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AN EOQ MODEL FOR PERISHABLE ITEMS UNDER STOCK AND TIME-DEPENDENT SELLING RATE WITH SHORTAGES

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ABSTRACT

In this paper, we study a deterministic inventory model for deteriorating items under time-dependent partial backlogging. Though lot of factors involving inventory affect the demand, among them time and stock are the most important factors. Therefore, we consider here the combined stock and time varying demand to make the theory more applicable in practice. We study the effects time dependent demand on the total profit and time factors. We prove that the optimal replenishment solution not only exists but is also unique. Numerical examples are given to illustrate the application of developed model.

Keywords: time-varying demand, deteriorating items, shortages.

1. INTRODUCTION

Academicians as well as industries have shown great interests in the development of inventory control and their uses. Since the development of the EOQ concept more than four decades ago T.M. Whitin [15], a substantial amount of researches have been conducted in this area of inventory lot sizing. Constant demand rate is not always suitable to many inventory items (e.g. electric goods, fashionable clothes, tasty foods, etc.) as they experience fluctuations in the demand rate. In the last few years, the inventory lot-sizing models with time-varying demand and deterioration have received considerable attention. Dave and Patel [5] first considered the inventory models for deteriorating items with linear increasing demand. The consideration of exponentially decreasing demand for deteriorating items was first analyzed by Hollier and Malc [10]. Haringa and Benkherouf [9] generalized Hollier and Mac's [10] model taking into account both exponentially growing and declining markets. Haiping and Wang [8] developed "An economic ordering policy model for deteriorating items with time proportional demand". H. Xu [17] proposed, 'Optimal inventory policy for perishable items with time proportional demand'. Goswami and Chaudhuri [6] established, "An EOQ model for deteriorating items with shortages and a linear trend in demand", Benkherouf and Mahmoud [2] developed an inventory model with deterioration and increasing time-varying demand and shortages, Wee [16] studied an, "Economic production lot size inventory model for deteriorating items with partial backordering," Silver [13] proposed a "Simple inventory object replenishment decision rule for a linear trend in demand". We know that the shortages in inventory systems are either completely backlogged or totally lost. However, it is more reasonable to characterize that the longer the waiting for the next replenishment, the smaller the backlogging rate would be for many products with growing sales. The length of waiting time for the replenishment is the main factor for determining whether the backlogging will be accepted or not, and the backlogging rate is expected to be time dependent. Abad [1] proposed several pioneer and inspiring backlogging rates to be decreasing functions of waiting time. Chang and Dye [3] developed an EOQ model for deteriorating items with time-varying demand and partial backlogging. Papachristos and Skouri [11] developed, "An optimal replenishment policy for deteriorating items with time-varying demand and partial exponential type-backlogging," Teng *et al.*, [14] who proposed an optimal recursive method for various inventory replenishment model with increasing demand and shortages also considered this area. In reality, products deteriorate continuously such as medicines, volatile liquids and others. Dave and Patel [5] developed a (T_1,S_i) Policy inventory model for deteriorating items with time proportional demand, Sachan [12] proposed a model, "On (T_1,S_i) policy inventory model for deteriorating items with time proportional demand", Yan *et al.*, [18] (1998) developed an Optimal production stopping and restarting times for an EOQ model with deteriorating Items. Goyal and Giri [7] presented a survey on "Recent trends in model of deteriorating inventory," which is more helpful to the researchers who are working in deteriorating items. In this paper, we discuss a deterministic inventory model with time and stock dependent demand under partial backlogging. Fashionable goods and hi-tech products time factor play an important role. So, here we propose time dependent demand together with stock dependency. This paper is presented as follows: In section 2, the notations and assumptions are given. In section 2 we present the mathematical model. In section 4, numerical examples are given to illustrate the model. Finally, we conclude the paper.

2. NOTATIONS AND ASSUMPTIONS

To develop the Mathematical model, the following notations and assumptions are being made:

2.1 Notations

K	the ordering cost per order
P	the purchase cost per unit
P'	the selling price per unit, where P' > P
θ	the deterioration rate
h	the holding cost per unit per unit time
s	the shortage cost per unit per unit time
π	the opportunity cost due to lost sales per unit
I(t)	the inventory level at time t, where $t \in [0,T]$
R(t)	the demand rate at time t, where $t \in [0,T]$
δ	the backlogging parameter, where $0 \le \delta \le 1$
T	the length of the replenishment cycle
T_1	the time at which the shortage starts, $0 \le T_1 \le T$
TP	the total inventory profit per unit time

2.2 Assumptions

The proposed model is developed under the same assumptions as adopted by C-Y- Dye and L-Y- Ouyang [4], except the one related to the time-dependent demand and the inflation and time-discounting.

a. The replenishment rate is infinite and lead time is zero.
b. The distribution of time to deterioration of the items follows exponential distribution with parameter θ (i.e. constant rate of deterioration).
c. The unit cost and the inventory carrying cost are known and constant.
d. The selling price per unit and the ordering cost per order are known and constant.
e. The demand rate function R(t), is deterministic and is a known function of time and instantaneous stock level I(t); the functional R(t) is given by

$$R(t) = \begin{cases} f(t) + \beta I(t), & I(t) > 0 \\ f(t), & I(t) \le 0 \end{cases}$$

Where β is a positive constant, $f(t)$ is a positive, Continuous function of $t \in (0, T]$.

f. Shortages are allowed and unsatisfied demand is backlogged at the rate of $\dfrac{1}{[1+\delta(T-t)]}$. The backlogging parameter δ is a positive constant, and $T1 \le t \le T$.
g. There is no repair or replacement of the deteriorated items during the production cycle.

3. MODEL FORMULATION

A typical behavior of the inventory in a cycle is depicted in the following Figure-1.

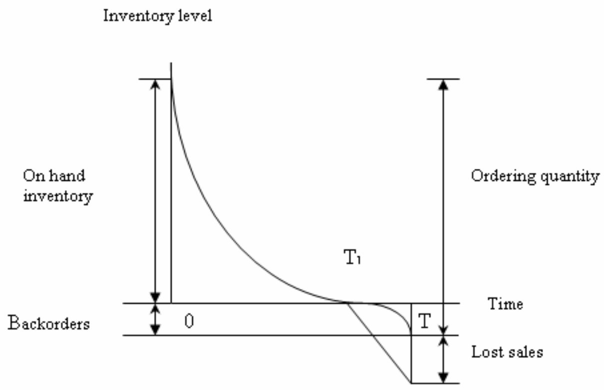

Figure-1.Graphical representation of inventory system.

The reduction of the inventory occurs due to the combined effects of the demand and deterioration in the interval $[0, T_1)$ and demand backlogged in the interval [T, t).The instantaneous states of the inventory level I(t) at time t ($0 \le t \le T$) can be described by the following differential equations:

$$\frac{dI(t)}{dt} = -f(t) - \beta I(t) - \theta I(t)\,;\quad 0 \le t \le T_1;\quad I(T_1)=0 \tag{1}$$

$$\frac{dI(t)}{dt} = \frac{-f(t)}{[1+\delta(T-t)]};\ T_1 \le t \le T;\quad I(T_1)=0 \tag{2}$$

The solutions of the above differential equations (1) and (2) are given respectively by

$$I(t) = \int_t^{T_1} e^{(\theta+\beta)(u-t)} f(u)\,du;\ 0 \le t \le T_1; \tag{3}$$

$$I(t) = -\int_t^{T_1} \frac{f(u)}{[1+\delta(T-u)]}\,du;\quad T_1 \le t \le T; \tag{4}$$

The profit per unit time of our model consists of the following elements:

- the setup cost per cycle K
- the holding cost per cycle (HC)
- the shortage cost per cycle (SC)
- the opportunity cost due to lost sales per cycle (OC)
- the purchase cost per cycle (PC_1, PC_2)
- the sales revenue per cycle (SR)

The purchase cost during the period $[0, T_1]$ and $[T_1, T]$ are given respectively by

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$$PC_1 = \int_0^{T_1} Pf(t)e^{(\theta+\beta)t}\,dt \tag{5}$$

$$PC_2 = \int_{T_1}^{T} P\frac{f(t)}{[1+\delta(T-t)]}\,dt \tag{6}$$

The holding cost for carrying inventory over the period [0, T_1] is given by

$$HC = \int_0^{T_1} h\,I(t)\,dt$$

$$= \int_0^{T_1} h\left[\int_t^{T_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt \tag{7}$$

The shortage cost over the period [T_1, T] is given by

$$SC = s\int_{T_1}^{T}\left[-I(t)\right]dt$$

$$= s\int_{T_1}^{T}\left(-\int_t^{T_1}\frac{f(u)}{[1+\delta(T-u)]}du\right)dt \tag{8}$$

The opportunity cost due to lost sales during the period [T_1, T] is given by

$$OC = \pi\int_{T_1}^{T} f(t)\left[1-\frac{1}{[1+\delta(T-t)]}\right]dt$$

$$= \pi\,\delta\int_{T_1}^{T}\frac{f(t)(T-t)}{[1+\delta(T-t)]}dt \tag{9}$$

The sales revenue over the period [0, T] is given by

$$SR = P'\int_0^{T_1}\left[f(t)+\beta\int_t^{T_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt + P'\int_{T_1}^{T}\frac{f(u)}{(1+\delta(T-u))}du \tag{10}$$

Therefore, the profit per unit time during the period [0, T] is given by

$$TP(T_1,T) = \frac{1}{T}\left[\begin{array}{l} P'\displaystyle\int_0^{T_1}\left[f(t)+\beta\int_t^{T_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt + \int_{T_1}^{T} P'\frac{f(t)}{[1+\delta(T-t)]}\,dt \\ -K-\displaystyle\int_0^{T_1} h\left[\int_t^{T_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt - \\ s\displaystyle\int_{T_1}^{T}\left(-\int_t^{T_1}\frac{f(u)}{[1+\delta(T-u)]}du\right)dt - \pi\int_{T_1}^{T} f(t)\left[1-\frac{1}{[1+\delta(T-t)]}\right]dt \\ -\displaystyle\int_0^{T_1} Pf(t)e^{(\theta+\beta)t}\,dt - \int_{T_1}^{T} P\frac{f(t)}{[1+\delta(T-t)]}\,dt \end{array}\right] \tag{11}$$

The solutions for the optimal values of T_1 and T (say T_1* and T*) can be found by solving the following equations simultaneously:

$$\frac{\partial TP(T_1,T)}{\partial T} = 0 \; and \; \frac{\partial TP(T_1,T)}{\partial T_1} = 0 \tag{12}$$

Provided they satisfy the conditions:

$$\left[\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right]_{at(T_1^*,T^*)} < 0, \left[\frac{\partial^2 TP(T_1,T)}{\partial T^2}\right]_{at(T_1^*,T^*)} < 0 \quad and$$

$$\left[\left[\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right]\left[\frac{\partial^2 TP(T_1,T)}{\partial T^2}\right]-\left[\frac{\partial^2 TP(T_1,T)}{\partial T_1\,\partial T}\right]^2\right]_{at(T_1^*,T^*)} > 0 \tag{13}$$

$$\frac{\partial TP(T_1,T)}{\partial T} = 0 \quad \Rightarrow$$

$$-\frac{1}{T}TP(T_1,T)+\frac{1}{T}\left(\begin{array}{l} P'\displaystyle\int_{T_1}^{T}\frac{-\delta f(t)}{[1+\delta(T-t)]^2}dt + P'f(T) - \pi\int_{T_1}^{T}\frac{\delta f(t)}{[1+\delta(T-t)]^2}dt - P\int_{T_1}^{T}\frac{-\delta f(t)}{[1+\delta(T-t)]^2}dt \\ -s\left\langle\displaystyle\int_{T_1}^{T}\left[\int_{T_1}^{t}\frac{-\delta f(u)}{[1+(\delta(T-u))]^2}du\right]dt + \left[\int_{T_1}^{T}\frac{f(u)du}{[1+\delta(T-u)]}\right]\right\rangle \end{array}\right) = 0 \tag{14}$$

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$$\frac{\partial TP(T_1,T)}{\partial T_1} = 0 \Rightarrow$$

$$\frac{1}{T}\left[\begin{array}{l} P'\left(\int_0^{T_1}\left(\beta e^{(\theta+\beta)(T_1-t)} f(T_1)\right)dt + f(T_1)\right) - P'\frac{f(T_1)}{[1+\delta(T-T_1)]} - h\left[\int_0^{T_1}\left(f(T_1)e^{(\theta+\beta)(T_1-t)}\right)dt\right] \\ -s\int_{T_1}^{T}\left(\frac{-f(T_1)}{[1+\delta(T-T_1)]}\right) - \pi\left[\frac{-\delta(T-T_1)f(T_1)}{[1+\delta(T-T_1)]}\right] - P\left(f(T_1)e^{(\theta+\beta)T_1} - \frac{f(T_1)}{[1+\delta(T-T_1)]}\right) \end{array}\right] = 0$$

i.e. $$\frac{(P'\beta - h - (\theta+\beta)P)}{(\theta+\beta)}\left(e^{(\theta+\beta)T_1} - 1\right) = \frac{-1}{(1+\delta(T-T_1))}\left(s + (P' - P + \pi)\delta\right)(T-T_1) \quad (15)$$

From the above results, we have the following propositions:

Proposition 1. If $(P'\beta - h - (\theta+\beta)P) \geq 0$, then the optimal solution of the maximum profit, TP (T_1*, T), does not exist.

Proof If $(P'\beta - h - (\theta+\beta)P) > 0$, we can get (15) holds if and only if (T-T_1) < 0. It is a contradiction. Similarly, if $(P'\beta - h - (\theta+\beta)P) = 0$, then from (15), we obtain T= T_1 and substituting this into (14), we get K=0 which is also a contradiction. Therefore, if $(P'\beta - h - (\theta+\beta)P) \leq 0$, then the optimal solution of the maximum profit, TP (T_1*, T), does not exist.

Proposition 2. If $(P'\beta - h - (\theta+\beta)P) < 0$, then the point ($T_1$*, T) which solves (14) and (15) simultaneously not only exist but is also unique.

Proof In order to prove the uniqueness of the solution, using (15), we take x = T-T_1 and let us take

$$F(x) = \frac{(P'\beta - h - (\theta+\beta)P)}{(\theta+\beta)}\left(e^{(\theta+\beta)T_1} - 1\right) + \frac{1}{(1+\delta x)}\left(s + (P' - P + \pi)\delta\right)x$$

$$F'(x) = \frac{1}{(1+\delta x)^2}\left\{\left(s + (P' - P + \pi)\delta\right)\right\} > 0$$

Hence f(x) is strictly increasing function in $x \in (0, \infty)$

$$F(0) = \frac{(P'\beta - h - (\theta+\beta)P)}{(\theta+\beta)}\left(e^{(\theta+\beta)T_1} - 1\right) < 0$$

$$\lim_{x\to\infty} F(x) = \frac{(P'\beta - h - (\theta+\beta)P)}{(\theta+\beta)}\left(e^{(\theta+\beta)T_1} - 1\right) + \frac{1}{\delta}\left(s + (P' - P + \pi)\delta\right) > 0$$

Therefore, there exists a unique $x^* \in (0,\infty)$ such that F(x*) = 0. So from this we conclude that once we get the value of T_1, we can find a T (>T_1) uniquely determined as a function of T_1.

$$\text{Let } G(T_1) = -\left[\begin{array}{l} P'\int_0^{T_1}\left[f(t) + \beta\int_t^{T_1} e^{(\theta+\beta)(u-t)} f(u)du\right]dt + \int_{T_1}^{T} P'\frac{f(u)}{[1+\delta(T-u)]}du \\ -K - \int_0^{T_1} h\left[\int_t^{T_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt - \int_{T_1}^{T} P\frac{f(u)}{[1+\delta(T-u)]}dt - \int_0^{T_1} Pf(t)e^{(\theta+\beta)t}dt \\ -s\int_{T_1}^{T}\left(-\int_t^{T_1}\frac{f(u)}{[1+\delta(T-u)]}du\right)dt - \pi\int_{T_1}^{T} f(t)\left[1 - \frac{1}{[1+\delta(T-t)]}\right]dt - \end{array}\right]$$

$$+T\left[\begin{array}{l} p'\int_{T_1}^{T}\frac{-\delta f(t)}{[1+\delta(T-t)]^2}dt+p'f(T)-p\int_{T_1}^{T}\frac{-\delta f(t)}{[1+\delta(T-t)]^2}dt-pf(T) \\ -s\left\langle\int_{T_1}^{T}\left[\int_{T_1}^{t}\frac{-\delta f(u)}{[1+(\delta(T-u))]^2}du\right]dt+\left[\int_{T_1}^{T}\frac{f(u)du}{[1+\delta(T-u)]}\right]\right\rangle-\pi\int_{T_1}^{T}\frac{\delta f(t)}{[1+\delta(T-t)]^2}dt \end{array}\right]$$

Then G (0) > 0, and $\lim_{T_1\to\bar{T}_1}$ G (T_1) < 0.

Where $\bar{T}_1 = \frac{1}{(\theta+\beta)}\ln\left(1-\frac{(\theta+\beta)((P'-P+\pi)\delta)}{\delta(P'\beta-h-(\theta+\beta)P)}\right)$

Since $\left(\frac{dT}{dT_1}-1\right)>0$, $\frac{\partial TP(T_1,T)}{\partial T_1}=0$ and $\frac{\partial^2 TP(T_1,T)}{\partial T^2}$ < 0, we have $\frac{dG(T_1,T)}{dT_1}$ < 0.

Therefore, there exists a unique $T_1{*} \in (0,\bar{T}_1)$ such that G (T_1*) = 0.

Proposition 3. If $(P'\beta-h-(\theta+\beta)P)$< 0, then the ($T_1$*, T) from equations (14) and (15) is the global maximum solution of the total relevant profit per unit time.

Proof Now, we examine the corresponding second order sufficient conditions for the optimal solutions. Since

$$\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}= f(T_1)\,(P'\beta-h-(\theta+\beta))\,e^{(\theta+\beta)T_1}+$$

$$\frac{f(T_1)}{(1+\delta(T-T_1))^2}\left(s+((P'-P+\pi)\delta)(T-T_1)\right) \tag{16}$$

$$\frac{\partial^2 TP(T_1,T)}{\partial T^2}\le -\frac{f(T_1)(s+(P'-P+\pi)\delta)(T-T_1)}{(1+\delta(T-T_1))^2})<0 \tag{17}$$

$$\frac{\partial^2 TP(T_1,T)}{\partial T\,\partial T_1}=\frac{\partial^2 TP(T_1,T)}{\partial T_1\partial T}=\frac{f(T_1)(s+(P'-P+\pi)\delta)(T-T_1)}{(1+\delta(T-T_1))^2} \tag{18}$$

$$\left[\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right]_{at(T_1*,T*)}<0,\left[\frac{\partial^2 TP(T_1,T)}{\partial T^2}\right]_{at(T_1*,T*)}<0.$$

And from the above results,

$$\left|\left[\frac{\partial^2 TP(T_1,T)}{\partial T^2}\right]\right|_{at(T_1*,T*)}=\left|\frac{\partial^2 TP(T_1,T)}{\partial T\partial T_1}\right|_{at(T_1*,T*)}$$

$$\left[\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right]_{at(T_1*,T*)}>\left|\frac{\partial^2 TP(T_1,T)}{\partial T\partial T_1}\right|_{at(T_1*,T*)}$$

$|H|=$

$$\left[\left[\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right]\left[\frac{\partial^2 TP(T_1,T)}{\partial T^2}\right]-\left[\frac{\partial^2 TP(T_1,T)}{\partial T_1\,\partial T}\right]^2\right]_{at(T_1*,T*)}>0.$$

Hence the Hessian matrix at $(T_1{*},T{*})$ is negative definite. So the stationary point for our optimization problem is a global maximum.

Solution procedure

Step 1: Solving (14) and (15) simultaneously, find T and T_1 values.

Step 2: using (11) find TP value.

4. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

In this section, numerical examples are proposed to illustrate the proposed model and its solution procedure. Example 1 below is presented for exponentially increasing demand function: f (t) = ae^{bt} (a > 0, b > 0). Example 2 below is presented for a constant demand function: f(t) = a(a >0). Sensitivity analysis for β,δ , a, b, is also reported for two types of functions mentioned above and are respectively displayed in Tables, 3 and 4, respectively.

Example 1

Let s = 3, K = 250, θ = .05, f (t) = ae^{bt} , h = 1.75, P' = 15, P = 5, $\pi = 5$, β =.2, δ = 5, a = 600, b = 3 in appropriate units. The optimal values are T =.5, T_1 =.1, TP = 12143.93. Numerical values are shown in Table-1.

Example 2

Let s = 3, K =250, θ =.05, f (t) = ae^{bt} , h = 1.75, P' =15, P = 5, $\pi = 5$, β =.2, δ = 5, a = 600, b = 0 in appropriate units. The optimal values are T =.5, T_1 =.4, TP =2513.26336. Numerical values are shown in Table-2.

Table-1. Optimal replenishment policy for an increasing demand function.

T	T_1	T_1 /T	TP
0.5000	.1000	.200	12143.93099

Table-2. Optimal replenishment policy for an increasing demand function.

T	T_1	T_1 / T	TP
0.500	0.400	0.800	2513.26336

In order to study how various parameters affects the optimal solution of the proposed inventory model, sensitivity analysis is performed. Keeping all the other parameters fixed and varying a single parameter at a time, for the same set of values we study the results. The results of the various parameters against the profit of our model are shown in Table 3 and Table 4.

Table-3. Sensitivity analysis for increasing demand.

Parameter	Value	TP
β	.09	11894.83
	.2	12143.93
	.3	12377.53
δ	3	12567.06
	5	12143.93
	7	11790.39
a	300	5821.96
	600	12143.93
	800	16358.58
b	2	8980.76
	3	12143.93
	6	32899.29

Table 4. Sensitivity analysis for constant demand.

Parameter	Value	TP
β	.09	2462.95
	.2	2513.26
	.3	2560.18
δ	3	2543.96
	5	2513.26
	7	2487.85
a	300	1006.63
	600	2513.26
	800	3517.69

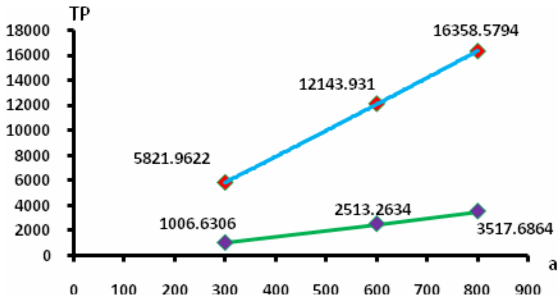
From the above numerical values we conclude that:

If the demand is an increasing function of time and for large values of "a" and "b" total profit is maximum. But if the demand is a constant, total profit is minimum compared to increasing demand function.

Managerial implications

- In order to increase the profit he should reduce all the cost parameters.
- Maximum stock dependency rate of the products will maximize the profit of the retailer.
- If the retailer increases the lost sales case it will reduce the benefit to him.

Graphical representation of **TP** against '**a**'



5. CONCLUSIONS

In this paper, we discussed an EOQ model for perishable items under stock and time-dependent selling rate and time-dependent partial backlogging. Here, we consider that demand is not only a function of stock but also fluctuates with time. For f (t) = a model reduces to that of C-Y-Dye and L-Y-Ouyang [4]. Finally, the sensitivity of the solution to changes in the values of different parameters has been discussed. From our numerical example one can easily conclude that total profit is maximum when the demand depends on time rather than constant demand.

The proposed model can be extended in several ways. We could extend the deterministic demand function to stochastic demand patterns, as a function of selling price etc.

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