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TRANSIENT ANALYSIS OF THREE-PHASE INDUCTION MACHINE USING DIFFERENT REFERENCE FRAMES

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ABSTRACT

Three-phase induction machines are generally used as motors for many industrial applications and all this is due to its simple construction and other advantages in contrast to other machines. Popularity of these motors has resulted into a lot of research including the transient behavior of the machine. Literature survey reveals that most of the researchers adopted only a single reference frame to estimate transient behavior of the machine. In this paper qd axis based modeling is proposed to analyze the transient performance of three-phase squirrel cage induction motor using stationary reference frame, rotor reference frame and synchronously rotating reference frame. Simulated results have been compared and verified with experimental results on a test machine. A close agreement between the simulated and experimental results proves the validity of proposed modeling. The proposed system has been developed using MATLAB/SIMULINK.

Keywords: modeling, induction motor, reference frames, simulation, transient analysis.

INTRODUCTION

During start-up and other severe transient operations induction motor draws large currents, produces voltage dips, oscillatory torques and can even generate harmonics in the power systems. In order to investigate such problems, the d, q axis model has been found to be well tested and proven to be reliable and accurate. [1-3] describes the basic concept of transient modeling of the machine. Dynamic behavior of the machine may be analyzed using any one of following the reference frames:

a) Stationary reference frame
b) Rotor reference frame
c) Synchronous reference frame

[4-5] recommends specific tests to estimate the machine parameters to proceed with transient modeling. [6-11] used the stationary or the synchronously rotating reference frames for the analysis of induction motors. [12] described the utility of various reference frames for different purposes. However, time span selected for simulation is found to be too small to study the complete behaviour of induction motor.

Researchers [13-15] have developed their own software packages for the modeling and simulation of induction machines. In [13], a software package was developed, using the FORTRAN programming language, for the steady-state and dynamic simulation of induction motor drives. Digital simulation of field-oriented control of an induction motor, again implemented in FORTRAN, is described in [14]. Both of these approaches required the development of numerical integration routines for the solution of the resultant state-space equations that describe the systems. A general-purpose program, the electromagnetic transients program (EMTP), is described in [15] for the digital simulation of field-oriented control of induction motor drives. This program has the facility to extend simulation to other machine types.

Now a days Matlab/Simulink is a very useful tool for modeling electrical machine and it may be used to predict the dynamic behavior of the machines [16-18]. In this paper Matlab/Simulink based modeling is proposed using all the three reference frames as mentioned above. On the bases of simulated results conclusions are drawn to justify the selection of a particular reference frame. During simulations sufficient time span is included to predict the complete behavior of the machine.

MATHEMATICAL MODELING

A three-phase induction motor can be modeled with qd axis theory. According to qdo axis modeling:

$$\left[F_{qdo}\right] = \left[T_{qdo}\right]\left[F_{abc}\right] \tag{1}$$

Where

$$\left[T_{qdo}\right] = 2/3\begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin\theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \tag{2}$$

The voltage balance equation for the q, d coils in arbitrary reference frames are as follows [2]:

$$\left[V^c\right] = \left[Z^c\right]\left[i^c\right] \tag{3}$$

Where $\left[V^c\right]$ and $\left[i^c\right]$ represents '4x1' column matrices of voltage and current and are given as $\left[V_{qs}^c\ V_{ds}^c\ V_{qr}^c\ V_{dr}^c\right]^T$ and $\left[i_{qs}^c\ i_{ds}^c\ i_{qr}^c\ i_{dr}^c\right]^T$ respectively.

and, impedance matrices (4x4), $\left[Z^c\right]$ is given as,

VOL. 4, NO. 8, OCTOBER 2009 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

$$[Z^c]=\begin{bmatrix} R_s+L_s p & \omega_c L_c & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s+L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega-\omega_r)L_m & R_r+L_r p & (\omega_c-\omega_r)L_r \\ -(\omega_c-\omega_r)L_m & L_m p & -(\omega_c-\omega_r)L_r & R_r+L_r p \end{bmatrix}$$

Stationary reference frame model

The speed of the reference frames is that of the stator, which is zero; hence,

$$\omega_C = 0 \tag{4}$$

Is substituted into equation (3). The resulting model is

$$[V]=[Z]\,[i] \tag{5}$$

Where $[V\ ]$ and $[i\ ]$ represents '4x1' column matrices of voltage and current and are given as $\left[V_{qs}\ V_{ds}\ V_{qr}\ V_{dr}\right]^T$ and $\left[i_{qs}\ i_{ds}\ i_{qr}\ i_{dr}\right]^T$ respectively.

And, impedance matrices (4x4), $[Z\ ]$ is given as,

$$[Z\ ]=\begin{bmatrix} R_s+L_s p & 0 & L_m p & 0 \\ 0 & R_s+L_s p & 0 & L_m p \\ L_m p & -\omega_r L_m & R_r+L_r p & -\omega_r L_r \\ \omega_r L_m & L_m p & \omega_r L_r & R_r+L_r p \end{bmatrix}$$

The torque equation is

$$T_e=\frac{3}{2}\frac{P}{2}L_m(i_{qs}i_{dr}-i_{ds}i_{qr}) \tag{6}$$

In case bus bar voltages are

$$V_{as}=V_m\cos(\omega_s t+\lambda)$$
$$V_{bs}=V_m\cos(\omega_s t+\lambda-2\pi/3)$$
$$V_{cs}=V_m\cos(\omega_s t+\lambda+2\pi/3) \tag{7}$$

And the motor terminals are connected directly to the bus, then using the Park's transformation of equation (2),

$$V_{qs}=V_m\cos(\omega_s t+\lambda)$$
$$V_{ds}=-V_m\sin(\omega_s t+\lambda) \tag{8}$$

Rotor reference frames model

The speed of the rotor reference frames is

$$\omega_c=\omega_r \tag{9}$$

and the angular position is

$$\theta_c=\theta_r \tag{10}$$

Substituting in the upper subscript r for rotor reference frames and equation (4) in the equation (3), the induction-motor model in rotor reference frames is obtained. The equations are given by

$$[V^r]=[Z^r]\,[i^r] \tag{11}$$

Where $[V^r]$ and $[i^r]$ represents '4x1' column matrices of voltage and current and are given as $\left[V^r_{qs}\ V^r_{ds}\ V^r_{qr}\ V^r_{dr}\right]^T$ and $\left[i^r_{qs}\ i^r_{ds}\ i^r_{qr}\ i^r_{dr}\right]^T$ respectively.

and, impedance matrices (4x4), $[Z^r]$ is given as,

$$[Z^r]=\begin{bmatrix} R_s+L_s p & \omega_r L_s & L_m p & \omega_r L_m \\ -\omega_r L_s & R_s+L_s p & -\omega_r L_s p & L_m p \\ L_m p & 0 & R_r+L_r p & -0 \\ 0 & L_m p & 0 & R_r+L_r p \end{bmatrix}$$

and the electromagnetic torque is

$$T_e=\frac{3}{2}\frac{P}{2}L_m(i^r{}_{qs}i^r{}_{dr}-i^r{}_{ds}i^r{}_{qr}) \tag{12}$$

The transformation from abc to qdo variables is obtained by substituting (10) into $[T_{abc}]$, defined in (2) as

$$[T^r{}_{abc}]=\frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r-\frac{2\pi}{3}\right) & \cos\left(\theta_r+\frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r-\frac{2\pi}{3}\right) & \sin\left(\theta_r+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{13}$$

The terminal voltage equation (7) becomes

$$V_{qs}=V_m\cos(s\omega_s t+\lambda)$$
$$V_{ds}=-V_m\sin(s\omega_s t+\lambda) \tag{14}$$

The q, d voltages are therefore of slip frequency and the q-axis rotor current behaves exactly as the phase a rotor current does.

Synchronously rotating reference frame model

The speed of the reference frame is

$\omega_c=\omega_s=$ Stator supply angular frequency (rad /sec). (15)

and the instantaneous angular position is

$$\theta_c=\theta_s=\omega_s t \tag{16}$$

By substituting (15) into (3), the induction motor model in the synchronous reference frames is obtained. By using the superscript e to denote this electrical synchronous reference frame, the model is obtained as

$$[V^e]=[Z^e]\,[i^e] \tag{17}$$

Where $[V^e]$ and $[i^e]$ represents '4x1' column matrices of voltage and current and are given as $\left[V^e_{qs}\ V^e_{ds}\ V^e_{qr}\ V^e_{dr}\right]^T$ and $\left[i^e_{qs}\ i^e_{ds}\ i^e_{qr}\ i^e_{dr}\right]^T$ respectively.

and, impedance matrices (4x4), $[Z^e]$ can be given as,

ARPN Journal of Engineering and Applied Sciences

$$[Z^e] = \begin{bmatrix} R_s + L_s p & \omega_s L_s & L_m p & \omega_w L_m \\ -\omega_s L_s & R_s + L_s & -\omega_s L_m & L_m p \\ L_m p & (w_s - \omega_r) L_m & R_r + L_r p & (\omega_s - \omega_r) L_r \\ -(\omega_s - \omega_r) L_m & L_m p & -(\omega_s - \omega_r) L_r & R_r + L_r p \end{bmatrix}$$

The electromagnetic torque is,

$$T_e = \frac{3}{2}\frac{P}{2} L_m (i^e{}_{qs} i^e{}_{dr} - i^e{}_{ds} i^e{}_{qr}) \qquad \text{(N.m)} \qquad (18)$$

The transformation from abc to dqo variables is found by substituting (16) into equation (2) and is given as

$$[T^e{}_{abc}] = \frac{2}{3} \begin{bmatrix} \cos\theta_s & \cos\left(\theta_s - \frac{2\pi}{3}\right) & \cos\left(\theta_s + \frac{2\pi}{3}\right) \\ \sin\theta_s & \sin\left(\theta_s - \frac{2\pi}{3}\right) & \sin\left(\theta_s + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad (19)$$

The terminal voltage in equation (7) become

$$V_{qs} = V_m \cos\lambda$$

$$V_{ds} = -V_m \sin\lambda \qquad (20)$$

This means the stator d, q voltages are dc quantities. The mechanical motion described by

$$p\omega_r = (T_e - T_L)/J \qquad (21)$$

RESULTS AND DISCUSSIONS

The proposed model has been tested on a three-phase induction motor (Appendix-I). Simulation results as obtained are shown in Figures 1 to 5. The comparison of simulated results with experimental results on a test machine (Appendix-I) has been shown in Figure-1. A close agreement between the simulated and experimental results (Appendix-II) confirms the validity of model adopted.

Figure-2 shows the predicted start-up characteristics, and found to be identical for each reference frame. Initially, the three-phase induction motor runs on no-load and after one second, it runs on full-load.

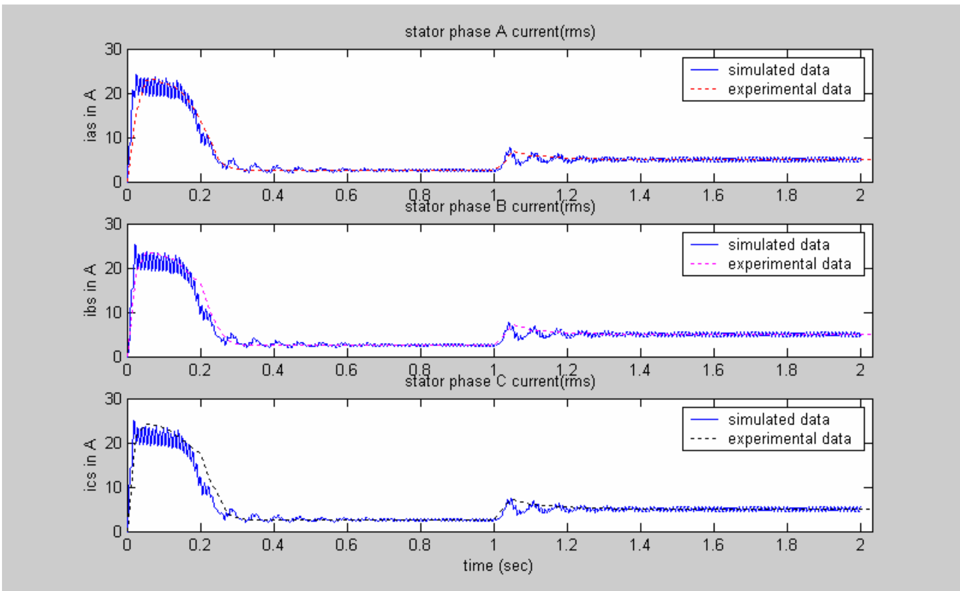

Figure-1. Stator phase currents in phase A, B and C.

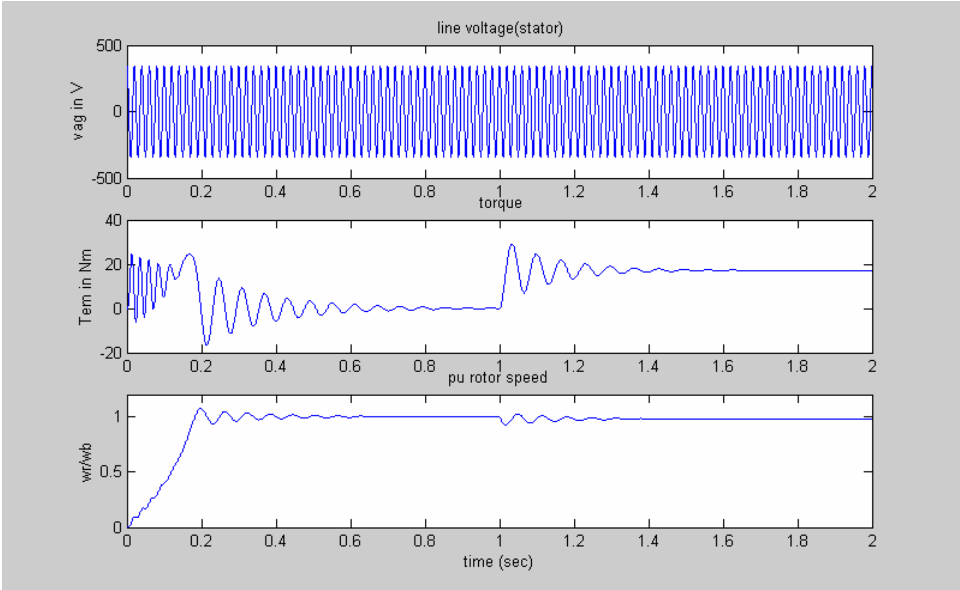

Figure-2. Stator line voltage, electro magnetic torque and rotor speed of a 3- hp induction motor.

With respect to stationary reference frame

In this reference frame the q- axis is fixed to and thus coincident with the axis of the stator phase a winding. This means that the mmf wave of the stator moves over this frame at the same speed as it does over the stator phase a windings. This reference frame‘s stator q-axis variables therefore behave in exactly the same way as do the physical stator phase A variables of the motor itself. Figure-3 shows the identical nature of the stator Phase A current and the stator q- axis current.

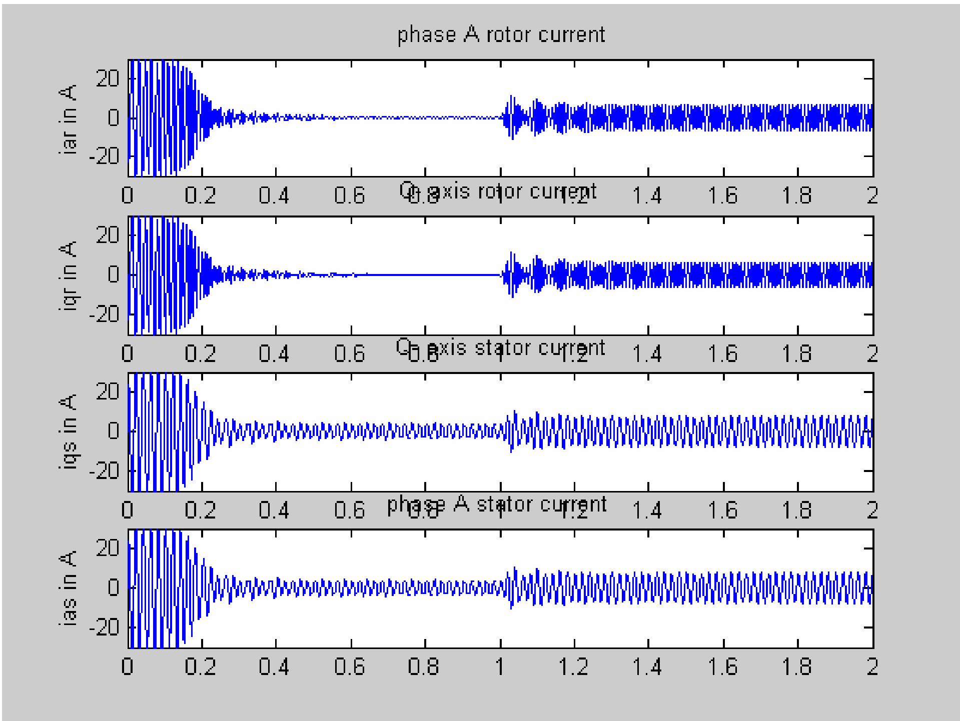

Figure-3. Predicted staring up current using stationary reference frame.

It would, therefore, be an advantage when studying transient occurrences involving the stator variables to use this reference frame.

With respect to rotor reference frame

Since in this reference frame the q- axis of the reference frame is moving at the same relative speed as the rotor phase a winding and coincident with its axis, it should be expected that the behaviour of the q- axis

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current and the phase a current would be identical. Figure-4 shows how the rotor phase a current and the rotor q-axis current are initially at 50 Hz, when the rotor is at standstill, but gradually changes to slip frequency at normal running speed.

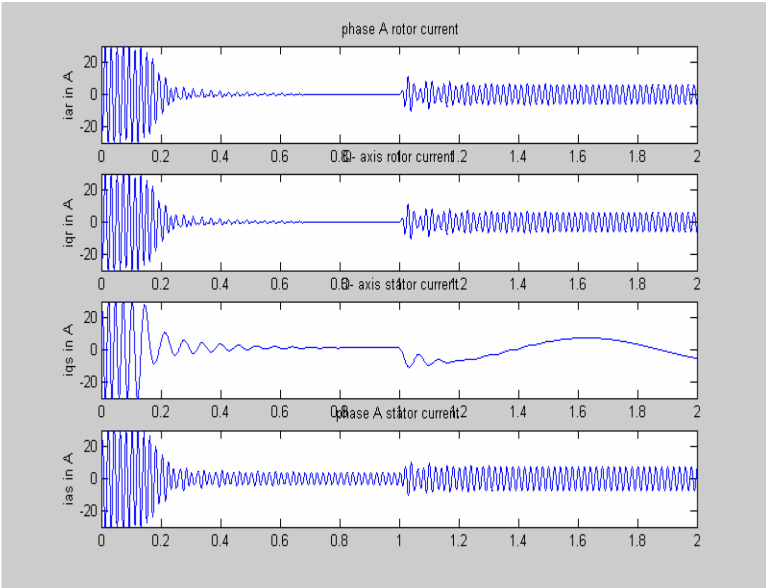

Figure-4. Predicted staring up current using rotor reference frame.

With respect to synchronously rotating reference frame

When the reference frame is rotating at synchronous speed, both the stator and rotor are rotating at different speeds relative to it. However, with the reference speed rotating at the same speed as the stator and rotor space field mmf waves, the stator and rotor q, d variables are constant quantities, whereas the actual variables are at 50 Hz and slip frequencies, respectively.

It is preferable to use this reference frame for stability of controller design where the motor equations must be linearized about an operating point, since in this reference frame the steady state variables are constant and do not vary sinusoidaly with time (Figure-5).

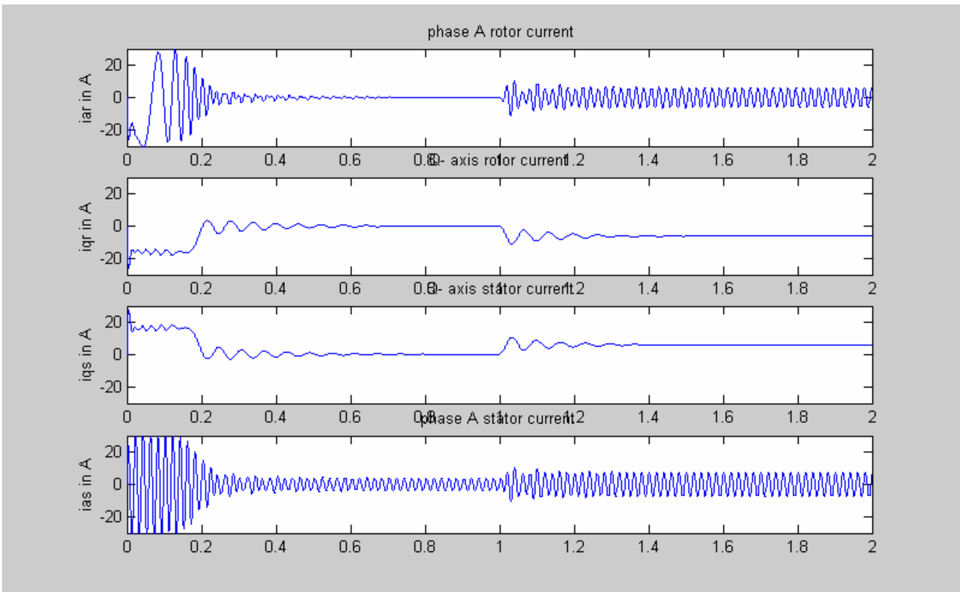

Figure-5. Predicted staring up current using synchronously rotating reference frame.

VOL. 4, NO. 8, OCTOBER 2009 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

CONCLUSIONS

In this paper an attempt has been made to predict the transient behavior of the three-phase induction motor using stationary reference frame, rotor reference frame and synchronously rotating reference frame. MATLAB/ SIMULINK is used to predict the transient performance. Sufficient time span is included to study the complete performance predicting characteristics of the induction motor. A critical observation of simulated results has been concluded as:

- Stator and rotor currents in phase A and in q-axis (ias, iqs, iar and iqr) in different reference frames follows the following pattern:

  Stationary reference frame: No difference between ias and iqs (refer Figure-3).

  Rotor reference frame: No difference between iar and iqr (refer Figure-4).

  Synchronously rotating reference frame: No resemblance between ias and iqs and iar and iqr (refer Figure-5).
- Rotor currents in phase 'A' does not appears to be identical for three reference frames as observed by Figures 3, 4 and 5; and
- However, solution of reference frame does not effects the speed rise, maximum inrush current and settling time for the machine.

From above it is observed that rotor reference frame is useful in case one is interested to look into the transient effects on the rotor side of the machine.

Nomenclatures

$V_{as,}V_{bs,}V_{cs}$	=	Bus bar voltages for phase A, B and C respectively
V_m	=	Maximum Voltage
$V^c qs, Vqds, V^r qds, V^e qds$	=	Stator q and d axes voltages in arbitrary, stationary, rotor and synchronously rotating reference frame
$V^c qdr, Vqdr, V^r qdr, V^e qdr$	=	Rotor q and d axes voltages in arbitrary, stationary, rotor and synchronously rotating reference frame
$i^c qds, iqds, i^r qds, i^e qds$	=	Stator q and d axes currents in arbitrary, stationary, rotor and synchronously rotating reference frame `
$i^c qdr, iqdr, i^r qdr, i^e qdr$	=	Stator q and d axes voltages in arbitrary, stationary, rotor and synchronously rotating reference frame
R_s	=	Stator Phase Resistance
L_s	=	Stator Self inductance
L_m	=	Mutual inductance
R_r	=	Rotor Phase Resistance
L_r	=	Rotor self inductance
$\omega_{c,}\omega_{r,}\omega_s$	=	Angular speed (rad/sec.) in arbitrary, rotor and synchronously rotating reference frame
$\theta_c, \theta, \theta_r, \theta_s$	=	Angular position in arbitrary, rotor and synchronously rotating reference frame
J	=	Inertia of Motor
T_e	=	Electrical Torque
T_L	=	Load Torque
p	=	Operator for differentiation
s	=	Slip

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Subscripts:	q	=	Quadrature axis
	d	=	Direct axis
	0	=	Zero axis
	s	=	Stator quantities
	r	=	Rotor quantities
Superscripts:	c	=	Quantity referred to arbitrary frame
	r	=	Quantity referred to rotating frame
	s	=	Quantity referred to synchronously rotating frame

Appendix-I:

3-hp, 3-phase, 50 Hz, 415 volts Induction Motor;

Stator Resistance,	R_s	=	4.44 ohms
Rotor Resistance,	R_r	=	0.9512 ohms
Stator & Rotor Inductance,	$L_s = L_r$	=	14.97 mH
Magnetizing Inductance,	Lm	=	267.4 mH
Moment of Inertia of test machine set up,	J	=	0.22 kgm2 (Appendix-III)

Appendix-II:

Experimental set-up for verification of simulated results:
A dc shunt generator is coupled with the induction motor is used to load the induction motor. The rating and parameters of the dc shunt machine are:

kw	:	3.7
Amperes	:	12
Rpm	:	1500
Volts	:	220
Armature resistance, Ra	:	2 Ω.
Field resistance, Rf	:	282 Ω.

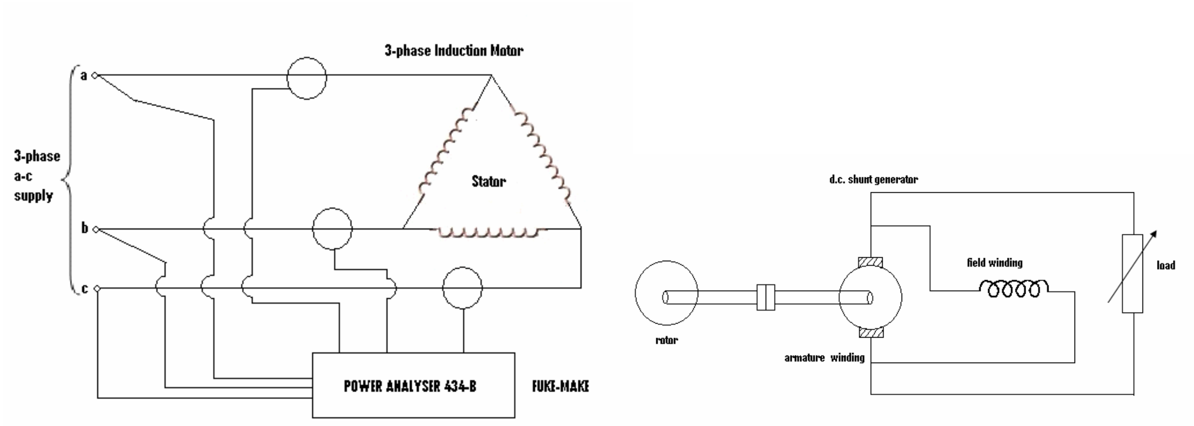

Figure-6. Experimental set-up for verification of simulated results.

Appendix-III:

Experimental set-up to determine the moment of inertia:
Circuit as shown in Figure-7 may be used to perform the retardation test on given motor with coupling with auxiliary machine. After taking the appropriate observations, following expressions may be used to determine moment of inertia. The power, P consumed in overcoming the rotational losses is given by,

$$P = J \quad x \quad \frac{4\pi^2}{3600} \quad x \quad N \quad x \quad \frac{dN}{dt}$$

Where N is operating speed of motor in rpm. dN / dt at normal rated speed can be found graphically using the oscillogram of speed vs. time.

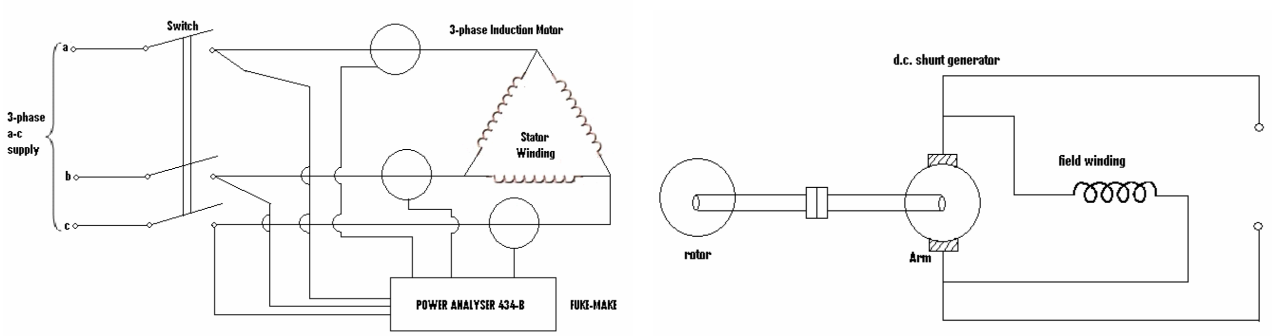

Figure-7. Experimental set-up to determine moment of inertia with external system.

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