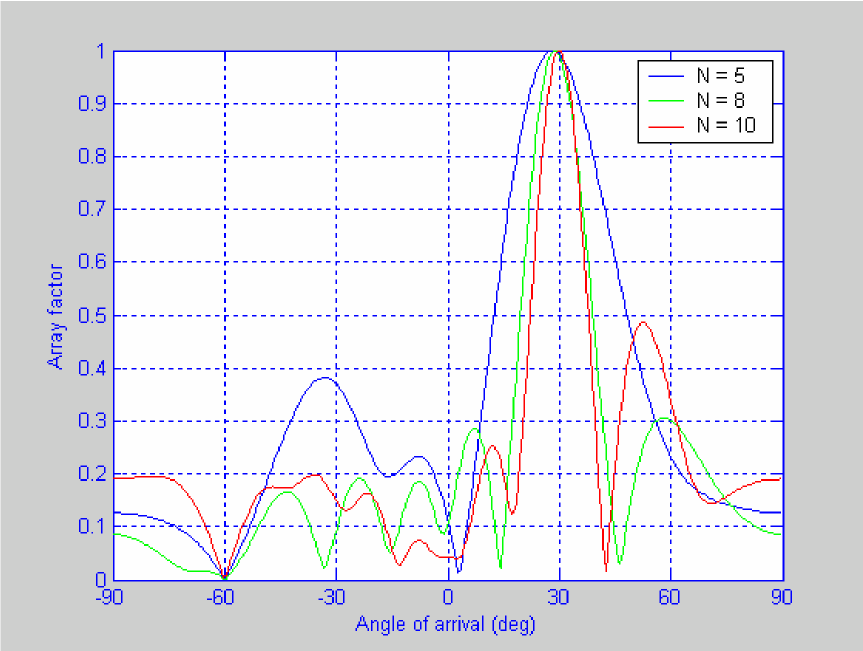

Figure-14. Array factor plots for RLS algorithm when the AOA of desired user at 30 degrees and the interferer at -60 degrees. (d = 0.5λ).

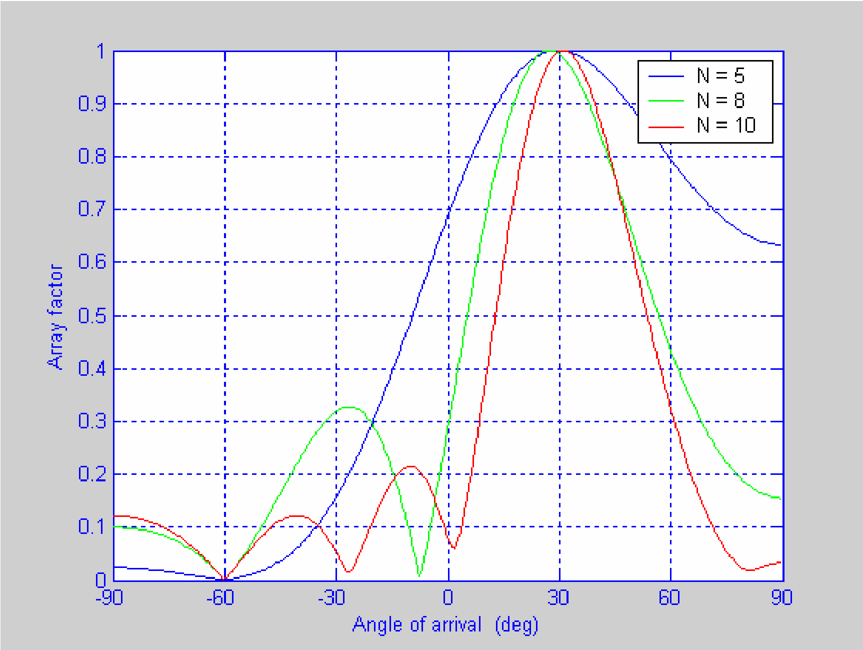

Figure-15. Array factor plots for RLS algorithm when the AOA of desired user at 30 degrees and the interferer at -60 degrees.(d = 0.2λ).

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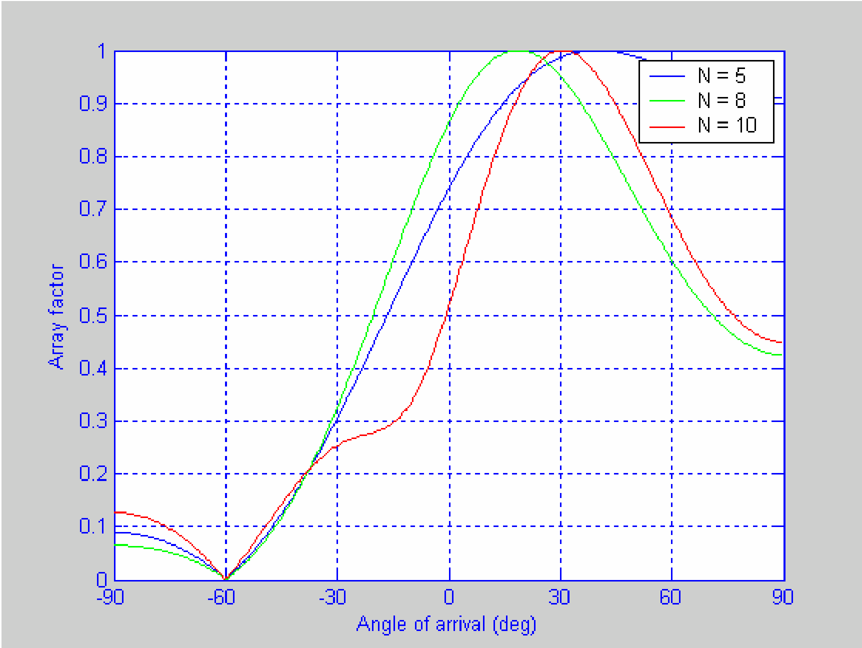

Figure-16. Array factor plots for RLS algorithm when the AOA of desired user at 30 degrees and the interferer at -60 degrees.(d = 0.125λ).

CONCLUSIONS

This paper discussed various adaptive beam forming algorithms like Least Mean Square (LMS) and Recursive Least Squares algorithms used in smart antennas. The convergence speed of the LMS algorithm depends on the eigen values of the array correlation matrix. In an environment yielding an array correlation matrix with large eigen values spread the algorithm converges with a slow speed. This problem is solved with the RLS algorithm by replacing the gradient step size μ with a gain matrix $\hat{R}^{-1}(n)$. The simulation results were provided to understand the convergence, stability, and the method of the adaptation of the algorithm. The results obtained from the simulations showed that the LMS had poor convergence compared to RLS, and the RLS algorithm is the most efficient and LMS is the slowest.

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