



ANALYTICAL STUDY OF SELF-EXCITED VIBRATION ON SINGLE DEGREE OF FREEDOM VIBRATORY-TILLAGE

Soeharsono¹ and Radite PA Setiawan²

¹Department of Mechanical Engineering, Trisakti University, Jakarta, Indonesia

²Department of Agricultural Machinery, Bogor Agricultural University Kampus IPB Dramaga Bogor, Indonesia

E-Mail: gatotsoeharsono@yahoo.com

ABSTRACT

Analytical and experimental study on vibratory tillage by adding external energy to the tillage tool has been widely conducted. Though this method has been shown to significantly reduce soil resistance, it will, unfortunately, increase the energy consumption excessively. Experimental study on vibratory tillage by self-excited vibration method has also been performed. This method can also reduce soil resistance though not as much as the former. No analytical study of the latter, however, can be found. This paper discusses analytical study of self-excited vibration of tillage-tool on vibratory tillage due to natural excitation of varying cutting forces. The objective of this discussion is to find dynamics parameter of vibratory tillage so the vibration of tillage-tool will be able to reduced draft force required for loosening soil density during tillage operation. The Vibration of vibratory tillage was modeled as a vibration with Single Degree of Freedom (SDOF) system. The tillage-tool was connected to an implement by an elliptic spring while the natural excitation of the varying cutting force was modeled as a periodic function, which can be expressed as a Fourier series. The elasticity of elliptic spring and the inertia of tillage tool were optimized such that the tillage-tool vibrates violently around its resonant frequency. This condition decreases both the soils resistance and the draft force required to loosen soil density due to self-excited vibration during tillage operation. The possibility of draft force reduction was investigated further by analyzing time response of the displacement and by analyzing the oscillating pathway of the tine tip.

Keywords: vibratory tillage, self-excited vibration, draft force reduction, soils resistance.

1. INTRODUCTION

Soil compaction reduces rooting, infiltration, water storage, aeration, drainage, and crop growth. To loosen the compact soil however high draft force and energy consumption is needed. It is well-known that tillage-tool vibration will reduce draft force during tillage operation if the maximum velocity of oscillation is higher than the velocity of the implement. Other advantages of this system is reduced clods size and increased number of crack in soil. This condition will facilitate penetration of plant roots, nutrients, water and air circulation in all soil that is necessary for plant growth.

There are two methods to oscillate tillage-tool [1]. The first method is done by adding mechanical energy while the second method is to take advantage of the elascity of tillage tolls combined with a varied cutting force so that the tillage-tool will vibrate in accordance with self excited vibration phenomenon. The first method has been studied analytically [2-6] and experimentally [4], [7-9]. This method would reduce soils resistance significantly when the velocity ratio (the ratio of maximum vibration speed of tillage tool tip to forward speed of tool carriage) is greater than unity. Unfortunately it uses a lot of excessive energy. The increase of energy consumption is caused by the amount of energy that is needed to drive inersia of tillage-tool and its mechanism. Yow J. *et al.*, [2] investigated a substantial reduction in draught force about 30 % at velocity ratio about 2 and the overall power consumption will then increase up to 50% above that for a non-oscillating tool. Butson *et al.*, [3,4] reported draft ratio (the ratio of average vibratory tool force to the non-vibratory tool force at the same tool

carriage velocity) about 0.58 at the velocity ratio about 3 but the power ratio (the ratio of average vibratory tool energy consumption to the non-vibratory tool energy consumption at the same tool carriage velocity) increased up to 3.2.

The second method has been studied experimentally [10-12]. All author reported that this method will reduce the draft force though not as much as the former. Decreasing the draft force depends only on the elasticity of the spring. Soeharsono *et al.*, [13] conducted an analitical study of self excited vibration on SDOF system of vibratory tillage. The excitation caused by varying cutting force was modeled as a periodic function which expands as a Fourier series. By varying the spring constant, Soeharsono *et al.*, [13] reported that the system will resonance at natural frequency closed to twice of the value of the excitation frequency.

This paper discusses analytical study of self excited vibration on a new model of vibratory tillage. The vibration of vibratory tillage was model as a vibration with Single Degree of Freedom (SDOF) system i.e. the oscillation of the tillage-tool θ . The tillage-tool was connected to tool carriage by a semi elliptic bar which had function as a main spring. The natural excitation of varying cutting force was modeled as a periodic function, which can be expressed as a Fourier series. The graph of natural excitation was based on the qualitative data of unconfined test conducted in soil mechanics laboratory of agricultural machinery department, Bogor Agricultural University. The friction between the tillage tool and tilled soil was modeled as viscous damping c . The inertia of tillage toll J was kept constant while elasticity of semi



elliptic spring k was varied. This condition affects the vibration of tillage-tool θ . This vibration represents in the graph of maximum displacement of tillage-tool as function of time t and graph of tillage tool displacement as function of spring constant k . At a certain condition, the tillage-tool vibrates violently so the velocity ratio will greater than unity. Restoring force on main spring pressed the tillage-tool to break up dense soil with high level of vibratory energy. The draft force reduction was investigated by analyzing the oscillating pathway of the tine tip. The draft force reduction was investigated by analyzing the oscillating pathway of the tine tip.

2. METHODOLOGY

2.1 Governing differential equation

The model of vibratory tillage is shown in Figure-1a. It consists of semi elliptic bar (which functions as main spring k) and tillage-tool J . The model was installed on a carriage with a forward speed of V_0 . The vibration of tillage-tool was induced by variation of soil resistance $F(t)$. During forward motion of carriage, tillage-tool press the soil and high resistance encounter, the tillage-tool displaces and the energy is stored as potential energy in the main spring. When soil breaks up, soil resistance decreases hence, the restoring force of the spring return the tillage-tool to the original position causing the tillage-tool oscillated following self excited vibration phenomenon.

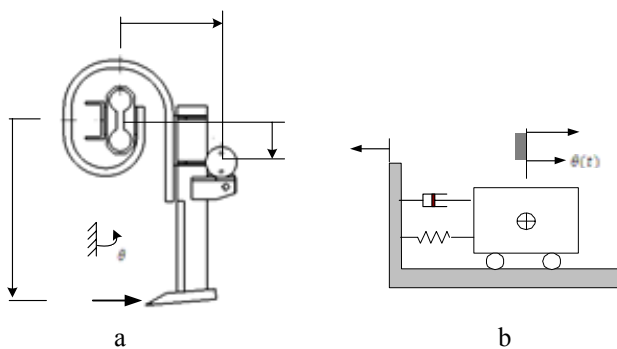


Figure-1. a) Physical model of vibratory tillage
b) Mathematical model.

Therefore, the vibration of vibratory tillage could be modeled as self excited vibration with SDOF system as shown in Figure-1b. Due to soils viscous-elastics behavior, the soil was modeled as viscous damper c_s and soil stiffness k_s while forward velocity of the implement (tool carriage) was V_0 . All structure were made from steel therefore the structural damping coefficient are neglected. If θ is angular displacement of tillage-tool, the differential equation of motion of the system is:

$$J\ddot{\theta} + c_s\dot{\theta} + k\theta = M(t) \quad (1a)$$

or

$$\ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2\theta = \frac{M(t)}{J} \quad (1b)$$

Where

$$M(t) = df(t)$$

$f(t)$: draft force

J : mass inertia of tillage-tool and main spring

k : angular stiffness of main spring

d : Distance from center of rotation O to draft force F_d

c_s : viscous damping coefficient of tilled soil

$M(t)$ is a periodic function with period of T and frequency of ω expresses as a Fourier series [13-15]:

$$M(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(n\omega t) + b_n \sin(n\omega t)\} \quad (2)$$

Where

$$a_0 = \frac{2}{T} \int_0^T M(t) dt, \quad a_n = \frac{2}{T} \int_0^T [M(t) \cos(n\omega t)] dt$$

$$b_n = \frac{2}{T} \int_0^T [M(t) \sin(n\omega t)] dt \quad (3)$$

Substitute equation 2 and 3 to equation 1 yield the angular response of

$$\theta(t) = \frac{a_0}{2k} + \sum_{n=1}^{\infty} (A_n \cos(n\omega t - \phi_n) + B_n \sin(n\omega t - \phi_n)) \quad (4)$$

Where

$$A_n = \frac{a_n/k}{\sqrt{\left(1 - \left(\frac{n\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi n\omega}{\omega_n}\right)^2}}$$

$$B_n = \frac{b_n/k}{\sqrt{\left(1 - \left(\frac{n\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi n\omega}{\omega_n}\right)^2}} \quad (5)$$

$$\phi_n = \tan^{-1} \left(\frac{2\xi n\omega/\omega_n}{1 - (n\omega/\omega_n)^2} \right) \quad (6)$$

$n = 1, 2, 3, \dots$

Equation (5) and (6) show that the system will vibrate violently if $\left(\frac{n\omega}{\omega_n}\right)^2$ closed to unity.

2.2 Dynamic parameter

The dynamic parameters of vibratory tillage in Figure-1 are as follow: The inertia of tillage-tool J was 3.789 kg-m^2 , damping factor $\xi = 10\%$ while the angular stiffness of main spring k was varied from 900 Nm/rad to 2900 Nm/rad . Other parameters were $d = 0.6 \text{ m}$, $a = 0.325 \text{ m}$ and $b = 0.136 \text{ m}$. The main spring was made from alloy steel which have yield strength about 400 MPa , other material were made from structural steel. Draft force $f(t)$ was modelled as a periodic function as shown in Figure-2. The draft force is divide into 4 step [1]. The vibration cycle starts from point 1 ($t = 0$) with the tillage-tool



stationary and about to move forward. From point 1 to point 2 the tillage-tool moves forward through tilled soil (loosen soil) and from point 2 to point 3 the tillage-tool move forward cut the untilled soil (dense soil). From point 3 to point 4 and 5 the tillage-tool moves backward through tilled soil. For each step, the model of draft force was based on the qualitative data of unconfined test which had conducted in soil physics laboratory of agricultural machinery department IPB:

- For tilled soil (loosen soil), the draft force is constant.
- For untilled soil (dense soil) the draft force is in the form of three order polynomial of time t .

Maximum draft force (point 3) was calculated with Gill model [16], for tillage depth of 0.25 m, the draft force is about 2500 N. Assumed that the period of draft force is 0.2 second (excitation frequency $f_e = 5$ Hz) so the draft force is modeled as:

$$F(t) = \begin{cases} 1000 \text{ N} & 0 < t < 0.05 \\ 12E6 t^3 - 3.6E6 t^2 + 360000 t^2 - 9500 \text{ (N)} & 0.05 < t < 0.1 \\ -40000 t + 6500 \text{ (N)} & 0.1 < t < 0.15 \\ 500 \text{ N} & 0.15 < t < 0.2 \end{cases} \quad (7)$$

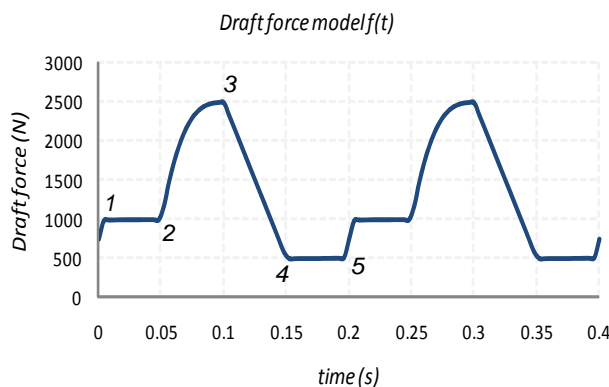


Figure-2. Draft force model for depth tillage of 0.25 m.

2.3 Simulation procedure

A detailed simulation of the SEV-Vibratory tillage was performing using MATLAB program and based on equation (1) to equation (7). The detail of simulation is shown in Table-1. The simulation was focused on steady state response of tillage-tool. Displacement of tillage-tool was plot as functions of spring stiffness k and plot in time domain. For high displacement of tillage tool, the stress of spring is check again, this is important to avoided excessive strain in the spring and to avoid the spring going to plastic. Draft force reduction was investigated by analyzing the graph of tool-tip path during tillage operation while soil quality was investigated by analyzing the graph of displacement of tillage tool.

Table-1. Dynamics parameter used for simulation.

Stiffness k (Nm/rad)	Inertia J (kg-m ²)	Damping factor ξ	mass m (kg)
900-29000	2.486	0.1	0

3. RESULTS AND DISCUSSIONS

3.1 Vibration of tillage-tool

Frequency was calculated first, for excitation period about 0.2 second, the excitation frequency f_e is 5 Hz. As spring stiffness varies from 900 Nm/rad. to 29000 Nm/rad., the fundamental frequency ω_n varies from 2.5 Hz to 13.75 Hz. The graph of maximum deflection of tillage-tool as function of spring stiffness caused by varying cutting force $f(t)$ is shown in Figure-3. The condition of velocity ratio greater than unity or the condition of draft force reduction is correlated to high tillage-tolls deflection, thus simulation is continuing with these conditions High displacement of tillage-tool is encounter at spring stiffness $k = 3740$ Nm/rad. ($\omega_n = \omega_e$) and at spring stiffness $k = 14960$ Nm/rad. ($\omega_n = \omega_e$). For spring stiffness $k = 3740$ Nm/rad., the displacement of tillage tool tip is about 880 mm and the stress in the main spring is about 780 MPa higher than yield strength of spring material. For spring stiffness $k = 14960$ Nm/rad., the displacement of tillage tool is about 124 mm and the stress in the main spring is about 430 MPa. Again this stress is higher than the yield strength of material.

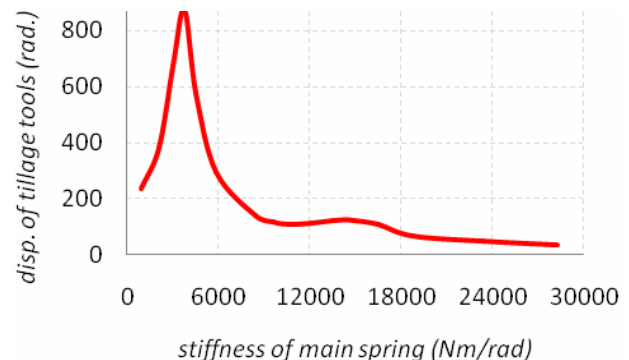


Figure-3. Displacement of tillage tool tip as function of spring stiffness.

For convenient, the spring stiffness around twice of its resonance frequency i.e. spring stiffness of 18750 Nm/rad. ($\omega_n = 2.34\omega_e$) was used for simulation. At this condition, the maximum displacement of tillage-tool tip is about 66 mm and the stress in the main spring is about 295 MPa lower than the yield strength of materials. At this condition, the steady state response of tillage-tool is shown in Figure-4. Positive gradient of graph mean that the tillage-tool moves backward while negative gradient of graph mean that the tillage-tool moves in forward direction. The graph shows that tillage-tool vibrates



periodically in excitation frequency and in its fundamental frequencies.

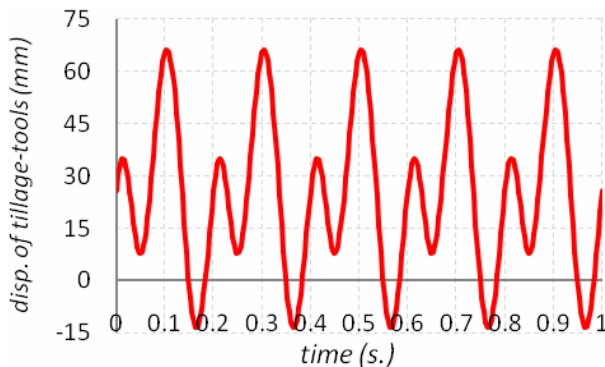


Figure-4. Steady state response of tillage-tool for $k = 18750 \text{ Nm/rad}$.

3.2 Draft force reduction

Draft force reduction was investigated by analyzing the graph of oscillating pathways of tillage-tool tip during tillage operation. An example of this graph is shown in Figure-5. The complete cycle consists of six

parts: (1) the backward moving phase (t_a, t_b = the time at which the tillage-tool moves in tilled soil); (2) The forward moving in tilled soil phase (t_b, t_c); (3) The firm soil cutting phase (t_c, t_d = the time at which the tillage-tool moves forward and cut the uncut soil); (4) the backward moving phase (t_d, t_e = the time at which the tillage-tool moved in tilled soil); (5) the forward moving in tilled soil phase (t_e, t_f); (6) the firm soil cutting phase (t_f, t_g = the time at which the tillage-tool moves forward and cut the uncut soil). The soil cutting force for backward moving phase is assumed to be 20 % of the maximum cutting force of non vibratory soil tillage. The soil cutting force for forward moving in tilled soil phase is assume to be 40 % of the maximum cutting force of non vibratory soil tillage and the cutting force for the firm soil cutting phase is assume to be 100 % of the maximum cutting force of non vibratory tillage. Therefore, the draft ratio (DR) of self excited vibration on single degree of freedom system vibratory tillage could model as:

$$DR = \frac{0.2 * (t_b - t_a + t_e - t_d) + 0.4 * (t_c - t_b + t_f - t_e) + (t_d - t_c + t_g - t_f)}{(t_g - t_a)} * 100 \% \quad (6)$$

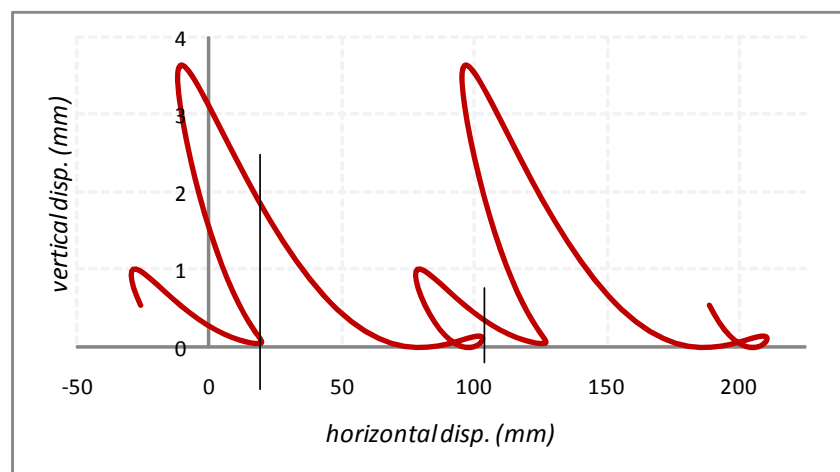


Figure-5. Example of graph of oscillating pathways of tillage-tool tip.

Others graph of oscillating path way of tillage tool tip for different velocity ratio λ are shown in Figure-6. Figure-6a shows graph of oscillating path way of tillage

tool tip for velocity ratio $\lambda = 1$, Figure-6b for $\lambda = 2$, Figure-6c for $\lambda = 4$ and Figure-6.d for $\lambda = 5$.

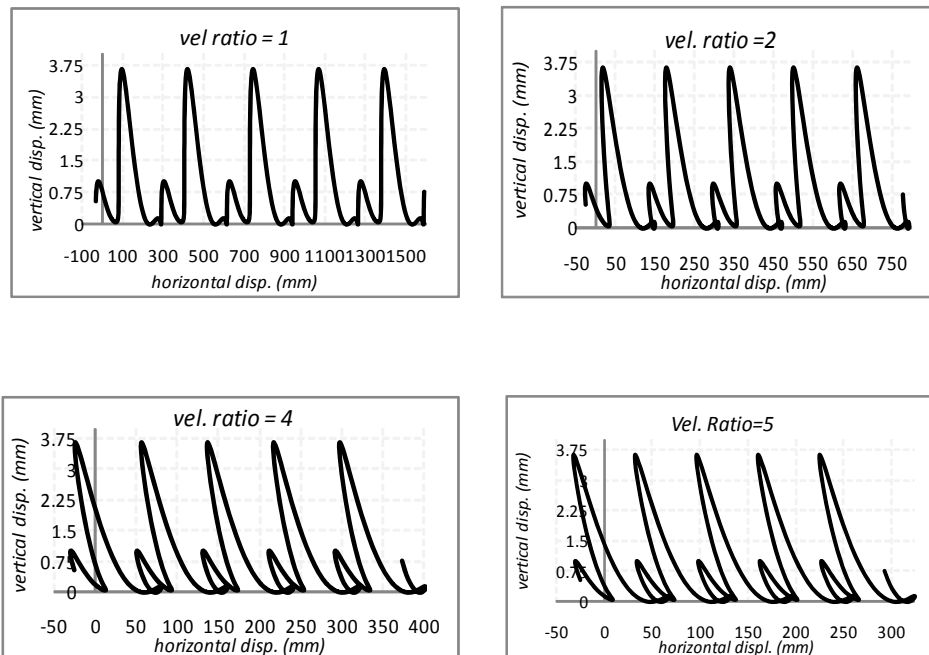


Figure-6. Oscillatory pathways of tillage tool tip. (a): $\lambda = 1$, (b) $\lambda = 2$, (c) $\lambda = 4$, (d) $\lambda = 5$.

Figure-6a shows that for the velocity ratio $\lambda=1$, the tillage-tool always contact to firm soil therefore the draft ratio is about 100 % or no draft force reduction was found during tillage operation. For velocity ratio greater than 1 (Figure-6b to Figure-6d), the tillage-tool moves backward through tilled soil and moves forward through tilled soil there for the draft ratio is smaller than one or draft force reduction was found during tillage operation.

The complete draft ratio as function of velocity ratios λ is shown in Figure-7. No draft reduction was found when the velocity ratio is less than one. As velocity ratio increase, the draft ratio will decrease. The draft force ratio is nearly constant about 45 % when the velocity ratio is greater than 4. This phenomenon is the same as the phenomenon on vibratory tillage by adding external energy to the tillage tool. The important conclusion regarding self-excited vibration is that there is no need for external energy to vibrate the tillage-tool; therefore this phenomenon can reduce energy consumption during tillage operation. Above a velocity ratio of unity, as it further increases, the force ratio correspondingly decreases.

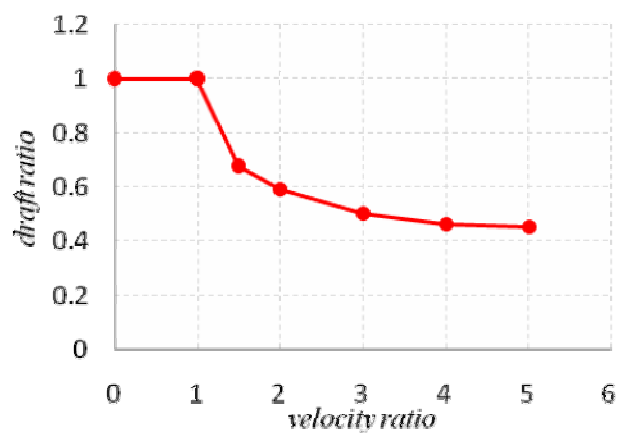


Figure-7. Draft ratio as function of velocity ratio λ .

4. CONCLUSIONS

In this work, the simulation of self excited vibration phenomenon on the vibratory tillage has been successfully demonstrated. The vibration of vibratory tillage was modeled as vibration with SDOF systems while the excited force was modeled as periodic function. The dynamics parameter was the stiffness of main spring $k = 18750 \text{ Nm/rad}$ and the inertia of tillage-tool $J = 2.486 \text{ kgm}^2$. The tillage-tool was vibrated periodically at excitation frequency and at its fundamental frequency. The value of resonance frequency was found twice than that the value of excitation frequency. The possibilities draft force reduction occurred when the system vibrates around its resonance frequency. In this case, two important conditions were note:



- No draft force reduction when the velocity ratio is less than unity. As velocity ratio increase the draft ratio decrease or draft force reduction increase.
- The draft ratio is nearly constant about 45 % when the velocity ratio is greater than 4.

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