VOL. 5, NO. 2, FEBRUARY 2010 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

COUETTE FLOW OF TWO IMMISCIBLE FLUIDS BETWEEN TWO PERMEABLE BEDS

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ABSTRACT

Couette flow of two viscous, incompressible, immiscible fluids in a channel bounded by permeable beds is investigated. The lower bed is of finite thickness with high permeability and the upper bed is of infinite thickness with low permeability. The flow in the lower permeable bed is described by Brinkman equation whereas the flow in the upper permeable bed is described by Darcy's law. The flow between the two beds is governed by Navier-Stokes equations. The velocity field and the mass flow rate are obtained. It is observed that the velocity is in increasing trend with the increment in the Reynolds number.

Keywords: Couette flow, immiscible fluids, permeable beds.

INTRODUCTION

Flow through and past porous media is a subject of widespread interest in Geophysics, Biology and Medicine. In many of these areas, fluid flow occurs in more than one phase. For example blood flow in arteries has been studied by many researchers considering blood as two phase flow (vide Chaturani [1]). In view of these several investigations on multiphase flows are reported by various researchers. Bird *et al.* [2] found an exact solution for the laminar flow of two immiscible fluids between two parallel plates. Bhattacharya [3] discussed the flow of immiscible fluids between rigid plates with a time dependent pressure gradient. Vajravelu *et al.* [4] have discussed the effect of magnetic field on unsteady flow of two immiscible conducting fluids between two permeable beds. Heat transfer in generalized Couette flow of two immiscible fluids through a porous channel is discussed by Bhargava and Sacheti [5]. Shijie Liu *et al.* [6] discussed the principles of single-phase flow through porous media. Couette flow of two immiscible viscous fluids with heat transfer using Brinkman model is studied by Singh *et al.*[7]. Transient Couette flow in a rotating non-Darcian porous medium parallel plate configuration is studied by Anwar Beg *et al.* [8]. Kadry Zakaria *et al.* [9] discussed Magnetohydrodynamics instability of interfacial waves between two immiscible cylindrical fluids.

Motivated by these studies Couette flow of two immiscible fluids between two permeable beds is investigated. In the lower bed the flow is governed by Brinkman equation where as the flow in the upper permeable is described by Darcy's law. The velocity field and mass flow rate are obtained. The effects of Reynolds number and Darcy number on the flow are discussed.

Nomenclature

x,y	cartesian coordinates
R	Reynolds number ($= \dfrac{Ch^2}{2\mu_2 U}$)
u_1	Velocity in zone I
u_2	Velocity in zone II
u_3	Velocity in zone III
Da	Darcy number, $\dfrac{k}{h^2}$
Da_1	Darcy number, $(\dfrac{k_1}{h^2})$
2h	height of the non porous channel
k_1	permeability of the lower bed
k_2	permeability of the upper bed
Q	Darcy velocity
ϕ	Viscosity factor
τ	Local shear stress
H	width of the lower porous finite bed
λ	$\sigma = \dfrac{H}{\sqrt{k}}$
u_{B1}	slip velocity at the upper bed
α	slip parameter
μ_1	viscosity of the fluid in zone 1
μ_2	viscosity of the fluid in zone 2
S	$\dfrac{\mu_2}{\mu_1}$ (ratio of viscosity)

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MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the Couette flow of two immiscible, incompressible fluids in a channel of height 2h bounded by two permeable beds of different permeabilities. The lower permeable bed is highly permeable with finite thickness H whereas the upper permeable bed is of lower permeability with infinite thickness. The flow geometry is described in Figure-1. The permeabilities of lower and upper beds are k_1 and k_2 respectively. x and y are the axial and vertical coordinates respectively with the origin at the centre of the channel and the positive direction of y being directed towards the upper permeable bed. A constant pressure gradient, C ($=-\partial p/\partial x$), acts at the mouth of the channel. The upper fluid (viscosity μ_1, density ρ_1) is assumed to occupy the upper half of channel (i.e.$0\le y\le h$), and this region is called zone 1. The lower fluid (viscosity μ_2 ($>\mu_1$), density ρ_2 ($>\rho_1$) occupies the region $[-(h + H) \le y \le 0]$ comprising the lower half of the channel and this region is named as zone 2. The governing equations in the channel section ($-h\le y\le h$) will be Navier-Stokes equations. The flow region in the lower permeable bed is called zone 3 and in this region the flow is governed by Brinkman equation. The flow region in the upper permeable bed is called zone 4 in which the flow is described by Darcy's law.

We introduce the non-dimensional variables

$$\eta^* = \frac{y}{h}$$

$$;u_1^* = \frac{u_1}{U};u_2^* = \frac{u_2}{U};u_3^* = \frac{u_3}{U};\bar{u}_{B1}^* = \frac{u_{B1}}{u};$$

$$Q^* = \frac{Q}{u} = u_4;\ \gamma=H/h;$$

$$Da=\frac{k}{h^2},\ \frac{y}{h}=1;\ \eta=1;u_1^* = u_{B1}^*+1;$$

$$\frac{du_1^*}{d\eta}=\frac{-\alpha}{\sqrt{Da}}(u_{B1}^*-Q^*)$$

Then we get the following sets of transformed governing equations and boundary conditions corresponding to various zones:

Governing Equations

Zone I ($0 \le \eta \le 1$)

$$\frac{d_2u_1}{d\eta^2}+2RS=0 \tag{1}$$

Zone II $(-1\le \eta \le 0)$

$$\frac{d_2u_2}{d\eta^2}+2R=0 \tag{2}$$

Zone III $(-(1+\tau)\le \eta \le -1)$

$$\frac{d_2u_3}{d\eta^2}-\lambda^2u_3=-2R \tag{3}$$

Boundary conditions

$$u_1=u_{B1}+1 \text{ And } \frac{du_1}{d\eta}=-\frac{\alpha}{\sqrt{Da}}(u_{B1}-Q) \text{ at } \eta=1 \tag{4}$$

Where $Q=2RSDa_1$

$$u_1=u_2 \text{ And } \frac{du_1}{d\mu}=S\frac{du_2}{d\mu} \text{ at } \eta=0 \tag{5}$$

$$u_2=u_3 \text{ And } \frac{du_2}{d\eta}=\phi_1\frac{du_3}{d\eta} \text{ at } \eta=-1 \tag{6}$$

$$u_3=0 \text{ At } \eta=-(1+\tau) \tag{7}$$

We note that the boundary condition (6) at the interface is introduced following Bhargava and Sacheti (1989)

SOLUTION OF THE PROBLEM

Solutions of momentum equations (1), (2) and (3), subject to relevant boundary and matching conditions referred above, have been obtained as

$$u_1=-RS\eta^2+c_1\eta S+c_2 \tag{8}$$

$$u_2=-R\eta^2+c_3\eta+c_4 \tag{9}$$

$$u_3=c_5\cosh(\eta\lambda)+c_6\sinh(\eta\lambda)+\frac{2R}{\lambda^2} \tag{10}$$

$$Q=u_4=\frac{-k}{\eta_1}\frac{\partial p}{\partial x}=\frac{cN}{\eta_1} \tag{11}$$

$$Q=2RsDa_1 \tag{12}$$

Where the constants c_1, c_2, c_3, c_4, c_5 and c_6 are defined in Appendix 1.

DISCUSSIONS AND CONCLUSIONS

Velocity profiles for the Couette flow of two immiscible fluids between two permeable beds are drawn in Figures 2 to 4 for various values of R, Da and ϕ. From Figure-2 we observe that the velocity increases with the increment in Reynolds number. From Figure-3 we infer that velocity increases with the increment in Darcy number. From Figure-4 we find that velocity increases with the decreasing value of viscosity factor. The Mass flow is evaluated numerically as a function of Da for various values of R and is shown in Figure-5. It is observed that the Mass flow rate is decreasing with increasing Darcy number for a given R. For a given Da, the mass flow rate is increasing with increment in R. Figure-6 is drawn to find the effect of the slip parameter α on the Mass flow rate. It is observed that the Mass flow rate decreasing with increasing Darcy number for a given α. For a given Da, the Mass flow rate increasing with increment in α.

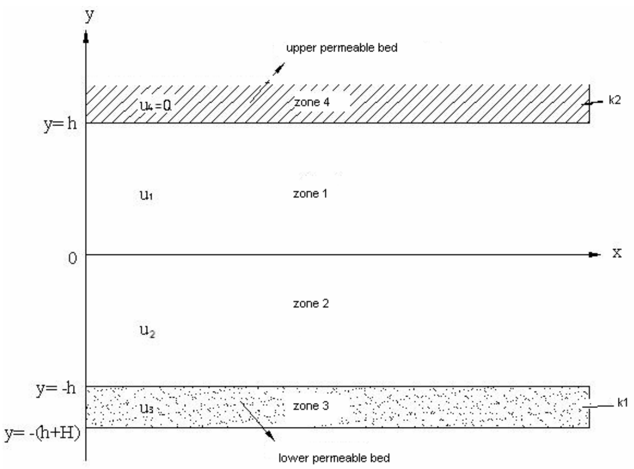

Fig 1 : Physical Model

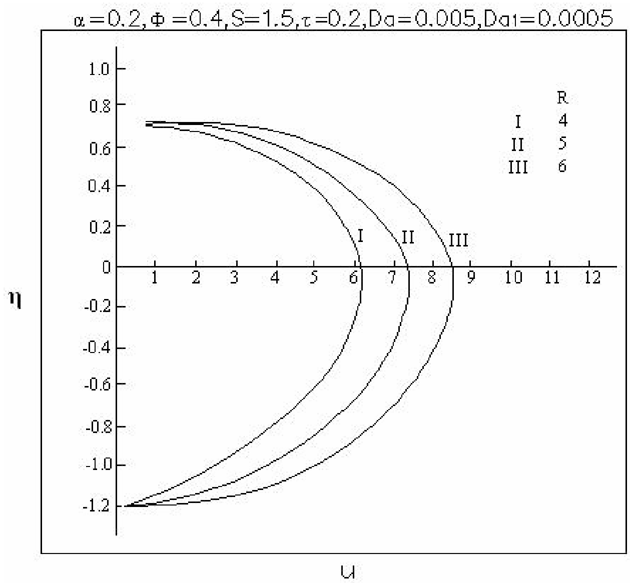

Fig 2 : Velocity profiles for R

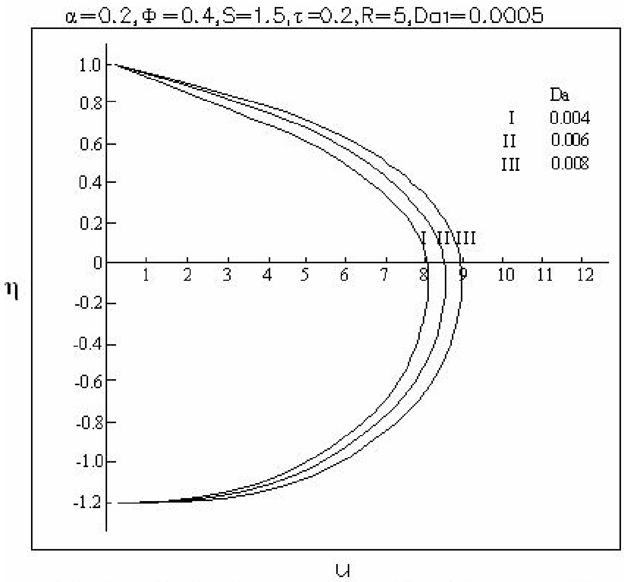

Fig 3 : Velocity profiles for different Da

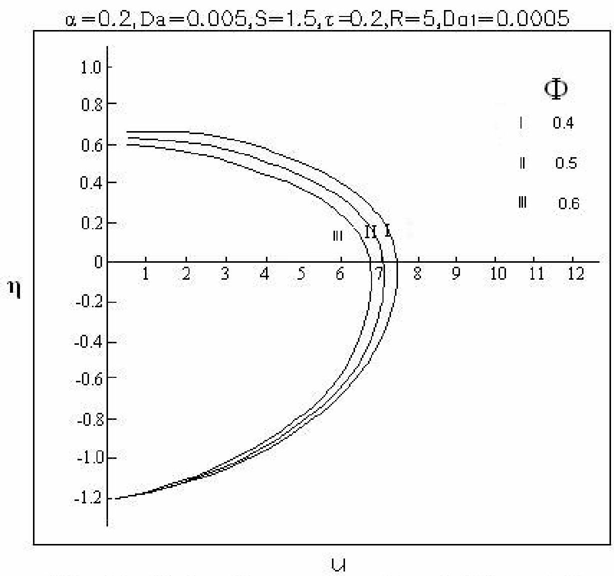

Fig 4 : Velocity profiles for different Φ

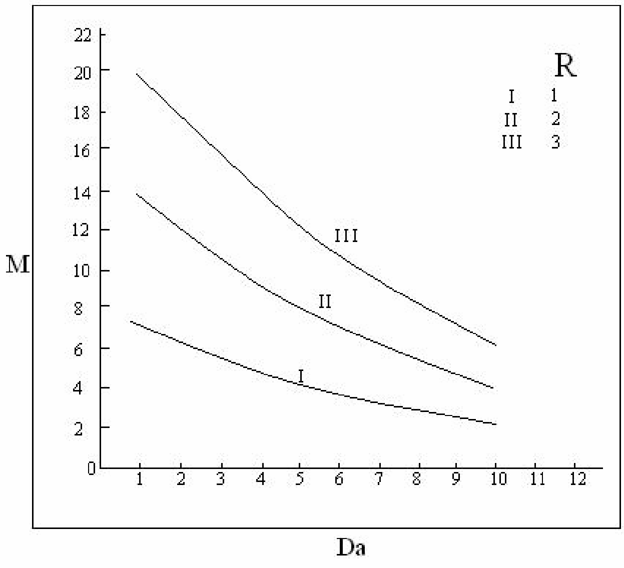

Fig 5 : Mass flow rate

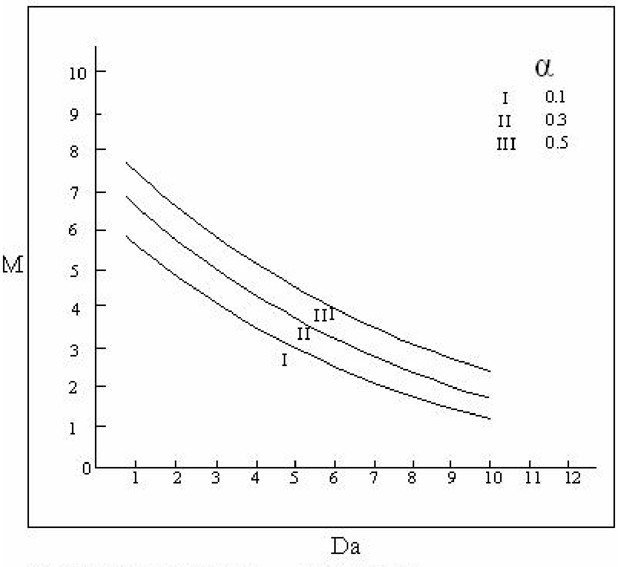

Fig 6 : Mass flow rate

ACKNOWLEDGEMENTS

One of the authors Dr. S. Sreenadh thanks DST, New Delhi for providing financial support through a major research project. The project number is SR/S4/MS: 503/07.

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APPENDIX-I

$G_1 = -c_5\lambda\phi\sinh\lambda + c_6\lambda\phi\cosh\lambda$

$G_2 = \cosh\lambda - \lambda\phi\sinh\lambda$

$G_3 = \lambda\phi\cosh\lambda - \sinh\lambda$

$D_1 = \cosh\lambda - \lambda\phi S\sinh\lambda - \lambda\phi(1+\frac{S\sqrt{Da}}{\alpha})\sinh\lambda$

$D_2 = \lambda\phi S\cosh\lambda - \sinh\lambda + \lambda\phi(1+\frac{S\sqrt{Da}}{\alpha})\cosh\lambda$

$D_3 = 1 + R + \phi + 3RS - \frac{2R}{\lambda^2} + 2RS(\frac{\sqrt{Da}}{\alpha})$

$D_4 = D_1\sinh(1+\tau)\lambda + D_2\cosh(1+\tau)\lambda$

$c_2 = c_4 = c_5G_2 + c_6G_3 - R + \frac{2R}{\lambda^2}$

$c_1 = c_3 = G_1 - 2R$

$c_5 = \frac{D_3}{D_1} - \frac{D_2D_3}{D_1D_4}\cosh(1+\tau)\lambda - \frac{2R}{\lambda^2}.\frac{D_2}{D_4}$

$c_6 = \frac{D_3}{D_4}\cosh(1+\tau)\lambda + \frac{2R}{\lambda^2}\frac{D_1}{D_4}$