# MODELING THE IMPACTS OF URBANIZATION ON RIVER FLOODING USING THE ST. VENANT EQUATIONS

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## ABSTRACT

Urbanization and expansion of structural developments into traditional flood prone areas of a natural river channel modifies the channel shapes, reduces the width of the flood prone areas relative to bankfull width at every point along the river channel and modifies the flood flows containment characteristics of the channel. These impacts are manifested the entrenchment ratio. In this study the Continuity equation in the St. Venant hydrodynamic equations was modified for the geomorphological parameter called the entrenchment ratio. The modified model was applied to investigate the extent of inundation of the floodplain by floods of various frequencies over the section of the Kaduna River adjoining the Kaduna City where urbanization has over the year caused the floodplain to be developed for residential accommodation, recreational and agricultural activities. Results obtained indicated that the 2yr, 5yr, 10yr, 25yr, 50yr, and 100yr floods when occur can cause maximum inundation of between 82.53% to 94.48% of the floodplain area between the Eastern Byepass bridge and the Kaduna South Waterworks with Ungwan Rimi, Kabala Doki and Kigo road extension as the most critical areas.

**Keywords**: urbanization, river flooding, St. Venant equations, floodplain development, entrenchment ratio.

## 1. INTRODUCTION

Urbanization is responsible for expansion of structural developments into traditional flood prone areas of urban settlements causing the floodplain to be encroached upon with structural and non structural developments which modifies the river channel geometry and flood flows through it. Such developments reduce the width of the floodprone areas gradually until it is reduced to the channel bankfull width and where this is not checked by the planning authorities, such development can advance into the river channel. The expansion of properties and agricultural development in all of the communities located near the main stream channel of the Kaduna River along Kaduna City, Nigeria has caused the floodplain areas to be developed for residential accommodation, recreational and agricultural activities and high rise block wall fences has been placed indiscriminately by developers to allow the use and development of these areas that originally provided zones for natural floodwater storage and conveyance. As a result, channel floodway zones have become constrained; the river channel geometry and its flows continuously modified; reduction in the width of the floodprone areas or floodplain and at several sections developments had advanced to the bankfull channel width. Consequences of these developments are many for instance flood passage through these areas can results in higher stages, higher velocities and loss of flood attenuation potential. The Kaduna 2003 flood disaster brought to play the urban development actions and river reactions to these developments when on Friday 6th September 2003, the Kaduna River overflew its banks and spill floodwaters into adjoin properties in the flooplain. Like any other floods in Nigeria (Etiosa, 2006; NWRI, 2008; Vanguard, 2005, 2006) the water stages in the channel and damages were unprecedented, lives were lost, properties worth about N500 million were destroyed while thousands of people were rendered homeless in the City by the ravaging flood which brought the socio-economic activities of the city to standstill for three consecutive days before the flood waters recedes. The flood event raises the challenges for the dynamic predictions of inundation areas; development of models for the propagation of flood waves on the floodplain; and the development of a rapid response and flood warning systems to control and manage flooding in Kaduna. The objective of this study therefore, was to develop a hydraulic model based on the St. Venant hydrodynamic flow equations; generalized the model for applications for flood simulations with variable top widths of flow as occasioned by urbanization impact on a natural river flowing through an urban settlement; and use the model to predict flood water stages and extent of flooded areas corresponding to the floods of various frequencies.

## 2. METHODOLOGY

### 2.1 The Saint Venant Equation

The basic equations that describe the propagation of a wave in an open channel are the Saint Venant's equation Chow (1985), consisting of the continuity and momentum equations presented in equation 1 and 2 respectively. In differential form the governing equations are written as:

Continuity equation:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

Momentum:

$$\frac{\partial Q}{\partial t}+\frac{\partial\left(Q^2/A\right)}{\partial x}+gA(\frac{\partial h}{\partial x}+S_f-S_x)=0 \qquad (2)$$

Where $Q$ the flow through the section is, $A$ is the flow area, $x$ is the longitudinal distance, $t$ time, $S_x$ channel bottom slope, q is lateral inflow into the channel and $S_f$ friction slope. $gA(S_f-S_x)=0$ is the kinematic wave; $gA(\frac{\partial h}{\partial x}+S_f-S_x)=0$ is the diffusion wave and dynamic wave represented by the complete momentum equation.

**2.2 Model formulation and development**

In the process of applying the Saint Venant equations to practical problems several researchers have found the need to modify the original equations to take into considerations additional factors which in their own opinion are not properly considered in the original development (Swiatek, 2007; Roger *et al*., 2000; Blackburn *et al*., 2002 and Siuta, 2006)). In order to deal with overbank flow in the floodplain, Fread (1975, 1976, and 1982) presents a modified form of the St. Venant equations as presented equations (3) and (4):

$$\frac{\partial Q}{\partial x}+\frac{\partial\left(A+A_o\right)}{\partial t}=0 \qquad (3)$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA(\frac{\partial z}{\partial x}+S_f+S_e)=0 \qquad (4)$$

Where $A$ is the active cross-sectional area of flow, $A_o$ the inactive or off channel cross sectional area, and $S_e$ the expansion and contraction slope?

Urbanization affects water movement in river system and modifies channel shapes which can lead to increase in flood peaks and more frequent flooding. Channel shape modification can be in the form of siltation of the river channel resulting in shallow depths of flow; contraction and expansion of the top width of flow arising from residential developments in the floodplain; and damage to the river planform by agricultural activities in the floodplain. Developments within the floodplain reduce the width of the floodprone areas and the vertical containment of floodwaters by the river channel at every point along the river channel. In geomorphological terms the vertical containment of floodwaters by the river channel is known as the entrenchment ratio. The vertical containment of floodwaters is otherwise known as the entrenchment ratio (the flood-prone area width ($W_{fpa}$) divided by bankfull channel width ($W_{bkf}$)) at the cross section under consideration). It also has the tendencies of increasing the sinuosity and the risk of flooding. The sinuosity represents the characteristics of the river plan shape and determines the degree of exchange of flow between the main channel and the floodplain (NEH, 2007 and Rosgen, 1996). The greater the sinuosity, the greater the difference between the flow direction of the main channel and the flood channel.

Consequently, to accurately model flood levels in a natural river channel flowing through an urban settlement, the Continuity equation in the Saint Venant equations need to be modified with a “floodplain width reduction factor called the entrenchment ratio” to incorporates the effect of physical development in the floodplain on the flood water stages in the floodplain (Alayande *et al*., 2010). With these considerations, the continuity equation in the Saint Venant equations become modified with “floodplain width reduction factor or the entrenchment ratio” to the form presented in equation (5) below.

$$\frac{\partial Q}{\partial x}+\frac{W_{fpa}}{W_{bkf}}\frac{\partial A}{\partial t}=0 \qquad (5)$$

Writing the continuity equation using the gravity oriented coordinates as in equation (6).

$$\frac{\partial Q}{\partial x}+\frac{W_{fpa}}{W_{bkf}}\frac{\partial A}{\partial z}\frac{\partial z}{\partial t}=0 \qquad (6)$$

Taking the channel top width as $B=\frac{\partial A}{\partial z}$ the continuity becomes

$$\frac{\partial Q}{\partial x}+\frac{W_{fpa}}{W_{bkf}}B\frac{\partial z}{\partial t}=0 \qquad (7)$$

Modifying the continuity equation with the entrenchment ratio $\frac{W_{fpa}}{W_{bkf}}$ thus generalized the equation for applications in overbank flow or flooding situation $\frac{W_{fpa}}{W_{bkf}}>1$ and non overbank flow situation $\frac{W_{fpa}}{W_{bkf}}=1$ thus enable the equation to be applicable under variable width of flooded area.

In the momentum equation $S_f=\frac{n^2Q|Q|}{A^2R^{4/3}}$ and putting $k=\frac{n}{AR^{2/3}}$ the momentum equation can be written as presented in equation (8).

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$$\frac{\partial Q}{\partial t}+\frac{\partial\left(Q^2/A\right)}{\partial x}+gA\left(\frac{\partial z}{\partial x}+k^2Q|Q|\right)=0 \tag{8}$$

Equations (7) and (8) form the model equations applied in this study.

### 2.3 Numerical solution of the model equations

The model equations were solved using the weighted four point implicit finite difference scheme presented in Figure-1 where each points in the scheme are identified or referenced by row and column (i,j) with i representing the distance between the river cross sections along the longitudinal axis of flow while j represent the time step. In the weighted four-point implicit scheme solution process, the spatial derivatives $\frac{\partial Q}{\partial x}$ and $\frac{\partial\left(\frac{Q^2}{A}\right)}{\partial x}$ as well as the variables other than the derivatives were approximated between adjacent time lines by finite difference quotients proportioned according to the factors θ and (1-θ). Θ = 0.55 is used after Fread (1978). Considering the schematic x-t solution plane in Figure- 1, the finite difference approximations for the terms in equations (7) and (8) for estimation of Q and h are derived as follows using the forward difference approximations:

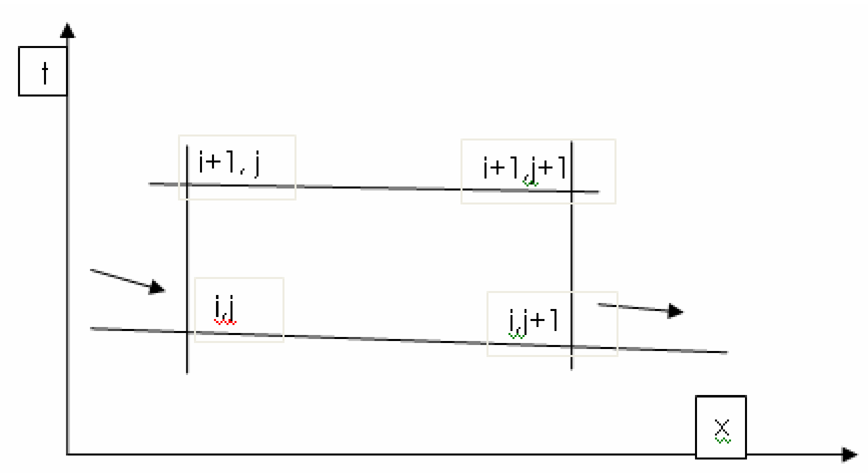


**Figure-1.** Finite differences scheme for the implicit method of solution of unsteady equations.

$$\left(\frac{\partial Q}{\partial x}\right)=\theta\frac{Q_{i+1}^{j+1}-Q_i^{j+1}}{\Delta x_i}+(1-\theta)\frac{Q_{i+1}^{j}-Q_i^{j}}{\Delta x_i}$$
$$=\theta\frac{Q_{i+1}^{j}+\Delta Q_{i+1}^{j}-\left(Q_i^{j}+\Delta Q_i^{j}\right)}{\Delta x}+\frac{Q_{i+1}^{j}-Q_i^{j}}{\Delta x_i}-\theta\frac{\left(Q_{i+1}^{j}-Q_i^{j}\right)}{\Delta x}$$
$$=\theta\frac{\Delta Q_{i+1}^{j}-\Delta Q_i^{j}}{\Delta x}+\frac{Q_{i+1}^{j}-Q_i^{j}}{\Delta x_i} \tag{9}$$

$$\left(\frac{\partial z}{\partial x}\right)=\theta\frac{z_{i+1}^{j+1}-z_i^{j+1}}{\Delta x_i}+(1-\theta)\frac{z_{i+1}^{j}-z_i^{j}}{\Delta x_i}$$
$$=\theta\frac{z_{i+1}^{j}+\Delta z_{i+1}^{j}-\left(z_i^{j}+\Delta y_i^{j}\right)}{\Delta x}+\frac{z_{i+1}^{j}-z_i^{j}}{\Delta x_i}-\theta\frac{\left(z_{i+1}^{j}-z_i^{j}\right)}{\Delta x}$$
$$=\theta\frac{\Delta z_{i+1}^{j}-\Delta z_i^{j}}{\Delta x}+\frac{z_{i+1}^{j}-z_i^{j}}{\Delta x_i} \tag{10}$$

$$\left(\frac{\partial\left(Q^2/A\right)}{\partial x}\right)=\theta\frac{\left(Q^2/A\right)_{i+1}^{j+1}-\left(Q^2/A\right)_i^{j+1}}{\Delta x_i}+(1-\theta)\frac{\left(Q^2/A\right)_{i+1}^{j}-\left(Q^2/A\right)_i^{j}}{\Delta x_i}$$
$$=\frac{\theta}{\Delta x_i}\left[\left(\left(Q^2/A\right)_{i+1}^{j}+\Delta\left(Q^2/A\right)_{i+1}^{1}\right)-\left(\left(Q^2/A\right)_i^{j}+\Delta\left(Q^2/A\right)_i\right)\right]+\frac{1}{\Delta x}\left[\left(Q^2/A\right)_{i+1}^{j}-\left(Q^2/A\right)_i^{j}\right] \tag{11}$$
$$\frac{-\theta}{\Delta x}\left[\left(Q^2/A\right)_{i+1}^{j}-\left(Q^2/A\right)_i^{j}\right]$$
$$=\frac{\theta}{\Delta x_i}\left[\Delta\left(Q^2/A\right)_{i+1}-\Delta\left(Q^2/A\right)_i^{1}\right]+\frac{1}{\Delta x}\left[\left(Q^2/A\right)_{i+1}-\left(Q^2/A\right)_i\right]$$

and the time derivatives as follows:

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$$\left(\frac{\partial Q}{\partial t}\right) = \frac{1}{2\Delta t}\left(\left(Q_{i+1}^{j+1} - Q_{i+1}^{j}\right) + (Q_i^{j+1} - Q_i^j)\right)$$
$$= \frac{1}{2\Delta t}\left(\left(Q_{i+1}^{j} + \Delta Q_{i+1} - Q_{i+1}^{j}\right) + (Q_i^{j} + \Delta Q_i - Q_i^j)\right)$$
$$= \frac{\Delta Q_{i+1} + \Delta Q_i}{2\Delta t} \quad (12)$$

$$\left(\frac{\partial A}{\partial t}\right) = \frac{1}{2\Delta t}\left(\left(A_{i+1}^{j+1} - A_{i+1}^{j}\right) + (A_i^{j+1} - A_i^j)\right)$$
$$= \frac{1}{2\Delta t}\left(\left(A_{i+1}^{j} + \Delta A_{i+1} - A_{i+1}^{j}\right) + (A_i^{j} + \Delta A_i - A_i^j)\right)$$
$$= \frac{\Delta A_{i+1} + \Delta A_i}{2\Delta t} \quad (13)$$

$$\left(\frac{\partial z}{\partial t}\right) = \frac{1}{2\Delta t}\left(\left(z_{i+1}^{j+1} - z_{i+1}^{j}\right) + (z_i^{j+1} - z_i^j)\right)$$
$$= \frac{1}{2\Delta t}\left(\left(z_{i+1}^{j} + \Delta z_{i+1} - z_{i+1}^{j}\right) + (z_i^{j} + \Delta z_i - z_i^j)\right)$$
$$= \frac{\Delta z_{i+1} + \Delta z_i}{2\Delta t} \quad (14)$$

and all non derivative terms are estimated as weighted averages between two adjacent time lines as follows:

$$gA = g\theta\frac{A_{i+1}^{j+1} + A_i^{j+1}}{2} + g(1-\theta)\frac{A_{i+1}^{j} + A_i^{j}}{2}$$
$$= g\theta\frac{A_{i+1}^{j} + \Delta A_{i+1} + \left(A_i^j + \Delta A_i^j\right)}{2} + g(1-\theta)\frac{A_{i+1}^{j} + A_i^{j}}{2}$$
$$= g\theta\frac{\Delta A_{i+1} + \Delta A_i^j}{2} + g\frac{A_{i+1}^{j} + A_i^{j}}{2} \quad (15)$$

$$\left(\frac{W_{fpa}}{W_{bkf}}B\right) = \theta\frac{\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j+1} + \left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}^{j+1}}{2} + (1-\theta)\frac{\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j} + \left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}^{j}}{2}$$
$$= \frac{\theta}{2}\left[\left(\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j} + \Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}\right) + \left(\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j} + \Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right)\right]$$
$$+ \frac{1}{2}\left[\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j} + \left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right] - \frac{\theta}{2}\left[\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j} + \left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right]$$
$$= \frac{\theta}{2}\left[\Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1} + \Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right] + \frac{1}{2}\left[\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j} + \left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right] \quad (16)$$

Similarly

$$gAk^2Q|Q| = \theta\frac{\left(gAk^2Q|Q|\right)_{i+1}^{j+1} + \left(gAk^2Q|Q|\right)_{i}^{j+1}}{2} + (1-\theta)\frac{\left(gAk^2Q|Q|\right)_{i+1}^{j} + \left(gAk^2Q|Q|\right)_{i}^{j}}{2}$$
$$= \frac{g\theta}{2}\left[\left(\left(Ak^2Q|Q|\right)_{i+1}^{j} + \Delta\left(Ak^2Q|Q|\right)_{i+1}\right) + \left(\left(Ak^2Q|Q|\right)_{i}^{j} + \Delta\left(Ak^2Q|Q|\right)_{i}\right)\right]$$
$$+ \frac{g}{2}\left[\left(Ak^2Q|Q|\right)_{i+1}^{j} + \left(Ak^2Q|Q|\right)_{i}^{j}\right] - \frac{\theta g}{2}\left[\left(Ak^2Q|Q|\right)_{i+1}^{j} + \left(Ak^2Q|Q|\right)_{i}^{j}\right]$$
$$= \frac{g\theta}{2}\left[\Delta\left(Ak^2Q|Q|\right)_{i+1} + \Delta\left(Ak^2Q|Q|\right)_{i}\right] + \frac{g}{2}\left[\left(Ak^2Q|Q|\right)_{i+1}^{j} + \left(Ak^2Q|Q|\right)_{i}^{j}\right] \quad (17)$$

The finite difference form of the continuity equation is derived by substituting the finite difference form of the derivatives and non derivatives of equation (7) as follows:

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$$\theta\frac{\Delta Q_{i+1}^{j}-\Delta Q_{i}^{j}}{\Delta x_i}+\frac{Q_{i+1}^{j}-Q_{i}^{j}}{\Delta x_i}+\begin{bmatrix}\frac{\theta}{2}\left[\Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}+\Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right]\\+\frac{1}{2}\left[\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j}+\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right]\end{bmatrix}\frac{\Delta z_{i+1}+\Delta z_i}{2\Delta t}=0 \tag{18}$$

Multiply through by Δx

$$=\theta\left(\Delta Q_{i+1}^{j}-\Delta Q_{i}^{j}\right)+\left(Q_{i+1}^{j}-Q_{i}^{j}\right)+\frac{\Delta x}{2\Delta t}\begin{bmatrix}\frac{\theta}{2}\left[\Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}+\Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right]\\+\frac{1}{2}\left[\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j}+\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right]\end{bmatrix}\left(\Delta z_{i+1}+\Delta z_i\right) \tag{19}$$

Where

$\Delta Q_{i+1}$ = change in flow at location (i+1) over time increment Δt
$\Delta Q_i$ = change in flow at location (i) over time increment Δt
$\Delta z_{i+1}$ = change in depth of flow at location (i+1) over time increment Δt
$\Delta z_i$ = change in flow at location (i) over time increment Δt
Equation (19) can be written in simplified form in terms of unknown $\Delta Q_{i+1}$, $\Delta Q_i$, $\Delta z_{i+1}$, and $\Delta z_i$ as equation (20)

$C_1\Delta Q_{i+1} + C_2\Delta Q_i + C_3\Delta z_{i+1} + C4\Delta z_i + C_5 = 0$ (20)

Where $C_1 = \theta = 0.55$

$C_2 = -\theta = -0.55$

$$C_3 = C_4 = \frac{\Delta x}{2\Delta t}\begin{bmatrix}\frac{\theta}{2}\left[\Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}+\Delta\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right]\\+\frac{1}{2}\left[\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i+1}^{j}+\left(\frac{W_{fpa}}{W_{bkf}}B\right)_{i}\right]\end{bmatrix}$$

$C_5 = Q_{i+1}^{j} - Q_{i}^{j}$

Similarly the finite difference form of the momentum equation is derived by substituting the finite difference form of the derivatives and non derivatives of equation (8) as follows:

$$\frac{\Delta Q_{i+1}+\Delta Q_i}{2\Delta t}+\left[\frac{\theta}{\Delta x_i}\left[\Delta\left(Q^2/A\right)_{i+1}-\Delta\left(Q^2/A\right)_{i}^{1}\right]+\frac{1}{\Delta x}\left[\left(Q^2/A\right)_{i+1}-\left(Q^2/A\right)_{i}\right]\right]$$
$$+\left(g\theta\frac{\Delta A_{i+1}+\Delta A_i^j}{2}+g\frac{A_{i+1}^j+A_i^j}{2}\right)\left(\theta\frac{\Delta z_{i+1}^j-\Delta z_i^j}{\Delta x}+\frac{z_{i+1}^j-z_i^j}{\Delta x_i}\right)$$
$$+\frac{g\theta}{2}\left[\Delta\left(Ak^2Q|Q|\right)_{i+1}+\Delta\left(Ak^2Q|Q|\right)_{i}\right]+\frac{g}{2}\left[\left(Ak^2Q|Q|\right)_{i+1}^{j}+\left(Ak^2Q|Q|\right)_{i}^{j}\right]$$
$$=\frac{\Delta Q_{i+1}+\Delta Q_i}{2\Delta t}+\frac{g\theta}{2\Delta x}\begin{bmatrix}\theta\left(\Delta A_{i+1}+\Delta A_i^j\right)\\+\left(A_{i+1}^j+A_i^j\right)\end{bmatrix}\left(\Delta z_{i+1}^j-\Delta z_i^j\right)+\frac{g}{2\Delta x}\begin{bmatrix}\theta\left(\Delta A_{i+1}+\Delta A_i^j\right)\\+\left(A_{i+1}^j+A_i^j\right)\end{bmatrix}\left(z_{i+1}^j-z_i^j\right)$$
$$+\left[\frac{\theta}{\Delta x_i}\begin{bmatrix}\Delta\left(Q^2/A\right)_{i+1}^{j}\\-\Delta\left(Q^2/A\right)_{i}^{1}\end{bmatrix}+\frac{1}{\Delta x}\begin{bmatrix}\left(Q^2/A\right)_{i+1}^{j}\\-\left(Q^2/A\right)_{i}^{j}\end{bmatrix}\right]+\frac{g\theta}{2}\begin{bmatrix}\Delta\left(Ak^2Q|Q|\right)_{i+1}^{j}\\+\Delta\left(Ak^2Q|Q|\right)_{i}^{j}\end{bmatrix}+\frac{g}{2}\begin{bmatrix}\left(Ak^2Q|Q|\right)_{i+1}^{j}\\+\left(Ak^2Q|Q|\right)_{i}^{j}\end{bmatrix} \tag{21}$$

Where

$\Delta Q_{i+1}$ = change in flow at location (i+1) over time increment Δt
$\Delta Q_i$ = change in flow at location (i) over time increment Δt
$\Delta z_{i+1}$ = change in depth of flow at location (i+1) over time increment Δt
$\Delta z_i$ = change in flow at location (i) over time increment Δt

Equation (21) can be written in simplified form in terms of unknown $\Delta Q_{i+1}$, $\Delta Q_i$, $\Delta z_{i+1}$, and $\Delta z_i$ as equation (22)

$C_6\Delta Q_{i+1} + C_7\Delta Q_i + C_8\Delta z_{i+1} + C_9\Delta z_i + C_{10} = 0$ (22)

Where $\theta = 0.5$

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$$C_6 = C_7 = \frac{1}{2\Delta t}$$

$$C_8 = \frac{g\theta}{2\Delta x}\begin{bmatrix} \theta\left(\Delta A_{i+1} + \Delta A_i^j\right) \\ + \left(A_{i+1}^j + A_i^j\right) \end{bmatrix}$$

$$C_9 = -\frac{g\theta}{2\Delta x}\begin{bmatrix} \theta\left(\Delta A_{i+1} + \Delta A_i^j\right) \\ + \left(A_{i+1}^j + A_i^j\right) \end{bmatrix}$$

$$C_{10} = +\frac{g}{2\Delta x}\begin{bmatrix} \theta\left(\Delta A_{i+1} + \Delta A_i^j\right) \\ + \left(A_{i+1}^j + A_i^j\right) \end{bmatrix}\left(z_{i+1}^j - z_i^j\right) + \left[\frac{\theta}{\Delta x_i}\begin{bmatrix} \Delta\left(Q^2/A\right)_{i+1}^j \\ - \Delta\left(Q^2/A\right)_i^1 \end{bmatrix} + \frac{1}{\Delta x}\begin{bmatrix} \left(Q^2/A\right)_{i+1}^j \\ - \left(Q^2/A\right)_i^j \end{bmatrix}\right] + \frac{g\theta}{2}\begin{bmatrix} \Delta\left(Ak^2Q|Q|\right)_{i+1}^j \\ + \Delta\left(Ak^2Q|Q|\right)_i^j \end{bmatrix} + \frac{g}{2}\begin{bmatrix} \left(Ak^2Q|Q|\right)_{i+1}^j \\ + \left(Ak^2Q|Q|\right)_i^j \end{bmatrix}$$

**2.4 The double sweep solution technique**

The implicit scheme is a numerical process that is stable for any magnitude of time increment Δt. The two resulting systems of linear algebraic equations of continuity and momentum must be solved at every computational point for every time step Δt. Linearizing the boundary conditions in terms of ΔQ and Δz, the systems of equations can be solved by several numerical methods of solutions of linear equations such as the Gaussian elimination, Gauss-Sidel etc. These methods involve a computational process that is cumbersome and high memory requirement for programming on a computer. The double sweep is a method for transferring a one-point boundary condition by means of a differential or difference equation corresponding to the given equation and is used for solving boundary value problems. It provides a more efficient approach in terms of computational operations and memory requirements for the computer programming. In the double sweep solution technique, it is assumed that at any point *n* there is a linear relationship between discharge increment ΔQ and stage increment Δz (Bamgboye et al.) of the form in equation (23).

$$\Delta Q_n = m_n \Delta z_n + K_n \tag{23}$$

Similar relationship exist for the point (n+1)

$$\Delta Q_{n+1} = m_{n+1}\Delta z_{n+1} + K_{N+1} \tag{24}$$

So that for the reach (n, n+1) the momentum equation (22) can be written as in equation (25)

$$C_{6n}\,\Delta Q_{n+1} + C_{7n}\Delta Q_n + C_{8n}\Delta Z_{n+1} + C_{9n}\Delta Z_n + C_{10n} = 0 \tag{25}$$

Substitute equation (23) in equation (25)

$$C_{6n}\,\Delta Q_{n+1} + C_{7n}(m_n\Delta Z_n + K_n) + C_{8n}\Delta Z_{n+1} + C_{9n}\Delta Z_n + C_{10n} = 0 \tag{26}$$

$$C_{6n}\,\Delta Q_{n+1} + C_{7n}m_n\Delta Z_n + C_{7n}m_nK_n + C_{8n}\Delta Z_{n+1} + C_{9n}\Delta Z_n + C_{10n} = 0 \tag{27}$$

$$C_{6n}\,\Delta Q_{n+1} + \Delta Z_n(C_{7n}m_n + C_{9n}) + C_{7n}m_nK_n + C_{8n}\Delta Z_{n+1} + C_{10n} = 0 \tag{28}$$

$$\Delta Z_n = -\frac{(C_{6n}\,\Delta Q_{n+1} + C_{7n}m_nK_n + C_{8n}\Delta Z_{n+1} + C_{10n})}{C_{7n}m_n + C_{9n}} \tag{29}$$

Equation (29) can be written as in equation (30)

$$\Delta Z_n = P_n\,\Delta Z_{n+1} + R_n\,\Delta Q_{n+1} + S_n \tag{30}$$

Where

$P_n = C_{8n} / Fn$

$R_n = C_{6n} / F_n$

$S_n = (C_{7n}K_n + C_{10n}) / F_n$

$F_n = -(C_{7n}m_n + C_{9n})$

Similarly, writing the continuity equation for reach (n, n+1), we have

$$C_{1n}\,\Delta Q_{n+1} + C_{2n}\Delta Q_n + C_{3n}\Delta Z_{n+1} + C_{4n}\Delta Z_n + C_{5n} = 0 \tag{31}$$

Substitute equations (23) and (30) in (31)

$$C_{1n}\,\Delta Q_{n+1} + C_{2n}[m_n(P_n\Delta Z_{n+1} + R_n\Delta Q_{n+1} + S_n) + K_n] + C_{3n}\Delta Z_{n+1} + C_{4n}(P_n\Delta Z_{n+1} + R_n\Delta Q_{n+1} + S_n) + C_{5n} = 0 \tag{32}$$

$$C_{1n}\,\Delta Q_{n+1} + C_{2n}m_n(P_n\Delta Z_{n+1} + R_n\Delta Q_{n+1} + S_n) + C_{2n}K_n + C_{3n}\Delta Z_{n+1} + C_{4n}P_n\Delta Z_{n+1} + C_{4n}R_n\Delta Q_{n+1} + C_{4n}S_n + C_{5n} = 0 \tag{33}$$

$$C_{1n}\,\Delta Q_{n+1} + C_{2n}m_nP_n\Delta Z_{n+1} + C_{2n}m_nR_n\Delta Q_{n+1} + C_{2n}m_nS_n + C_{2n}K_n + C_{3n}\Delta Z_{n+1} + C_{4n}P_n\Delta Z_{n+1} + C_{4n}R_n\Delta Q_{n+1} + C_{4n}S_n + C_{5n} = 0 \tag{34}$$

$$\Delta Q_{n+1}(C_{1n} + C_{2n}m_nR_n + C_{4n}R_n) + \Delta Z_{n+1}(C_{2n}m_nP_n + C_{3n} + C_{4n}P_n) + (C_{2n}m_nS_n + C_{2n}K_n + C_{4n}S_n + C_{5n}) = 0 \tag{35}$$

Equation (35) can be written in the form presented in equation (36)

$$\Delta Q_{n+1} = m_{n+1}\,\Delta Z_{n+1} + K_{n+1} \tag{36}$$

Where

$$m_{n+1} = (C_{2n}m_nP_n + C_{3n} + C_{4n}P_n)/T_{n+1}$$

$$K_{n+1} = (C_{2n}m_nS_n + C_{2n}K_n + C_{4n}S_n + C_{5n})/T_{n+1}$$

$$T_{n+1} = -(C_{1n} + C_{2n}m_nR_n + C_{4n}R_n)$$

equations (30) and (36) provide the means of computing the next value of water surface elevation $Z^{t+\Delta t}$ and $Q^{t+\Delta t}$ for all the modeled points n=1 to n = r-1 when all the boundary conditions are linearized locally. This requires that the boundary conditions should be of the form:

a. At the boundary n = 1, the relationship between Q and H or as in this application Q and Z presented by equation (36) must be known so that the coefficient $m_1$ and $k_1$ can be determined.
b. At the downstream boundary n = r, the change in water surface elevation Δz must be known so that the coefficient $m_r$ and $k_r$ can be determined.

**2.5 Model computerization**

The model equations were solved on Microsoft Excel 2007. Microsoft Excel is versatile software that provides professionals with unlimited capabilities to analyze and perform basic and complex operations on spreadsheet data through its built-in functions like any programming languages such as Quick Basic, Visual Basic, Pascal, FORTRAN, C-language, etc. Many commercial software also comes in Excel templates, thus with Excel one requires the ability to use the built-in functions and tools. In engineering modeling as in this application, a template was created for the solution of the simultaneous equations (30) and (36) by writing valid Excel equations for the constants in these equations for each nodal point in the model corresponding to cross section locations along the river.

**2.6 Applications of the Saint Venant equation to Kaduna River**

The solution of the St. Venant equations for the reaches of the Kaduna River adjoining the City of Kaduna was accomplished by solving equations (7) and (8) using equations (30) and (36). In the solution process, the terms "forward sweep" and "backward sweep" were used to describe two independent procedures whereby the upstream and downstream boundary conditions were transferred from one point to another by means of difference equation (30) and (36).

The solution technique starts at the upstream boundary (section n = 1) whereby the coefficients $m_n$ and $k_n$ were specified by the upstream boundary condition. In the forward sweep procedure the equation (36) was written for section n+1 and the coefficients $m_{n+1}$ and $k_{n+1}$ determine by combining equations (30) and (36) such that:

$$m_{n+1} = [C_{2n}m_n(\frac{-C_{8n}}{C_{7n}m_n + C_{9n}}) + C_{3n} + C_{4n}(\frac{-C_{8n}}{C_{7n}m_n + C_{9n}})]/[-(C_{1n} + C_{2n}m_n(\frac{-C_{6n}}{C_{7n}m_n + C_{9n}}) + C_{4n}(\frac{-C_{6n}}{C_{7n}m_n + C_{9n}})] \qquad (37)$$

$$K_{n+1} = [(C_{2n}m_n(\frac{-(C_{7n}K_n + C_{10n})}{C_{7n}m_n + C_{9n}}) + C_{2n}K_n + C_{4n}(\frac{-(C_{7n}K_n + C_{10n})}{C_{7n}m_n + C_{9n}}) + C_{5n})]/[-(C_{1n} + C_{2n}m_n(\frac{-C_{6n}}{C_{7n}m_n + C_{9n}}) + C_{4n}(\frac{-C_{6n}}{C_{7n}m_n + C_{9n}})] \qquad (38)$$

The coefficients $m_{n+1}$ and $k_{n+1}$ were thus completely specified in terms of parameters determined from known conditions at the previous section *n*. With this, the equation of continuity (equation (36)) can be written for all model points *n = r* in which case we have two unknown ΔQ and ΔZ at each point. The backward sweep, starting from the downstream boundary section *n = r* to upstream section *n=1*, provide a sequential solution for the set of equations formed during the forward sweep. The value of $\Delta z_r$ is known from downstream boundary condition. The flows and water stages for each time, nΔt, are then computed at each section by adding the incremental changes ΔQ and ΔZ to the previous values of the flow and stage Q and Z.

**2.6.1 Model boundary limits**

In selecting the two extremities of the modeled reach consideration was given to points with control section such as bridges and gauged section such that the flow across which can be estimated with some degree of accuracy. To this effect, the upstream boundary was taken at the Eastern Byepass Bridge at Malali while the downstream boundary was taken at the Western Bypass Bridge downstream the Kaduna South Waterworks gauging station.

**2.6.2 Model data input**

**2.6.2.1 Channel hydraulic geometry**

The variables in the model equations consisted essentially of water stage, discharge and Mannings roughness coefficient and other channel geometry parameters. Hydraulic geometry data on the reach of the Kaduna under investigation were extracted from survey data carried out across. A total of thirty eight cross sections were surveyed consisting of 25 in Reach 2 and 13 in Reach 3 and the cross sections were spaced along the longitudinal direction in a manner to capture the changes along the channel and extending across the width of the floodplain at the section. The cross section were plotted using the GRAPH function of Microsoft Excel 2007 and the channel geometry parameters related to bankfull and flood dimensions presented in Table-1 were calculated on EXCEL WORKSHEET at each cross section to provides the baseline data for use in the model.

**2.6.2.2 Rating curves and rating tables**

Rating Curves and Tables were established for each cross section using the Manning's formula and channel hydraulic geometry parameters. Kaduna River is gauged at Kaduna South Waterworks and the 2003 flood level was marked at the Railway Bridge about 905m upstream the

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Kaduna South Waterworks at 573.29m above mean sea level (amsl).

**Table-1.** Channel geometry parameters related to bankfull and flood dimensions.

| Parameters | |
|---|---|
| **Bankfull dimensions** | **Flood dimensions** |
| X-section area (m.sq.) | Flood prone area width |
| Width | Width left floodplain |
| Mean depth | Width left floodplain encroached |
| Max depth | Width right floodplain |
| | Width right floodplain encroached |
| | Low bank height |
| | Max riffle depth |
| | Bank height ratio (low bank height/max riffle depth) |
| | Flood prone area elevation |
| | Maximum level in the cross section |

With a right bank valley slope of 0.042% and distance of 905m to the cross section at the Kaduna South Waterworks, the corresponding level at this cross section is 573.29-0.042%*905 or 572.91m amsl. Extending the rating curve at Kaduna South Water Works to 572.91m gives the corresponding discharge of 3,485.31m$^3$/sec corresponding to the 2003 flood level. A Manning's coefficient of 0.035 gave the discharge of 3,485.31m$^3$/sec at 572.91m amsl and was used to calculate the discharges at other levels in the cross section. This stepwise level transfer between two immediate cross sections was carried out for all the cross sections and the Manning's coefficient determined for that cross section was used to calculate the discharges for other levels in the cross section.

**2.6.2.3 Flood frequency data**

The analysis of the extreme rainfall and streamflow databases created from available rainfall at Kaduna Airport and Kaduna South Waterworks indicated that flooding along the Kaduna City adjoining the Kaduna River are rainfall induced and the river channel are expected to be on higher risks of flooding when the channel is flowing at bankfull capacity coincides with high rainfall. Consequently the flood frequency analysis was carried out separately on the extreme flow and rainfall databases by fitting the Log Pearson Type III distribution to the database to determine floods levels corresponding to 200, 100, 50, 20, 10, 5 and 2 years annual recurrence intervals using the Microsoft EXCEL as presented in Table-2. The flood magnitudes for the maximum daily flow were used as model flows.

**2.7 Model run and verification**

The model was run for extreme flood flows for 2yr, 10yr, 25yr, 50yr, 100yr and 200yr recurrence interval and the 2003 flood discharge. In order to compute the successive values of water surface elevation ΔH and ΔQ for all the modeled points from upstream to downstream locations, all the boundary conditions were linearized locally. The linearization of the boundary conditions at the boundaries was accomplished using the slope and intercept functions in Microsoft Excel to determine the coefficient $m_1$ and $k_1$ at the upstream and $m_r$ and $k_r$ at the downstream as in equation (37) and (38).

**Table-2**. Flood frequency analysis for Kaduna River at Kaduna South Waterworks (Log Pearson type III Estimated Flood Flows (m$^3$/sec).

| | **Return period (years)** | | | | | | |
|---|---|---|---|---|---|---|---|
| | 2 | 5 | 10 | 25 | 50 | 100 | 200 |
| Max daily Q | 1,578.60 | 2,181.72 | 2,607.43 | 3,175.96 | 3,621.59 | 4,086.07 | 4,573.47 |
| Max 5days Q | 1,218.57 | 1,535.55 | 1,607.96 | 1,641.22 | 1,649.64 | 1,652.41 | 1,654.22 |
| Max 7 days Q | 1,108.94 | 1,343.03 | 1,350.97 | 1,403.08 | 1,406.81 | 1,407.59 | 1,408.20 |

The model parameters were laid out in Microsoft Excel Worksheet, each line for each cross section from the upstream to downstream. The model was initialized with the 2yr flood and run for other floods of 5yr, 10yr, 25yr, 50yr, 100yr and 200yr year annual recurrence interval. The slope and intercept values were entered into the model as $m$ and $k$ and the differential water stages (ΔH) between the initial flow (2 yr-floods) and model flow for the downstream location. The successive change in values of water surface elevation ΔQ and ΔH for all the modeled points from downstream to upstream locations were subsequently calculated in a backward sweep.

**3. RESULTS AND DISCUSSIONS**

The model results were verified by comparing the modeled flood stages at each cross section with the measured water stages at the same discharge as presented in Figures 2 to 9. Figure-8 is the presentation of the comparison at the Kaduna South Waterworks gauging station. The comparison of the top width of flow for the 2yr, 5yr, 10yr, 25yr, 50yr, and 100yr model run and the maximum width of the flood prone area measured for each cross sections indicated that the 2yr, 5yr, 10yr, 25yr, 50yr, and 100yr floods when occur, the level of flood plain inundation could be as much as 82.53% to 94.48% of the floodplain area between the Eastern Byepass bridge and

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the Kaduna South Waterworks with Ungwan Rimi, Kabala Doki and Kigo road extension as the most critical areas.

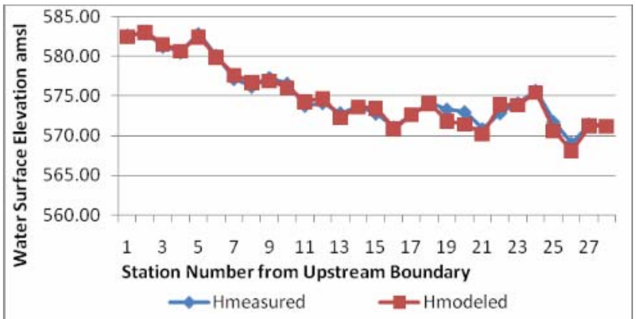

**Figure 2** Comparison of Predicted and Measured Water Stages for 5yr Flood

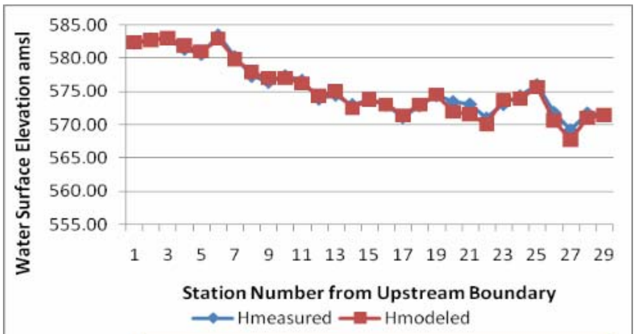

**Figure 3** Comparison of Predicted and Measured Water Stages for 10yr Flood

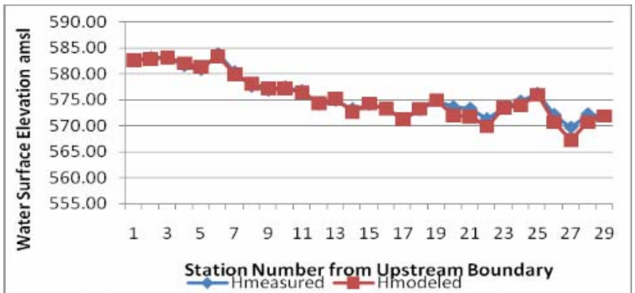

**Figure 4** Comparison of Predicted and Measured Water Stages for 25yr Flood

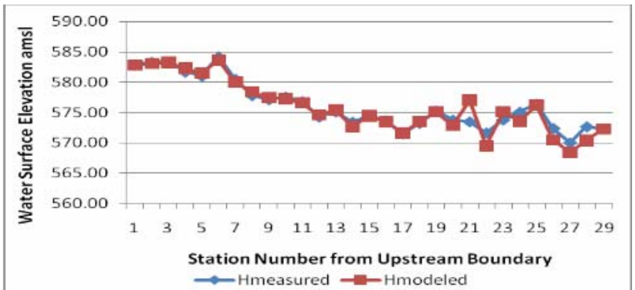

**Figure 5** Comparison of Predicted and Measured Water Stages for 50yr Flood

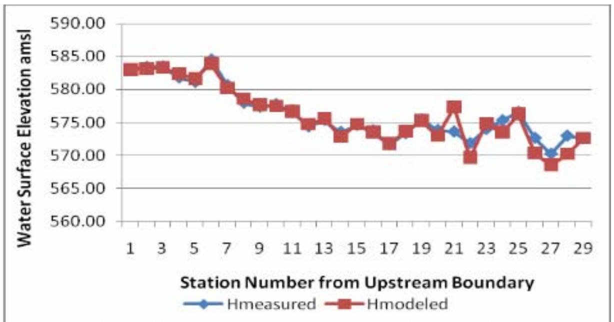

**Figure 6** Comparison of Predicted and Measured Water Stages for 100yr Flood

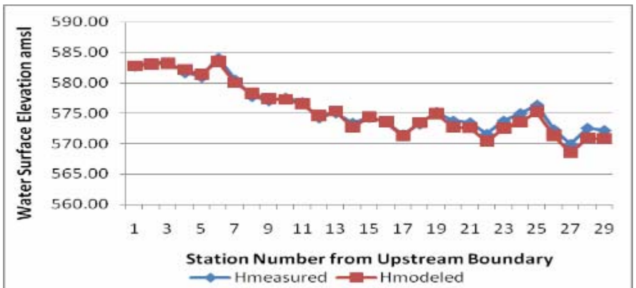

**Figure 7** Comparison of Predicted and Measured Water Stages for Year 2003 Flood

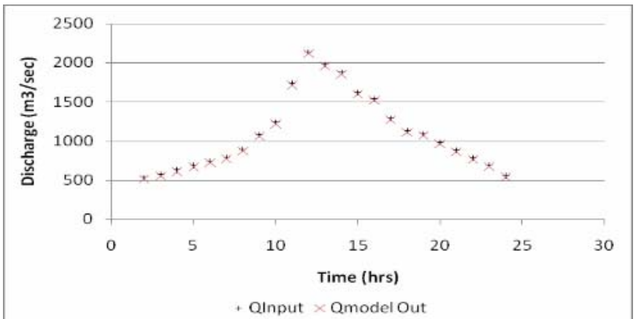

**Figure 8** Comparison of Predicted Discharge and Measured Discharge at the Kaduna South Waterworks

## 4. CONCLUSIONS

The Saint Vennat hydrodynamic equations had been modified for applications into modeling impact of urbanization on the flow of flood waters in a natural river channel onto the floodplains under variable top width of flows. The modified model was applied to the section of the Kaduna River adjoin the Kaduna city where urbanization has over the year caused the floodplain to be developed for residential accommodation, recreational and agricultural activities. This situation if persisted without proper flood protection works will endanger both lives and properties in the floodplain. Existing flood protection measures cannot and will never put to check the menace of flooding along Kaduna River. A concerted effort in the form holistic approach towards controlling the flood is urgently required. It is, therefore, recommended that the Kaduna State Government should immediately put in place a policy to regulate infrastructural development along the Kaduna floodplain as a short term measure and construct dyke along the banks to shield already developed area from flood water as a long term measure.

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