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COUPLED FINITE-INFINITE ELEMENTS MODELING OF BUILDING FRAME-SOIL INTERACTION SYSTEM

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ABSTRACT

The soil-structure interaction analysis of structures is a complex and broad area of research in structural and geo-technical engineering. It deals with study of mechanics of interaction between foundation, soil and superstructure or its parts buried in soil to investigate the interaction behaviour. In common structural design practice the foundation loads from structure analysis are obtained without considering allowance for soil settlements. The foundation settlements are estimated assuming a perfectly flexible structure. A powerful numerical tool like finite element method can be used to analyze the composite system. The finite element modeling of the domain of the building frame-soil interaction system needs variety of isoparametric elements with different degrees of freedom. The superstructure is discretized with conventional isoparametric elements while the soil mass with coupled finite-infinite elements having different decay patterns to model the far field behaviour. This paper presents the modeling of plane frame-foundation beam-soil system for elasto-plastic interaction analysis considering the entire system to act as a single integral compatible structural unit using finite-infinite elements. The forces in the frame members (beams and columns) and the foundation beam have been evaluated and compared with conventional frame analysis.

Keywords: Soil-structure interaction, elasto-plastic analysis, yield criteria, infinite elements, isoparametric elements, decay pattern.

1. INTRODUCTION

The solution of the problem of building frame-foundation beam-soil mass interaction system needs a proper physical modeling and numerical analysis to access the more realistic and accurate structural behaviour of the composite system. The powerful numerical tool like finite element method can be used to analyze the problem considering the superstructure, foundation and the soil mass to act as single integral compatible structural unit. The material nonlinearity involved in the problem of soil structure-interaction also needs a special numerical treatment. The discretization of the domain of interaction system needs variety of isoparametric elements with different degrees of freedom.

The finite element idealization of the unbounded domain of the soil mass using only finite elements proves to be computationally uneconomical and expensive. The use of coupled finite-infinite elements with proper location of truncation boundary provides accurate and computationally economical solutions.

In the present investigations, the elasto-plastic interaction analysis of plane building frame-soil system has been presented considering the superstructure to behave in linear elastic manner while the soil mass to behave in elasto-plastic manner. Various yield criteria for the soil mass are considered for the elasto-plastic interaction analysis. The forces in the frame members and the foundation beam have been evaluated and the results of interaction analysis are compared with non-interaction analysis.

2. FINITE ELEMENT MODELING BUILDING FRAME-SOIL SYSTEM

The superstructure, which includes the floor beams, columns and the foundation beam, is discretized using three noded beam bending elements with three degrees of freedom per node (u, v, θ). The unbounded domain of the soil mass is discretized by eight noded plane strain finite elements with two degrees of freedom per node (u, v) coupled with six noded infinite elements with two degrees of freedom per node (u, v). A three noded doubly infinite element is used as corner element in the finite-infinite mesh. Different types of decay patterns can be used for the infinite elements to model the far field behaviour. Table-1 depicts various types of elements and their shape functions.

The modeling of interface between the foundation beam elements and the finite soil elements is achieved using six noded isoparametric interface elements. The element has three degrees of freedom (u, v,θ) for the upper three nodes and two degrees of freedom (u, v) for the bottom three nodes.

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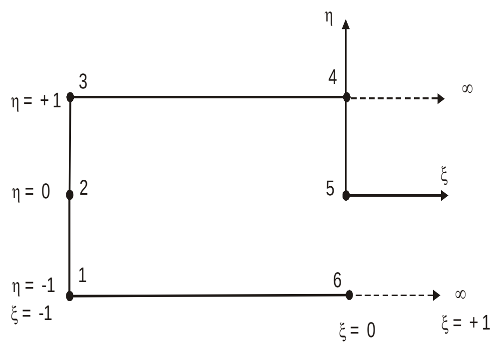
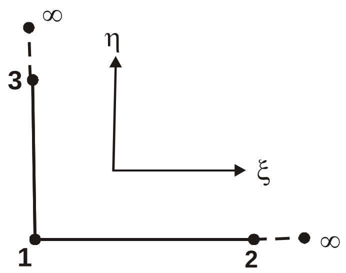
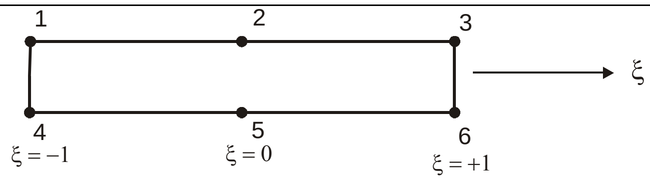
Table-1. Shape functions for isoparametric finite and infinite elements.

Element type	Element figure	Shape functions
3-Noded beam element		$N_1 = \xi(1-\xi)/2$ $N_2 = (1-\xi^2)$ $N_3 = \xi(1+\xi)/2$
6- Noded infinite element with 1/r type decay		$N_1 = \frac{\xi\eta(1-\eta)}{(1-\xi)}$ $N_2 = \frac{-2\xi(1-\eta^2)}{(1-\xi)}$ $N_3 = \frac{-\xi\eta(1+\xi)}{(1-\xi)}$ $N_4 = \frac{(1+\xi)\eta(1+\eta)}{2(1-\xi)}$ $N_5 = \frac{(1+\xi)(1-\eta^2)}{(1-\xi)}$ $N_6 = \frac{-(1+\xi)\eta(1-\eta)}{2(1-\xi)}$
3- Noded doubly infinite element with 1/ r type decay		$N_1 = \frac{(\xi\eta+3)(-1-\xi-\eta)}{(1-\xi)(1-\mu)}$ $N_2 = \frac{2(1+\xi)}{(1-\xi)(1-\mu)}$ $N_3 = \frac{2(1+\eta)}{(1-\xi)(1-\mu)}$
6- Noded modified interface element (Zero thickness) $N_1 = N_4$ $N_2 = N_5$ $N_3 = N_6$		$N_1 = \xi(1-\xi)/2$ $N_2 = (1-\xi^2)$ $N_3 = \xi(1+\xi)/2$

3. INFINITE ELEMENTS AND FORMULATION APPROACH

An 'Infinite element' may be defined as an element in which one or more dimensions extend to infinity. Such elements find their wide applicability in practically all unbounded continua problems. The infinite elements with different types of decay patterns are able to model the far field behaviour accurately.

A simple procedure was proposed [1] to derive static infinite element from a linear isoparametric element using two approaches termed as 'Displacement descent formulation' and 'Coordinate ascent formulation'. An infinite element with displacement descent formulation requires numerical integration over a semi-infinite range for calculation of the elemental properties, which is quite inconvenient.

In the coordinate ascent formulation, the shape functions of coordinate transformation are derived so that an infinite element in the physical plane is mapped into a more convenient shape in the natural plane. Now, the conventional shape functions are used for the nodal function transformation. Thus, an infinite element in the physical plane is compressed to be a finite element of a regular shape in natural plane and, therefore, the numerical integration ranges over -1 to +1. Thus, infinite elements with co-ordinate ascent formulation can be integrated conveniently using Gauss-Legendre numerical integration scheme.

These elements are attractive from the point of view of their application to real engineering problems as well as their implementation in any finite element code. However, the location of truncation boundary between finite and infinite elements is the most important aspect in the analysis of a problem. Infinite element formulations have been incorporated in finite element computer program with great advantage. The results obtained from finite cum infinite element meshes are more accurate. Not only these meshes give results that are comparable with the result from classical mechanics, but the computational effort is also reduced by a large extent in comparison to purely finite element results.

3.1 Decay function

The role of the decay function is to ensure that the behaviour of the element at infinity is a reasonable reflection of physics of the problem. The shape functions of the finite element are multiplied by a decay function to obtain the shape functions for an infinite element.

Let the shape function of the original (parent) element be M_i, where i = 1 to n, where, 'n' is the number of nodes in the element. This shape function will not appropriately describe the behaviour of the far field variables. Therefore, the decay functions are introduced which modify the finite element shape functions:

$$N_i(\xi,\eta) = f_i(\xi,\eta)\, M_i(\xi,\eta) \qquad (1)$$

The decay function must have value of unity at its own node i.e.

$$f_i(\xi_i,\eta_i) = 1 \qquad (2)$$

There is no requirement that the decay function should take any special value at the other nodes. In addition, N_i must tend to the far field value at infinity. However, it must have a realistic description of the problem under study. Two types of decay functions are available:

(i) Exponential decay functions: This formulation is suggested by Peter Bettess [2]. The formulation comprises of a series of shape functions analogues to Lagrange polynomials but including an exponential decay term for elements extending to infinity. The main requirements of the shape functions are that it should be realistic and lead to finite integration over the element domain. The decay in the positive ξ and η directions is expressed as:

$$f_i(\xi,\eta) = \exp\left(\frac{\xi_i - \eta_i - \xi - \eta}{L}\right) \qquad (3)$$

(ii) Reciprocal decay functions: The decay function, $f_i(\xi,\eta)$ is expressed as:

$$f_i(\xi,\eta) = \exp\left(\frac{\xi_i - \xi_0}{\xi - \xi_0}\right)^m \left(\frac{\eta_i - \eta_0}{\eta - \eta_0}\right)^n \qquad (4)$$

Where, (ξ_0,η_0) is some origin point known as pole. The pole must be outside the infinite element. The exponent, 'm' and 'n' must be greater than the highest power of ξ in M_i.

3.2 Elemental formulation

For the evaluation of the stiffness matrix for an infinite element, the original (parent) element shape function is used to define the mapping [3] through the Jacobian and an infinite shape function (obtained from the product of the original shape function and the decay function) to define far field behavior. Once the infinite shape functions are evaluated, the element stiffness is obtained in the usual manner as those for conventional isoparametric finite elements.

4. ELASTO PLASTIC INTERACTION ANALYSIS SOFTWARE

A computer program in FORTRAN-90 has been developed for elasto-plastic interaction analysis of frame-foundation beam-soil system. It includes library of different types of elements needed for the discretization of the interaction system. The beam element included in the program is a modified form of the beam-bending element [4], which includes one additional degree of freedom to take care of axial deformation in the frame members. The discretization of the infill panels uses conventional eight noded isoparametric elements, whereas the coupled finite-infinite elements are used to discretize the soil mass [5].

The software takes into account the elasto-plastic behaviour of soil mass by considering different yield criteria. Various yield criteria for the soil mass are transformed into convenient form for their easy implementation in finite element code. The Gauss-Legendre scheme is employed for the evaluation of element stiffness of finite and infinite elements both. The elasto-plastic interaction analysis is carried out using mixed (incremental-iterative) method. A frontal solver by [6] has been reorganized and made compatible to solve linearized simultaneous equations arising from a discretization of the domain with variety of elements.

5. ELASTO-PLASTIC INTERACTION ANALYSIS OF FRAME-SOIL SYSTEM

5.1 Problem under investigation

In the present investigations, the linear elastic interaction analysis (LIA) and elasto-plastic interaction analysis (EPIA) of two-bay two-storey plane frame-foundation beam-soil system (FS) have been carried out considering the frame to behave in linear elastic manner whereas the subsoil in elasto-plastic manner. The geometrical properties of the frame and soil parameters for elasto-plastic analysis [5] are provided in Figure-1.

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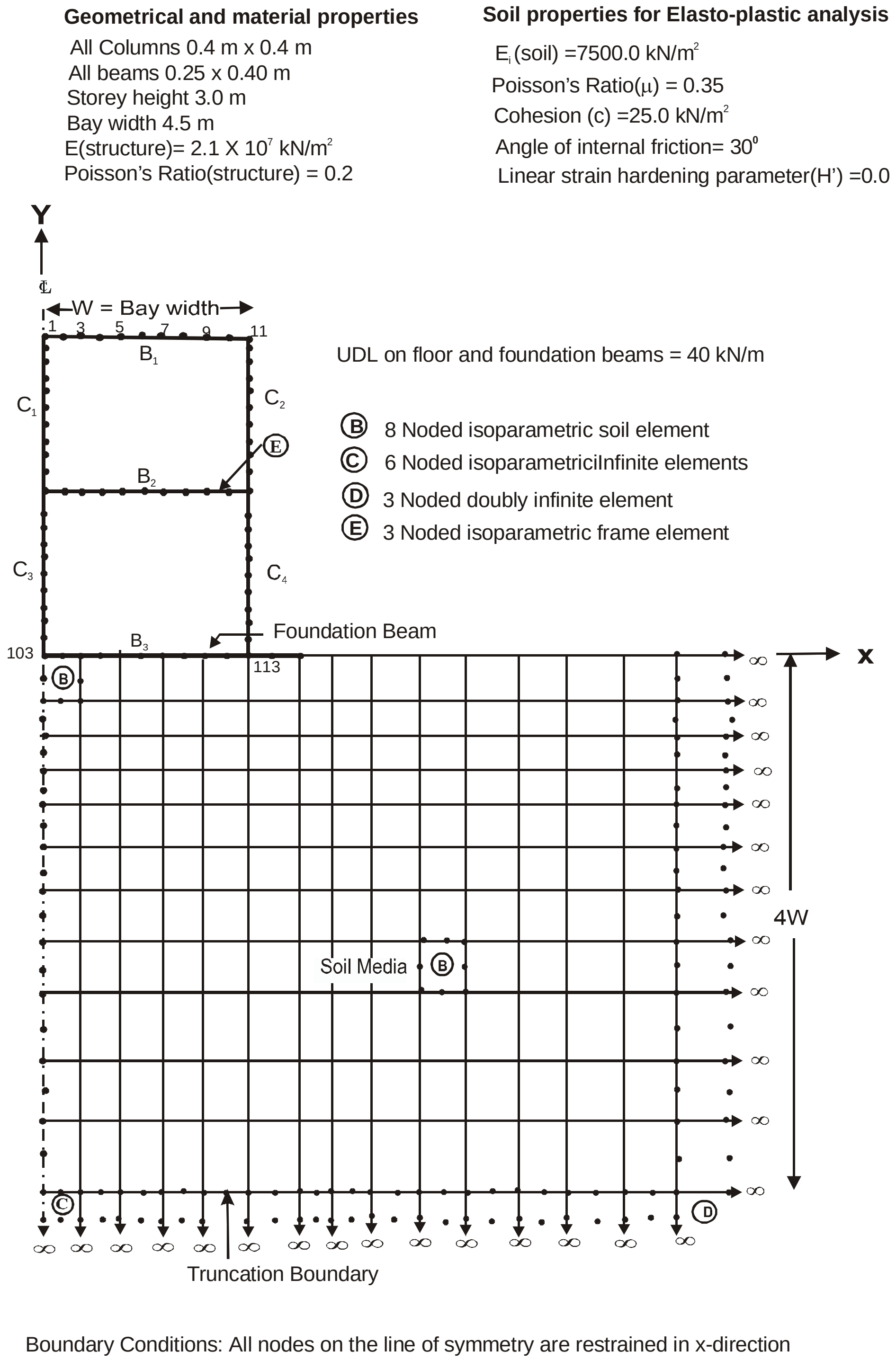

Fig. 1. Finite-infinite element discretization of plane frame-soil system

The floor beams and the foundation beam carry uniformly distributed load of 40 kN/m, which includes dead load and live load. The elasto-plastic constitutive relationship [7] of the soil mass is considered. In any coupled finite-infinite element formulation, the most important aspect is the location of truncation boundary (the common junction between the finite and infinite element layer), which is found by trial and error [6]. The non-interaction analysis (NIA) is carried out considering the columns fixed at their bases. The results of EPIA are compared with those obtained due to NIA. Figure-1 shows the discretization of the interaction system. Since the system is symmetrical with respect to geometry and loading, only half of the structural-foundation beam-soil system is considered and meshed for carrying out the interaction analysis.

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5.2 Interaction analysis

The computational algorithm adopted for elasto-plastic interaction analysis is quite identical [7]. The floor beams and the foundation beam carry uniformly distributed load of 40 kN/m. The elasto-plastic analysis of the interaction system is carried out using mixed (incremental-iterative) method. In this analysis, the initial load was decided in such a manner, which causes local failure in some finite elements of the soil mass (i.e. load factor of unity which corresponds to 40 kN/m). The vertical load is applied in thirteen increments (50, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 5, and 5% of 40 kN/m). The load increments are chosen depending upon the nature of the stress-strain curve and material properties of the soil mass and this requires trial and error. The norm of residual force for convergence is chosen for the interaction analysis. A tolerance limit of 1% is selected for residual forces. The elasto-plastic interaction analysis has been carried out considering the subsoil to yield according to the yield criteria [11] depicted in Figure-2.

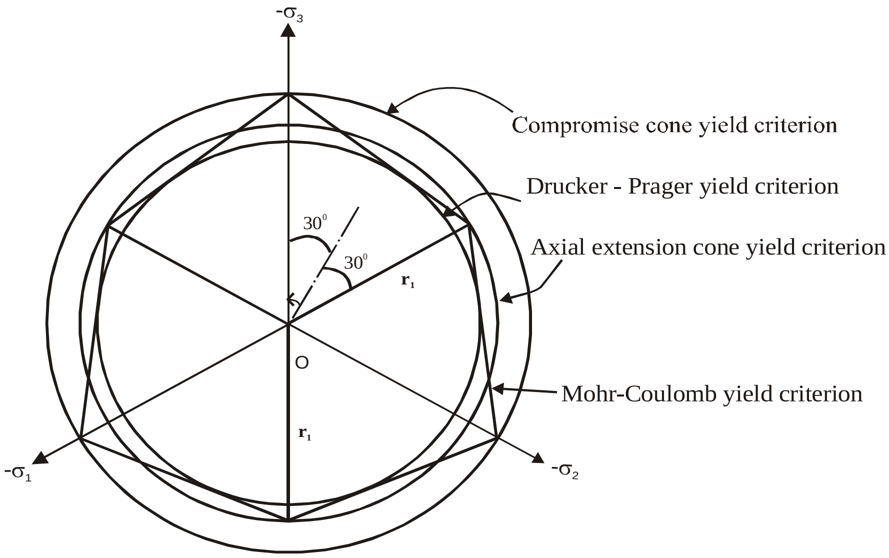

Figure-2. Two dimensional representations of various yield criteria.

5.2.1 Axial forces in columns

Table-2 shows the value of axial force in the columns due to various analyses for plane frame-soil system. The comparison of axial forces due to NIA and LIA reveals that the interaction effect causes redistribution of the forces in the column members. The inner columns are relieved of the forces and corresponding increase is found in the outer columns due to differential settlements of the soil mass.

Table-2. Axial force (kN) in columns of plane frame-foundation beam- soil interaction system.

Load factor (1)	**Storey level** (2)	**Member** (3)	**NIA** (4)	**LIA-FS** (5)	**EPIA-FS** (CC) (6)	**% Diff.** (4 - 6)
0.8	II	C_1	76.16	55.89	55.78	-26.75
		C_2	67.83	88.44	88.28	+30.16
	I	C_3	149.94	103.30	103.14	-31.14
		C_4	138.06	184.16	184.99	+34.00
	II	C_1	95.21	69.87	69.86	-26.62
1.0		C_2	84.79	110.55	110.37	+30.16
	I	C_3	187.43	129.13	129.06	-31.14
		C_4	172.58	230.96	231.35	+34.05
1.2	II	C_1	114.25	83.84	83.89	-26.62
		C_2	101.75	132.66	132.44	+30.16
	I	C_3	224.91	154.95	155.05	-31.14
		C_4	207.09	277.15	277.15	+34.05
	II	C_1	152.33	111.79	113.08	-26.62
1.6		C_2	135.66	176.88	175.54	+30.16
Collapse	I	C_3	299.88	206.60	209.91	-31.14
		C_4	276.13	369.53	366.86	+34.05

FS- Plane frame-foundation beam-soil system; CC-Compromise cone yield criterion.

Table-2 also shows axial force in the column due to EPIA considering compromise yield criterion. The axial forces at lower load factors (0.8 and 1.0) due to LIA and EPIA are almost the same. The axial forces due to all yield criteria are found to be in close agreement.

5.2.2 Bending moment in outer columns

Table-3 depicts the values of bending moment in outer columns of plane frame-soil system due to various analyses. The comparison of NIA and LIA reveals that the interaction effect causes significant increase in bending moments in the outer columns. This is because of the transfer of moments from the interior columns to the outer columns due to differential settlements of the soil mass. The significant increase of nearly 230% is found due to LIA at the roof level of the outer column of the first storey and nearly 101% for the top storey. At lower load factors the results provided by LIA and EPIA are nearly same. At load factor corresponding to collapse, the bending moments in the outer columns are nearly 1.6 times to that obtained at lower load factor of 0.8.

Table-3. Bending moments (kN-m) in outer columns of plane frame-soil interaction system.

Load factor (1)	**Storey level** (2)	**Member** (3)	**NIA** (4)	**LIA-FS** (5)	**EPIA-FS** (CC) (6)	**% Diff.** (4 - 6)
0.8	II	C_2	40.82	82.20	82.26	+101.51
			31.01	50.91	50.94	+64.22
	I	C_4	17.24	56.72	56.86	+229.24
			8.83	91.4	91.57	**
1.0	II	C_2	51.03	102.75	102.76	+101.30
			38.77	63.64	63.66	+64.22
	I	C_4	21.56	70.90	70.98	+229.24
			11.04	114.25	114.16	**
1.2	II	C_2	61.23	123.30	123.12	+101.07
			46.52	76.36	76.37	+64.20
	I	C_4	25.87	85.08	84.88	+228.10
			13.25	137.10	136.20	**
1.6 Collapse	II	C_2	81.65	164.40	161.72	+98.06
			62.03	101.82	101.44	+63.53
	I	C_4	34.50	113.44	109.40	+217.10
			17.66	182.80	172.01	**

FS- Plane frame-soil system; CC-Compromise cone yield criterion; ** Very high difference in values

5.2.3 Bending moments in floor beams

Table-4 depicts the values of bending moments in the floor beams due to various analyses. The comparison of NIA and LIA suggests that the interaction effect causes transfer of bending moments from the inner end of the beam to the outer end at all floor levels due to differential settlement of soil mass.

The reversal in the sign of the bending moment is observed at the junction between the beams of first storey with interior column. In addition to this, the interaction effect also causes shifting of location of maximum positive bending moment towards the outer end in all floor beams. A significant increase of nearly 123% is found at the outer end of first floor beam and nearly 101% in the top floor beam due to LIA.

Table-4 shows that the results obtained due to EPIA at lower load factors are nearly same to that provided by LIA. At load factor corresponding to collapse, the bending moments in the floor beams are nearly 1.6 times to that obtained at lower load factor of 0.8.

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Table-4. Bending moments (kN-m) in floor beams of plane frame-soil interaction system.

Load factor (1)	**Storey level** (2)	**Member** (3)	**NIA** (4)	**LIA-FS** (5)	**EPIA-FS** (CC) (6)	**% Diff. 7** (4 - 6)
0.8	II	B_1	59.59	9.32	9.24	**
			-40.82	-82.19	-82.26	101.34
	I	B_2	56.23	-8.76	-9.82	*
			-48.26	-107.71	-107.80	123.17
1.0	II	B_1	74.49	11.66	11.62	**
			-51.03	-102.74	-102.77	101.34
	I	B_2	70.29	-10.96	-10.94	*
			-60.33	-134.64	-134.63	123.17
1.2	II	B_1	89.38	13.99	14.19	**
			-61.23	-123.28	-123.13	101.09
	I	B_2	84.34	-13.15	-13.02	*
			-72.39	-161.56	-161.25	122.75
1.6 Collapse	II	B_1	119.18	18.65	+22.0	**
			-81.65	-164.38	-161.73	98.07
	I	B_2	112.46	-17.53	-17.34	*
			-96.52	-215.42	-210.84	-118.44

FS- Plane frame-soil system; CC-Compromise cone yield criterion; ** Very high difference in values;* Reversal in sign.

5.2.4 Bending moments in foundation beam

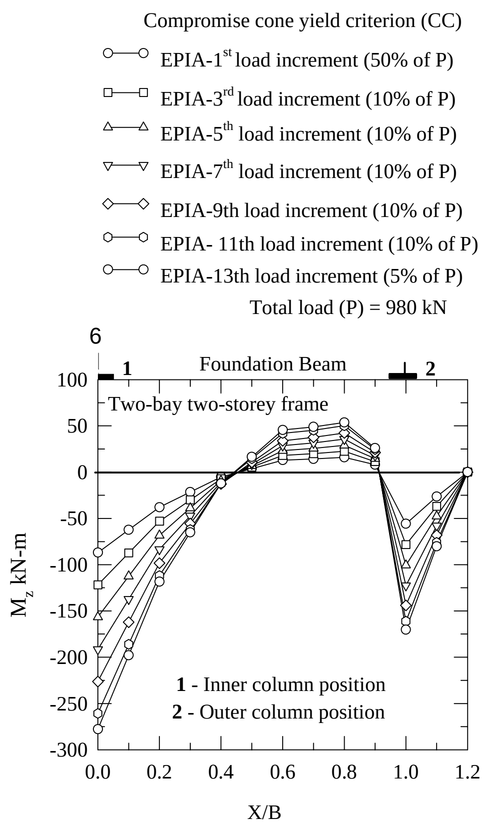

Fig. 3 Variation of BM's in foundation beam for elasto-plastic analysis

Figure-3 depicts the distribution of bending moments along the foundation beam of plane frame-soil system due to LIA and EPIA for load factor of unity. The variation resembles the behavior of the beam subjected to column loads from top and upward soil pressure beneath.

Both interaction analyses almost depict the same behavior and the values of bending moments are nearly the same.

6. CONCLUSIONS

In the present study, an attempt has been made to investigate the interaction behavior of plane frame-foundation beam-soil system with proper modeling using variety of isoparametric elements. The forces in various frame members due to interaction analysis are considerably different from those obtained due to conventional frame analysis. The results obtained by coupled finite-infinite element meshes are more accurate and the computational economy is achieved to large extent in comparison to using purely finite elements. The proposed research work leads to a more rational approach for accurate analysis and design of building frames.

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