STABILITY ANALYSIS OF TWO LOBE HYDRODYNAMIC JOURNAL BEARING WITH COUPLE STRESS LUBRICANT

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ABSTRACT

A generalized Reynolds equation has been derived for carrying out the stability analysis of a two lobe hydrodynamic bearing operating with couple stress fluids that has been solved using the finite element method. A non-dimensional parameter, '*l*' has been used to indicate the length of the long chain polymer added to the bulk Newtonian fluid. It has been observed that the dynamic characteristics, i.e. the stiffness and the damping coefficients, are greatly influenced with the variation of the couple stress parameter '*l*'. The threshold speed of the journal, obtained as a solution to the linearized equations of motion is used to demonstrate the increased stability of the journal bearing system.

Keywords: journal bearing, couple stress fluid, dynamic analysis, stability.

Notation

∇	gradient operator
$(\dot{\ })$	differenttaion with respect to dimensionless time 't'
B	body force vector
c	radial difference between the journal and the lobe
C	body couple vector
$\bar{C}_{mn}$	non dimensional damping coefficients, $\partial \bar{W}_m / \partial \bar{\dot{n}}$, (m,n=x,y)
e	eccentricity
ε	eccentricity ratio e/c
$\bar{h}$	non dimensional film thickness, $\bar{h}$ =1+εCosθ
$\bar{K}_{mn}$	non dimensional stiffness coefficients, $\partial \bar{W}_m / \partial \bar{n}$,(m,n=x,y)
l	couple stress parameter, $l=(\eta/\mu)^{1/2}$
L	length of the bearing
$\bar{M}_j, \bar{M}_c$	non dimensional mass and critical mass of the journal
p	pressure
$\bar{p}$	non dimensional film pressure, $pc^2 / \mu\omega R^2$
R	radius of the journal
U,V	tangential and normal velocity component of the journal surface
$\bar{W}, \bar{W}_h, \bar{W}_v$	bearing load and bearing load along the horizontal and the vertical directions
$\bar{x}, \bar{y}, \bar{z}$	non dimensional coordinate, x/R, y/c, z/R
u,v,w	fluid velocity along the x, y and z axis
V	velocity vector
η	material constant responsible for the couple stress property
μ	viscosity coefficient of a Newtonian lubricant
$\bar{e}_p$	non dimensional ellipticity ratio of two lobe bearing,0.3
ρ	density of fluid
ϕ	attitude angle
ω	angular velocity

1. INTRODUCTION

Most of the machine elements today operate in the thin film lubrication zone. Under these operating conditions, lubricating films in which the properties are different from those of the bulk lubricant are of great rheological significance. These films are created by adding a small amount of additive in the form of long chain polymers and have a considerable influence on the performance of a bearing effecting the wear and failure during operation. The conventional lubrication theory neglects the size effects of the fluid particles and is not capable of modeling these complex fluids. Among all the theories developed to incorporate the intrinsic motion of the material constituents [1-2], the Stokes micro-continuum theory [2] is the simplest allowing the polar effects such as the presence of couple stresses (stresses produced by the spin of the micro elements), body couples and non-symmetric tensors.

Stability analysis of journal bearing system is a topic of intensive research. Several papers are available which deal with the performance enhancement both due to geometrical changes of the journal bearing system and due to the changes in the lubricating fluid. Li *et al.,* [3] carried out linear and nonlinear transient analyses of rigid rotors in elliptical, offset half, three-lobe and four-lobe journal bearings using a fast Fourier transform analysis. Allaire *et al.,* [4] predicted that out of the four bearings (elliptical, offset half, three-lobe and four-lobe) studied, elliptical bearings had the most violent whirl vibration, while the offset half bearings exhibited the least amount of sub-synchronous vibration. Richie [5] developed a fast technique to obtain the nonlinear transient response of a finite journal bearing. Chandrawat and Sinhasan [6] and Jain *et al.*, [7] studied the transient response of flexible journal bearing. The linear and nonlinear trajectories both predict limit cycles for rigid circular journal bearings with a Newtonian lubricant when $\overline{M}_j$ is equal to $\overline{M}_c$. Mehta *et al.,* [8-12] have done the stability analysis of the multilobe pressure dam bearing for a newtonian fluid under both steady and turbulent flows lubrication conditions.

A number of studies have been carried out using the Stokes microcontinuum theory to investigate the effects of the couple stress parameter, '*l'*, on the performance of different types of fluid film bearings. These studies include work done by Lin [13-15] and Mokhiamer *et al.,* [16] and on rolling-element bearings by Sinha and Singh [17] and Das [18]. The characteristics of pure squeeze film bearings has been analyzed by Ramanaiah and Sarkar[19], Ramanaiah [20], Bujurke [21], Lin[22,23] while slider bearings have been studied by Ramanaiah [24] and Bujurke *et al*., [25] .

A review of the literature indicates that the available studies on the transient response of journal bearings are for bearings lubricated with Newtonian and non-Newtonian lubricants. As the performance characteristics of plain circular bearings with couple stress fluids is significantly different [13-15], it is expected that the transient response of two lobe bearing systems will also be affected. To best of the authors' knowledge, stability analyses of two lobe journal bearings lubricated with couple stress lubricants has not been reported.

The present study is mainly concerned with the stability analysis of a hydrodynamically lubricated two lobe bearing. The positive pressure zone for a finite journal bearing satisfying the Reynolds boundary condition is established and non-dimensional pressure is obtain using the Finite Element Method. The non-dimensional pressure so obtained is used to evaluate various bearing characteristics (i.e. eccentricity, attitude angle, stiffness and damping coefficients.). To show the couple stress effects on the performance of the system, the results for the different couple stress parameters are compared to that of the Newtonian lubricant case.

2. ANALYSIS

2.1 Basic equations

From the Stokes micro-continuum theory, for an incompressible fluid with couple stresses

Momentum Equation:

$$\rho\frac{DV}{Dt} = -\nabla p + \rho F + \frac{1}{2}\rho\nabla xC + \mu\nabla^2 V - \eta\nabla^4 V \quad (1)$$

Continuity Equation:

$$\nabla V = 0 \quad (2)$$

Where the vectors V, F and C denote the velocity, body force and body couple per unit mass, respectively; p is the pressure, ρ is the density, μ is the classical viscosity, and η is a new material constant peculiar to fluids with couple stresses.

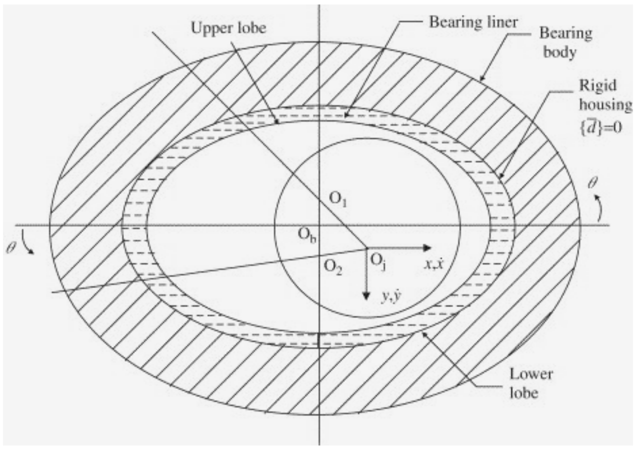

Figure-1. Geometry and coordinate system used.

Figure-1 presents the geometry and co-ordinate system used for analysis of a bearing with a journal of radius R rotating with angular velocity ω. Assume that the fluid film is thin, body forces and body couples are absent, and fluid inertia is small as compared to the viscous shear. Then the field equations governing the motion of the lubricant given in Cartesian coordinates reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

$$\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4} \quad (4)$$

$$\frac{\partial p}{\partial y} = 0 \quad (5)$$

$$\frac{\partial p}{\partial z} = \mu\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4} \quad (6)$$

Boundary conditions at the surface of bearing (y = h)

$u(x,h,z) = 0$, $v(x,h,z) = 0$, $w(x,h,z) = 0$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=h} = 0, \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=h} = 0 \quad (7)$$

Boundary conditions at the surface of Journal (y = 0)

$u(x,0,z) = U$, $v(x,0,z) = V$, $w(x,0,z) = 0$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = 0, \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=0} = 0 \quad (8)$$

Where U and V are tangential and normal velocity components of the journal surface at an angular position,θ. Integrating equation (4) and integrating equation (6) with respect to 'y' and applying the boundary conditions, we get the modified Reynolds equation as:

$$\frac{\partial}{\partial x}\left(\frac{h^3-12l^2\left[h-2l\tanh\left(\frac{h}{2l}\right)\right]}{6\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3-12l^2\left[h-2l\tanh\left(\frac{h}{2l}\right)\right]}{6\mu}\frac{\partial p}{\partial z}\right)=6U\mu\frac{\partial h}{\partial x}+6\mu h\frac{\partial u}{\partial x}+12\mu V$$

Now introducing non dimensional variables and parameters, dimensionless Reynolds Equation can be written as

$$\frac{\partial}{\partial \bar{x}}\left(f(\bar{h},\bar{l})\frac{\partial \bar{p}}{\partial \bar{x}}\right)+\frac{1}{4\beta^2}\frac{\partial}{\partial \bar{z}}\left(f(\bar{h},\bar{l})\frac{\partial \bar{p}}{\partial \bar{z}}\right)=6\frac{\partial \bar{h}}{\partial \bar{x}}+\frac{12}{\omega}\frac{\partial \bar{h}}{\partial \bar{t}}$$

Where

$$f(\bar{h},\bar{l})=\bar{h}^3-12\bar{l}^2\left[\bar{h}-2\bar{l}\tanh\left(\frac{\bar{h}}{2\bar{l}}\right)\right] \quad (10)$$

The hydrodynamic pressure field in the journal bearing is established by solving equation (10) satisfying the following boundary and symmetry conditions:

$$\left.\frac{\partial \bar{p}}{\partial \bar{x}}\right|_{(\bar{x}=\bar{x}2,\bar{z}=0)}=0,\ \left.\frac{\partial \bar{p}}{\partial \bar{z}}\right|_{(\bar{z}=0)}=0,\ \bar{p}(\bar{x}2,\bar{z})=0,$$

$$\bar{p}(0,\bar{z})=0,\ \bar{p}\left(\bar{x},\pm\frac{1}{2}\right)=0 \quad (11)$$

2.2 Film thickness

For an aligned bearing the fluid film thickness of the nth lobe is expresses as:

$$h^n=1-(X_J-X_L^n)\cos\theta-(Z_J-Z_L^n)\sin\theta$$

Where $(X_J, Z_J)=(\varepsilon\sin\phi,-\varepsilon\cos\phi)$, represents the coordinates of the journal centre and $(X_L{}^n, Z_L{}^n)$ represents the centers of the lobes.

2.3 Bearing performance characteristics

2.3.1. Load carrying capacity

The fluid film reaction components in 'y' and 'x' directions are given by

$$\bar{W}_x=\int\bar{p}\cos\theta.d\bar{\theta}.d\bar{y}$$

$$\bar{W}_y=\int\bar{p}\sin\theta.d\bar{\theta}.d\bar{y} \quad (12)$$

For vertical load support, the following conditions are satisfied in the journal centre equilibrium position

$$\bar{W}_v=W \text{ And } \bar{W}_h=0 \quad (13)$$

2.3.2 Attitude angle

Angle between the load line and the line joining the bearing and journal centers is defined as the attitude angle. The attitude angle is established by satisfying the condition of vertical load support, Equation (13), using an iterative process for a given load or eccentricity ratio.

2.3.3 Stiffness coefficients

If the journal center is displaced from its static equilibrium position, then in general, the difference in the components of the fluid-film reactions in the disturbed position and in the static equilibrium position gives the out of balance components of the fluid-film force. The non-dimensional fluid-film stiffness coefficients are defined as

$$\begin{bmatrix}\bar{K}_{xx} & \bar{K}_{xz}\\ \bar{K}_{zx} & \bar{K}_{zz}\end{bmatrix}=-\begin{bmatrix}\partial\bar{W}_x/\partial\bar{x} & \partial\bar{W}_x/\partial\bar{z}\\ \partial\bar{W}_z/\partial\bar{x} & \partial\bar{W}_z/\partial\bar{z}\end{bmatrix} \quad (14)$$

The first subscript of the stiffness coefficients denotes the direction of force and the second, the direction of displacement.

2.3.4 Damping coefficients

The damping coefficients are defined as

$$\begin{bmatrix}\bar{C}_{xx} & \bar{C}_{xz}\\ \bar{C}_{zx} & \bar{C}_{zz}\end{bmatrix}=-\begin{bmatrix}\partial\bar{W}_x/\partial\bar{u} & \partial\bar{W}_x/\partial\bar{v}\\ \partial\bar{W}_z/\partial\bar{u} & \partial\bar{W}_z/\partial\bar{v}\end{bmatrix} \quad (15)$$

The first subscript of the damping coefficients denotes the direction of force and the second, the velocity.

2.3.5 Threshold speed

The threshold speed at which the journal bearing system becomes unstable is found as the roots to the characteristic equation:

$$\omega^4\lambda^4+\omega^2(\bar{C}_{xx}+\bar{C}_{zz})\lambda^3+\left[\omega^2(\bar{K}_{xx}+\bar{K}_{zz})+\bar{C}_{xx}\bar{C}_{zz}-\bar{C}_{xz}\bar{C}_{zx}\right]\lambda^2$$
$$+(\bar{C}_{xx}\bar{K}_{zz}+\bar{C}_{zz}\bar{K}_{xx}-\bar{C}_{xz}\bar{K}_{zx}-\bar{C}_{zx}\bar{K}_{xz})\lambda+(\bar{K}_{xx}\bar{K}_{zz}-\bar{K}_{xz}\bar{K}_{zx})=0 \quad (16)$$

A bearing becomes unstable at a speed ω, if any root to the above equation becomes positive.

3. SOLUTION SCHEME

Equation (10) is solved by Finite Element Method using Galerkin's Approach. The domain is discretized into elements and the condition of mesh convergence has been satisfied. The finite element equations contributed by all the elements have been assembled in the global stiffness and the global force matrix which has been modified to meet the boundary conditions mentioned in equation (11). The Reynolds

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boundary condition has been satisfied by adjusting the trailing edge of the fluid film. For any eccentricity ration, the above process is repeated to get an attitude angle for vertical load at the equilibrium condition. The journal from this state is perturbed to obtain the stiffness and the damping coefficients (equations 14-15).

4. RESULTS

In the present study the effect of couple stress parameter on the dynamic performance of two lobe bearing is analyzed.

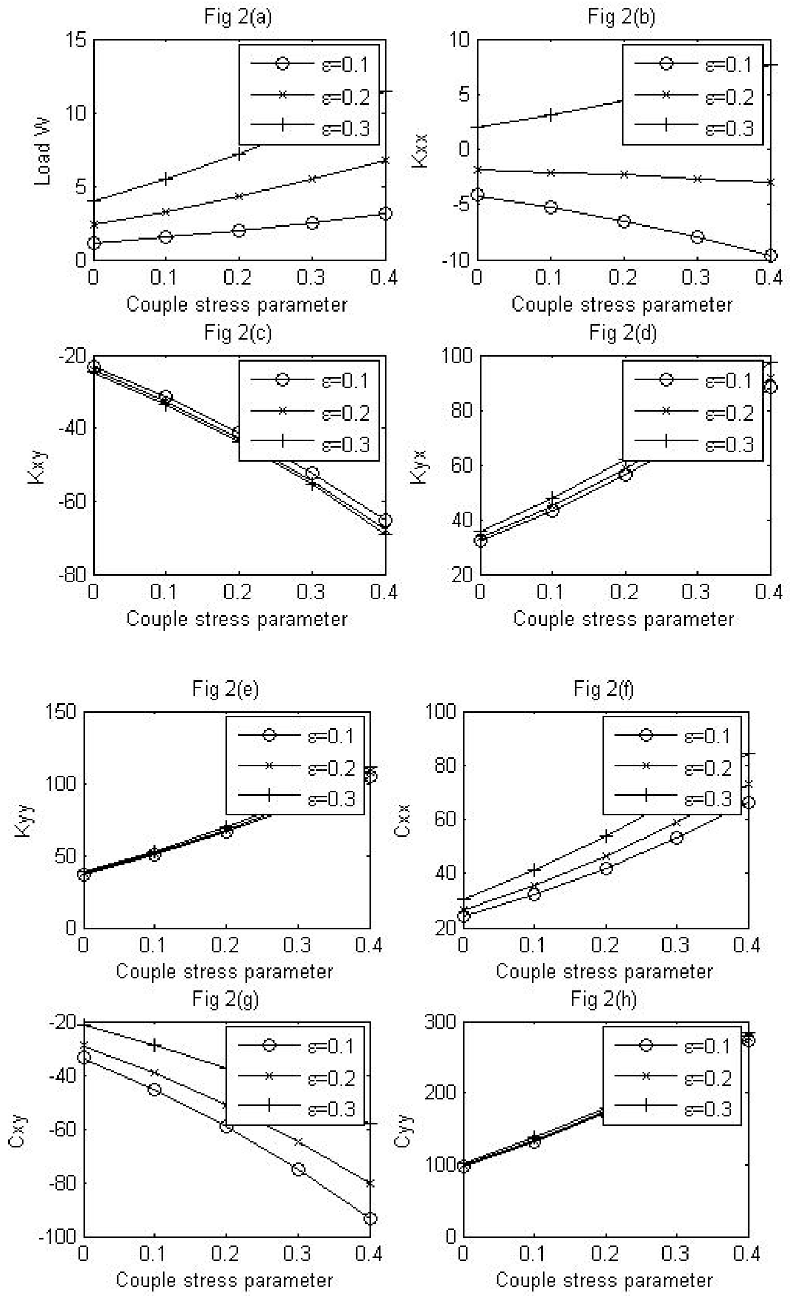

Figure-2(a-h). Dynamic characteristics of two lobe bearing vs. couple stress parameter '*l*' at different eccentricity ratios.

The results obtained for the two lobe bearings with eccentricities 0.05 to 0.35 at an ellipticity ratio of 0.3 are plotted for the couple stress factor ranging from 0 (Newtonian fluid) to 0.4 in steps of 0.1. Figure-2(a) shows the variation of bearing load 'W' versus the couple stress parameter '*l*' over a range of eccentricities.

It is observed that as the eccentricity ratio increases, the couple stress parameter '*l*' produces increased influence on the bearing load 'W'. This influence is observed to be more apparent at higher values of the couple stress parameter. It is also observed that the bearing load reduces asymptotically with the decrease in couple stress parameter for all values of eccentricity ratios. This is apparent as the generalized Reynolds equation (10) reduces to a classical Reynolds equation applicable for a Newtonian fluid as the couple stress parameter 'l' becomes zero.

Figure-2 (b-e) presents the influence of couple stress parameter 'l' on the stiffness coefficients K_{xx}., K_{xz}, K_{zx} and K_{zz} for various eccentricity ratios. The low values of couple stress parameters are observed to have marginal effect on these coefficients especially at low values eccentricity ratios. However at higher values of couple stress parameter 'l', the stiffness parameters change at a rapid rate with the increase in the eccentricity ratios.

A similar pattern is apparent in the variation of damping coefficients C_{xx}., C_{xz}, C_{zx} and C_{zz} which are presented in Figure-2 (f-h). A general conclusion can be drawn from these analyses that since the bearing load increases with increasing values of couple stress parameter 'l', the stiffness and the damping coefficients also show a similar trend producing a larger restoring force when the journal is displaced from the equilibrium position.

To show the influence of couple stress fluid on the stability of the journal bearing system, analysis were carried out over a much wider range of eccentricity ratio and couple stress parameter 'l'. Figure-3 shows the variation of threshold speed against different values of couple stress parameter 'l' for various eccentricity ratios. The Figure clearly show an increase in the stable operating zone as the couple stress parameter 'l' or the eccentricity ratio increases. At low values of eccentricity ratios the increase in threshold speed with change in couple stress parameter is significantly less as compared to the increase obtained at higher eccentricity ratios. This increase can be as high as five times or more at eccentricities 0.4 and above.

At eccentricity ratios more than 0.5 it was observed that any introduction of couple stress fluid resulted in the journal bearing system becoming infinitely stable as all roots to the equation (16) were found to be negative for any value of operating speed 'ω'. For Newtonian fluids however the bearing system had a finite threshold speed even at eccentricity ratio more that 0.5.

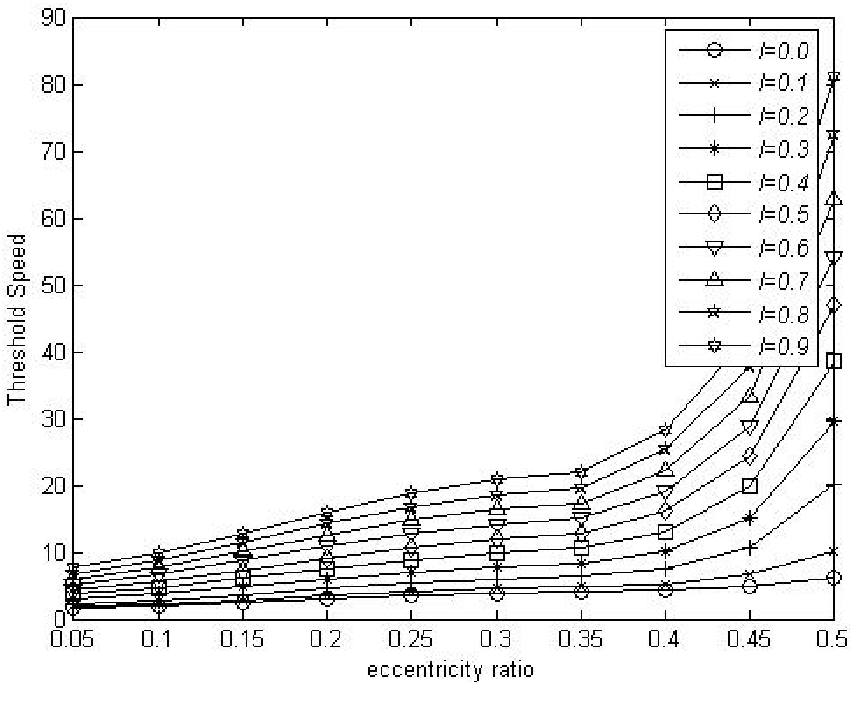

Figure-3. Threshold speed vs. eccentricity at different couple stress parameters 'l' at ellipticity ratio of 0.3.

5. DISCUSSIONS

Important inferences can be made about the use of couple stress fluid and are listed below.

a) The use of couple stress fluid increases the load capacity which means that for any given load the bearing system will be operating at a lower eccentricity ratio.

b) The use of couple stress fluid increases the "safe operating zone" signifying that the journal will finally return to the static operating state when perturbed from the equilibrium condition.

c) It may be noted that the analysis for the threshold speed have been carried out assuming a linearized behaviour of the journal system indicating that even after external disturbances the journal continues to operate in the close vicinity of the static equilibrium state. For conditions where the journal is perturbed to a large extent, it may be necessary to carry out the non-linear analysis.

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