NUMERICAL SIMULATION OF HYDRODYNAMIC FIELD FROM PUMP-TURBINE RUNNER

A. Iosif and I. Sârbu
Department of Building Services, Politehnica University of Timisoara, Romania
E-Mail: ioan.sarbu@ct.upt.ro

ABSTRACT

One of the fundamental hypotheses of turbomachine design is to assume that the stream surfaces in the machine bladed zones are of revolution type. This can be obtained by accepting the flow as axisymmetric in the hypothesis of incompressible and ideal fluid and by determining the hydrodynamic field constituted by the stream lines and equipotential ones, respectively, in the meridian plane. In this paper is developed a computational model of the hydrodynamic field in the meridian plane by the boundary element technique. The results are obtained by solving a boundary-limit conditions problem for Stokes' equation for the φ velocity potential. From the connection between the functions φ and ψ is determined the stream function ψ and the velocity field along stream lines. The proposed computational model is applied to a pump-turbine runner and the numerical results are compared to those obtained by the finite element method.

Keywords: model, pump-turbine runner, numerical simulation, axisymmetric motion, boundary element method, velocity distributions.

1. INTRODUCTION

Many times, the mathematical formulation of some physical phenomena will lead to partial differential equations that together with the corresponding to the limit conditions give the so called limit problems. In most limit problems there is the impossibility of constructing analytical solutions which has led to the elaboration of numeric methods for the purpose of obtaining some approximated solutions. From this point of view the Boundary Element Method (BEM) was developed.

In this paper the dimensionless formulation in the φ potential functions will be used to provide a generality of the results. Therefore in Figure-1 in the meridian plane are presented: the analysis domains and the boundary conditions in pump case, respectively water turbine.

2. BOUNDARY ELEMENT METHOD FORMULATION

Within the BEM, taking into account the formulated hypothesis we well adopt a cylindrical coordinate system (r,θ z). The dimensionless way of dealing with the problem in the φ potential functions will impose the next change of variable and of function:

$$\begin{aligned} z^* &= zL_{ax.}^{-1} \\ r^* &= rL_{ax.}^{-1} \\ \varphi^* &= 2\pi L_{ax.}Q^{-1}\varphi \end{aligned} \tag{1}$$

where Q, $L_{ax.}$ represent the fluid flow rate and the axial extension of analysis domain.
Given that the axisymmetric domain is notated with $\overline{\Omega}^*$ and its boundary with $\overline{\Gamma}^*, d\overline{\Gamma}^* = r^* d\theta d\Gamma^*$, the following expression of integral equation on the boundary will result [5]:

$$\begin{aligned} & c(\zeta)\varphi^*(\zeta) + \int_{\Gamma^*} \varphi^*(x)\overline{q}^{**}(\zeta,x)r^*(x)\,\mathrm{d}\Gamma^* = \\ & = \int_{\Gamma^*} \frac{\partial\varphi^*(x)}{\partial n^*}\overline{\varphi}^{**}(\zeta;x)\,\mathrm{d}\Gamma^* \end{aligned} \tag{2}$$

in which Γ^* is the Ω^* domain boundary in the axial semiplane in conformity with Figure-1 and $\overline{\varphi}^{**}(\zeta,x)$ and $\overline{q}^{**}(\zeta,x)$ are the following integrals:

$$\begin{aligned} \overline{\varphi}^{**}(\zeta,x) &= \int_0^{2\pi} \varphi^{**}(\zeta,x)\,\mathrm{d}\theta(x) \\ \overline{q}^{**}(\zeta,x) &= \int_0^{2\pi} \frac{\partial\varphi^{**}(\zeta,x)}{\partial n^*}\mathrm{d}\theta(x) \end{aligned} \tag{3}$$

The fundamental $\varphi^{**}(\zeta,x)$ for Laplaceэs equation in three-dimensional case, where ζ is the source point and x is field point:

$$\varphi^{**}(\zeta,x) = \left\{ \begin{matrix} r^{*2}(\zeta) + r^{*2}(x) - 2r^*(\zeta)r^*(x)\times \\ \times\cos[\theta(\zeta)-\theta(x)] + [z^*(\zeta)-z^*(x)]^2 \end{matrix} \right\}^{-\frac{1}{2}} \tag{4}$$

allows calculating the integrals (3) knowing that the normal derivative expression of the fundamental solution is determined with no difficulty. The first integral from (3) has the form [4], [6]:

$$\overline{\varphi}^{**} = \frac{4K(m)}{(a+b)^{\frac{1}{2}}} \tag{5}$$

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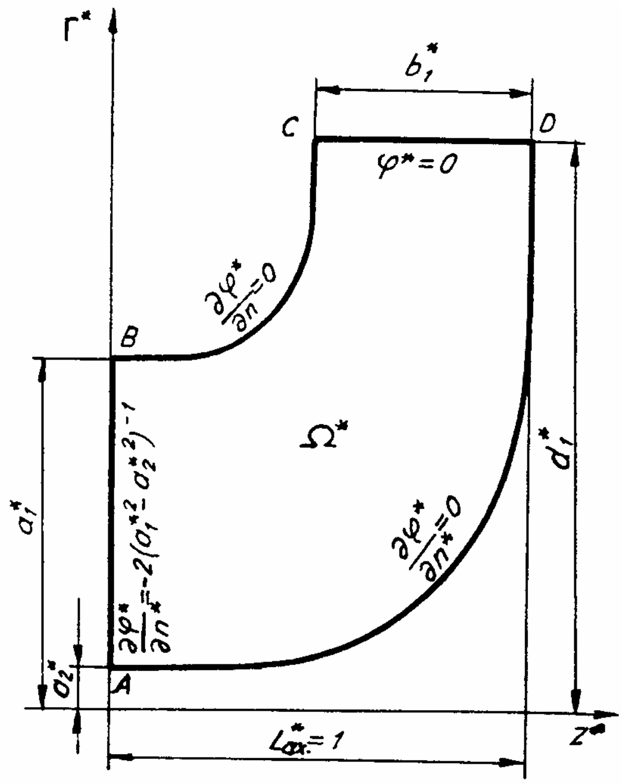
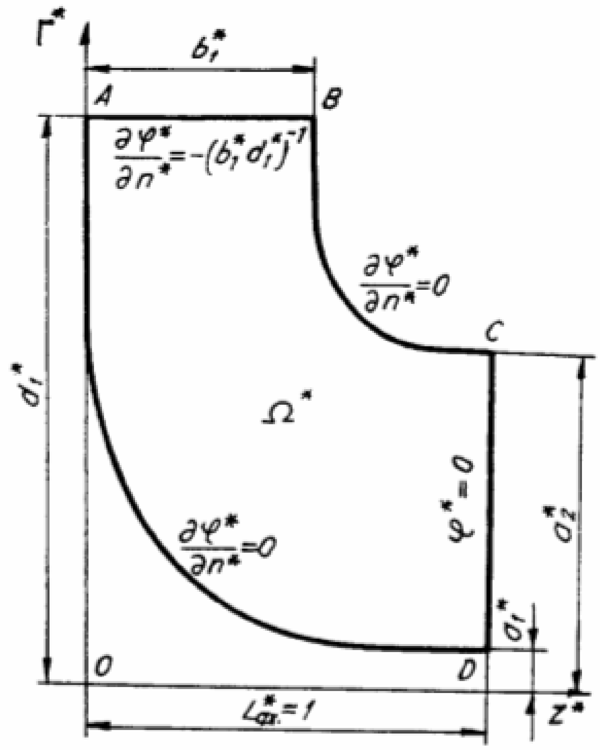


a) Pump



b) Turbine

Figure-1. Analysis domains and the boundary conditions.

in which the notation below have been used:

$$m = 2b(a+b)^{-1}$$
$$a = r^{*2}(\zeta) + r^{*2}(x) + \left[z^*(\zeta) - z^*(x)\right] \tag{6}$$
$$b = 2r^*(\zeta) r^*(x)$$

and the complet elliptic integral of first kind $K(m)$ is replaced with polinom of approximation [1], [3]:

$$K(m) = \sum_{j=0}^{4}\left[a_j m_1^j - b_j m_1^j \ln(m_1)\right] + \varepsilon(m) \tag{7}$$

where $m_1 = 1 - m$, and $\varepsilon(m)$ is the error terms.

The second integral from (3) in conformity with [5] has the expression:

$$\overline{q}^{**} = \frac{4}{(a+b)^{\frac{1}{2}}}\left\{\frac{1}{2r^*(x)}\left[\frac{r^{*2}(\zeta) - r^{*2}(x) + [z^*(\zeta) - z^*(x)]^2}{a-b}\times E(m) - K(m)\right] n^*_{r^*}(x) + \frac{z^*(\zeta) - z^*(x)}{a-b} E(m) n^*_{z^*}(x)\right\} \tag{8}$$

where $E(m)$ represents the complete elliptic integral of second kind. This is replaced with the next polinom of approximation:

$$E(m) = 1 + \sum_{j=1}^{4}\left[c_j m_1^j - d_j m_1^j \ln(m_1)\right] + \varepsilon(m) \tag{9}$$

The discretization of the Γ^* boundary in N constant boundary elements that have the Γ_j^* boundary makes the following discretized form of the (3) equation obtainable:

$$2\pi\varphi_i^* + \sum_{j=1}^{N} H_{ij}\varphi_j^* = \sum_{j=1}^{N} G_{ij}\left(\frac{\partial\varphi^*}{\partial n^*}\right)_j \tag{10}$$

The coefficients H_{ij} and G_{ij} are given by:

$$H_{ij} = \int_{\Gamma_j^*} \overline{q}^{**}(\zeta, x) r^*(x) d\Gamma^*(x)$$
$$G_{ij} = \int_{\Gamma_j^*} \overline{\varphi}^{**}(\zeta, x) r^*(x) d\Gamma^*(x) \tag{11}$$

also φ_j^* and $\left(\frac{\partial\varphi^*}{\partial n^*}\right)_j$ are notations for the values on the j element of the φ^* function and its normal derivative.

The integral equation (10) after implementation of the boundary conditions will lead to a linear system of N equations with N variables. If on the Γ_j^* boundary of the constant element j are considered for the z^*, r^* variables following parametric equations:

$$z^* = A\xi + B; \quad r^* = C\xi + D; \quad \xi \in [-1, 1] \tag{12}$$

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for coefficients G_{ij} are obtained expressions which contain integrals that can be numerically evaluated using a Gauss quadrature. These are [3], [7]:

$$G_{ij} = 2l_j \int_{-1}^{1} N(\xi)K(m)d\xi; \quad i \neq j \tag{13}$$

$$G_{ii} = 2l_j \int_{-1}^{1} K^*(m)N(\xi)\mathrm{d}\xi - - 2l_j \int_{0}^{1} H(\xi)\ln\left(\frac{1}{\xi}\right)\mathrm{d}\xi; \quad i = j \tag{14}$$

In the relation above $K^*(m)$ comes from the decomposing proposed for $K(m)$:

$$K(m) = K^*(m) + G(\xi)\ln(\xi) \tag{15}$$

The coefficients H_{ij} are calculated regarding the following expressions that can be numerically evaluated using a Gauss quadrature:

$$H_{ij} = \int_{-1}^{1} E(m)N_1(\xi)\mathrm{d}\xi - - \int_{-1}^{1} K(m)N_2(\xi)\mathrm{d}\xi; \quad i \neq j \tag{16}$$

$$H_{ii} = \int_{-1}^{1} N^*(\xi)\left[E(m) - K^*(m)\right]\mathrm{d}\xi + + \int_{0}^{1} H^*(\xi)\ln\left(\frac{1}{\xi}\right)\mathrm{d}\xi; \quad i = j \tag{17}$$

The φ_i^* values in $\forall \zeta_i \in \Omega^*$ are determinated with the help of following integral representation written under discretized form:

$$\varphi_i^* = (4\pi)^{-1}\left[\sum_{j=1}^{N} G_{ij}\left(\frac{\partial \varphi^*}{\partial n^*}\right)_j - \sum_{j=1}^{N} H_{ij}\varphi_j^*\right] \tag{18}$$

The coefficients G_{ij} and H_{ij} from (18) are determined with the relations (13) and (16) remembering that ζ is replaced with $\zeta_i \in \Omega^*$. On the basis of the BEM there were elaborated in FORTRAN programming language the computer programs FIELFR and FICTAXS for IBM-PC compatible systems. The first solves the equation (10) and the second, on the basis of the integral representation (18) determines the values φ_i^* in $\forall \zeta_i \in \Omega^*$. The it calculates, through numerical derivative the values of components $v^*_{z^*}$, $v^*_{r^*}$ belonging to the velocity v^* in $\forall \zeta_i \in \Omega^*$, determines streamlines $\psi^* = ct.$ and equipotential $\varphi^* = ct.$ also the velocity and pressure field along the streamlines taking into account the relations [8]:

$$\overline{v}^* = \frac{v^*}{v^{*AB}}; \quad \overline{p} = 1 - \overline{v}^{*2} \tag{19}$$

3. NUMERICAL RESULTS

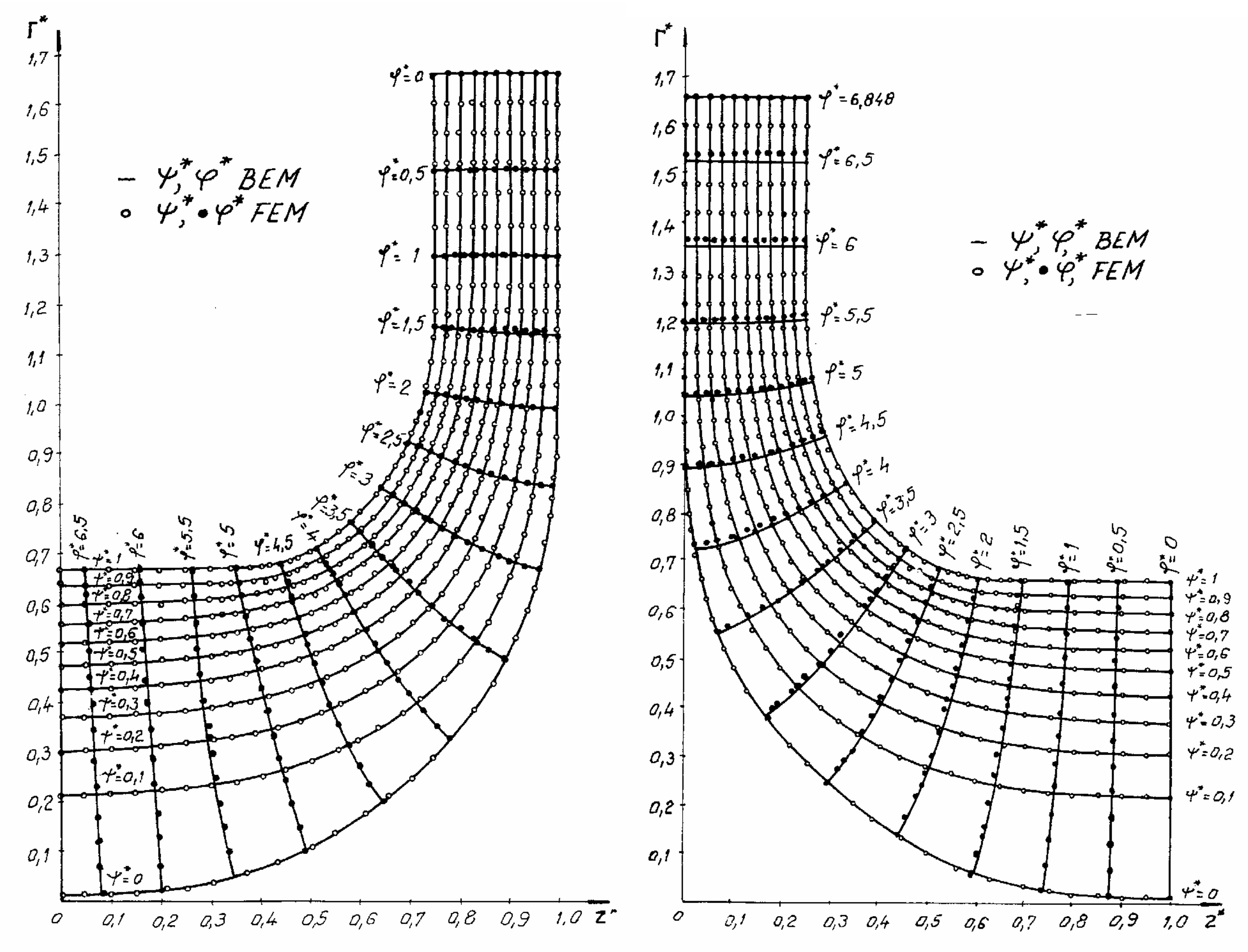
In Figures-2 is presented the hydrodynamic field in the pump and turbine cases. Notable is the fact the besides the results obtained with BEM in the figures below are also shown those calculated with the Finite Element Method (FEM) [9], [10], [11].

In Figures-3 and 4 are presented the velocity and pressure distributions along the streamlines in the pump and turbine cases. These are obtained taking into account eq. (19) with the observation that $\overline{v}^* = v^*/v^*_{ies.}$ where $v^*_{ies.}$ is the velocity corresponding to the last point on the streamlines found on the CD boundary.

From Figures 3 and 4 of interest are the values $\overline{v}^*_{max.}$, $\overline{p}_{min.}$ and length l^* of the streamline $\psi^* = 1$ also for the pump case as for the turbine case. These values are reported in Table-1. From this table is noticeable that values $\overline{p}_{min.} = -1.045$ (pump) $\overline{p}_{min.} = -0.9$ (turbine); obtained with the BEM and respectively $\overline{p}_{min.} = -0.8$; $\overline{p}_{min.} = -0.7$ computed with FEM shows that the operating in pumping regime is less convenient cavitationally. In Table-2 there were centralized the values of the $\overline{v}^{*CD}$ velocity and of the $\overline{p}^{CD}$ pressure from exit in the pump case, respectively $\overline{v}^{*AB}$, $\overline{p}^{AB}$ operating as a water turbine, values obtained with BEM and FEM for ψ^*=0.0; 0.2; 0.6; 0.8; 1.0.

Table-1. Maximal velocities and minimal pressures.

Case	ψ^*	BEM			FEM		
		$\overline{v}^*_{max.}$	$\overline{p}_{min.}$	l^*	$\overline{v}^*_{max.}$	$\overline{p}_{min.}$	l^*
Pump	1	1.43	-1.045	0.46	1.34	-0.8	0.45
Turbine	1	1.38	-0.9	1.07	1.3	-0.7	1.07



a) Pump b) Turbine

Figure-2. Hydrodynamic field.

Table-2. Velocities and pressures on the boundary CD and AB.

ψ^*	Pump				Turbine			
	BEM		FEM		BEM		FEM	
	$\overline{v}^{*CD}$	$\overline{p}^{CD}$	$\overline{v}^{*CD}$	$\overline{p}^{CD}$	$\overline{v}^{*AB}$	$\overline{p}^{AB}$	$\overline{v}^{*AB}$	$\overline{p}^{AB}$
0.0	0.524	0.725	0.52	0.73	0.690	0.650	0.595	0.646
0.2	0.524	0.725	0.52	0.73	0.565	0.680	0.570	0.675
0.4	0.524	0.725	0.52	0.73	0.540	0.710	0.545	0.703
0.6	0.524	0.725	0.52	0.73	0.530	0.720	0.530	0.720
0.8	0.524	0.725	0.52	0.73	0.525	0.724	0.525	0.724
1.0	0.524	0.725	0.52	0.73	0.520	0.730	0.520	0.729

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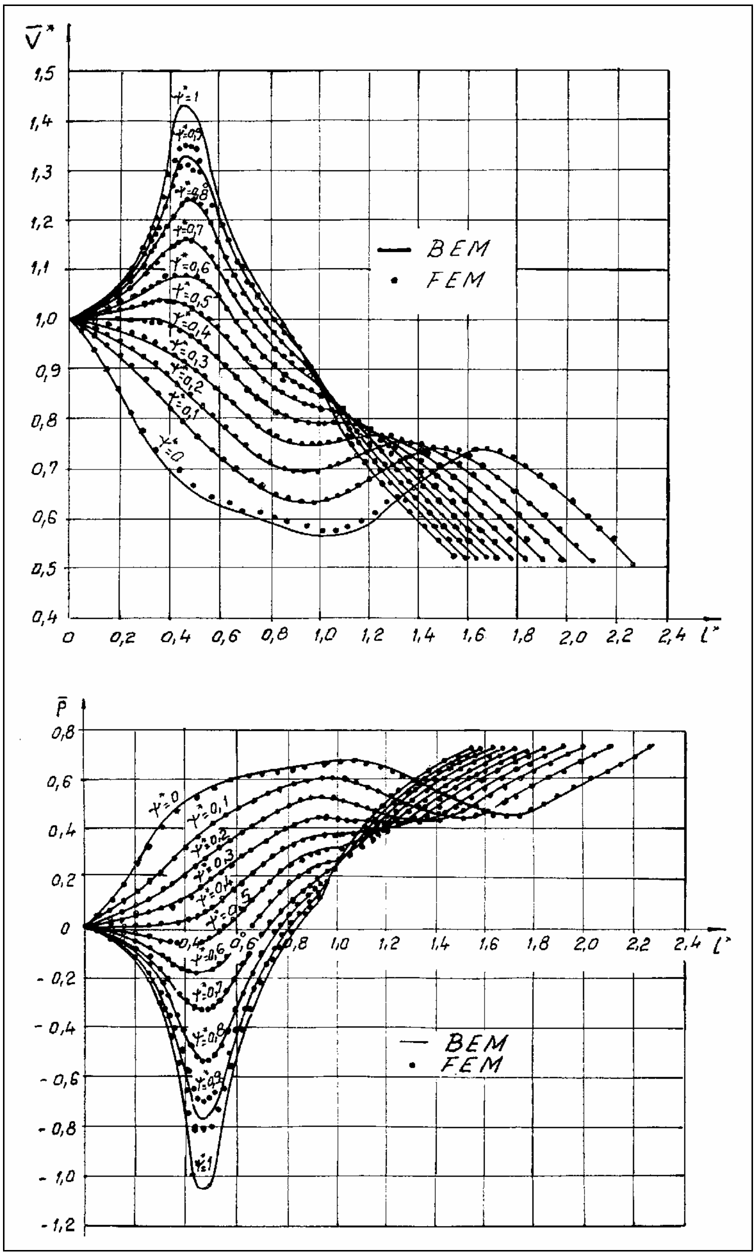

Figure-3. Velocity and pressure distributions along streamlines in the pump case.

4. CONCLUSIONS

Figures 3 and 4 suggest the way in which kinetic energy becomes potential energy, along the streamline established for the runner with no blades, meaning the field defined by the solid boundaries $\psi^* = 0$, $\psi^* = 1$ and also the input and output boundaries.

The dimensionless values of the minimum presure $\overline{p}_{\min.} = -1.045$ (pump); $\overline{p}_{\min.} = -0.9$ (turbine) obtained with BEM and $\overline{p}_{\min.} = -0.8$; $\overline{p}_{\min.} = -0.7$ obtained with FEM show that the operation in pumping regime is the most unfavourable cavitationally.

It has been observed that the values $\overline{p}^{CD} = 0.725$ (BEM); $\overline{p}^{CD} = 0.73$ (FEM) obtained in the pumping regime are the same for all ψ^* values and in the turbine case $\overline{p}^{AB} = 0.65...0.73$ (BEM); $\overline{p}^{AB} = 0.648...0.729$ (FEM) variates according to ψ^*.

The method presented offers the possibility of determining the position of the point pertaining to the streamline ψ^*=1, from which the pressure $\overline{p}_{\min.}$ is obtained. This result can be the basis for accomplishing a geometrical optimization of the solid boundary ψ^*=1, thus resulting a convenient value for $\overline{p}_{\min.}$ which would diminish as much as possible the unsuccessful operating of the runner from a cavitational point of view.

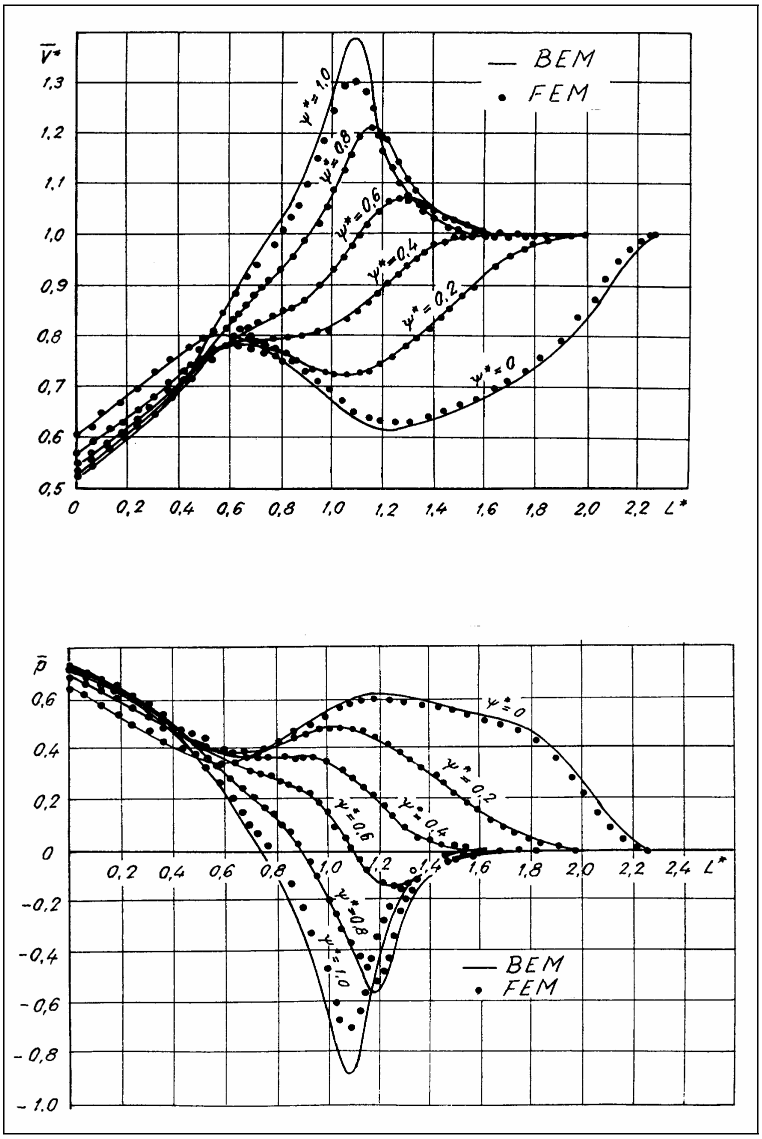

Figure-4. Velocity and pressure distributions along streamlines in the turbine case.

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