



# AN IMPROVED CREEP AND SHRINKAGE BASED MODEL FOR DEFLECTIONS OF COMPOSITE MEMBERS REINFORCED WITH CARBON FIBER REINFORCED BARS

M. A. Faruqi, S. Bhadra D. Sun, and J. Sai

Department of Civil and Architectural Engineering, Texas A & M University, Kingsville, USA

E-Mail: [m-faruqi@tamuk.edu](mailto:m-faruqi@tamuk.edu)

## ABSTRACT

Corrosion of steel and resulting degradation of concrete severely affects the serviceability and safety of structures. Millions of dollars are spent every year for the repair and rehabilitation of such structures. Carbon fiber reinforced polymers (CFRP) provide excellent choice to resist corrosion. In this paper an improved analytical model to predict the creep and shrinkage based deflections of CFRP reinforced concrete flexural members is presented. Deflection values are compared with experimental data from literature and commonly used models. Our results show that the proposed model correlates well with the experimental results and provides much better prediction than the American Concrete Institute's (ACI) procedure and a common literature model.

**Keywords:** model, structures, steel, concrete, CFRP reinforcement, flexural member, creep, shrinkage, deflection.

## 1. INTRODUCTION

Reinforced concrete structures, exposed to atmosphere are susceptible to corrosion of reinforcing steel. This subsequently affects the serviceability and safety of structures. Exposure to severe and harsh climatic conditions aggravates the situation. Retrofitting measures for the rehabilitation of structures can incur huge cost. Fiber Reinforced Polymers (FRP) provides excellent resistance to corrosion. Long term deflection and curvature are governed by creep and shrinkage. Sometimes these time-dependent deformations can be so large that it could pose a severe threat to the serviceability of the structure.

In this paper, an improved analytical model to predict the creep and shrinkage based deflections of CFRP reinforced concrete flexural members is presented. Deflection values are assessed with experimental data from literature, a commonly used literature model, and ACI procedure.

## 2. PROPOSED APPROACH

Experimental data [1] on deflection and common analytical approaches [1, 2] for predicting the time dependent behavior of CFRP reinforced concrete beams have been used as a comparison in this research. The applicability of these common models is limited due to their inability to account for relative humidity.

This basic research consists of two parts. First part involves the development of a proposed analytical model and the second part compares the model values with the experimental data and other models.

### 2.1 Proposed model

Symbols used in the model are listed in the nomenclature section. Total deflection at any particular time is the sum of instantaneous and time dependent deflections. The instantaneous deflection is computed by elastic analysis. Time dependent curvature and deflections have been determined using creep and shrinkage

coefficients. Section properties have been determined using age adjusted elastic modulus method. Mean curvature at the end of loading period is:

$$\psi(t_0) = (1 - \lambda_1)\psi_1 + \lambda_1\psi_2 \quad [1]$$

$$\lambda_1 = \begin{cases} 1 - 0.75 \left( \frac{M_{cracked}}{M_{applied}} \right) \\ 0 \end{cases}$$

For a cracked and uncracked section

$$\psi_1 = M_a / E_c I_{gross}$$

$$\psi_2 = M_a / E_c I_{effective}$$

Parabolic variation is assumed for instantaneous central deflection. The approximate instantaneous maximum deflection at center is provided by:

$$\Delta_i = \frac{\psi(t_0)L^2}{50} \quad [2]$$

Mean curvature at time  $t_s$  is provided by:

$$\psi(t_s) = (1 - \lambda_2)(\psi_1 + \Delta\psi_1) + \lambda_2(\psi_2 + \Delta\psi_2) \quad [3]$$

$$\lambda_2 = \begin{cases} 1 - 0.25 \left( \frac{M_{cracked}}{M_{applied}} \right)^2 \\ 0 \end{cases}$$

Change of curvature is provided by:

$$\Delta\psi_1 = \alpha [\varphi(t, t_s)\psi_1 + \varepsilon_{cs}(t, t_s)y_c / r_c^2] \quad [4]$$



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$$\Delta \psi_2 = \alpha [\varphi(t, t_s) \psi_2 + \varepsilon_{cs}(t, t_s) y_c / r_c^2] \quad [5]$$

Age adjusted section properties are:

$$\alpha = I_{cr} / I_{effective} \quad [6]$$

$$I_{cr} = bc_{tr}^3 / 3 + A_f^{tr} (c_{tr} - d')^2 + A_f^{tr} (d - c_{tr})^2 \quad [7]$$

$c_{tr}$  values are obtained by setting the following equation to zero:

$$bc_{tr}^2 / 2 + A_f^{tr} (c_{tr} - d') - A_f^{tr} (d - c_{tr}) = 0$$

$$A_f^{tr} = \{2\eta'(t, t_s) - 1\} A_f'$$

$$A_f^{tr} = \eta'(t, t_s) A_f' \quad [8]$$

$$\eta'(t, t_s) = E_{CFRP} / E_c'(t, t_s)$$

$$E_c'(t, t_s) = 57000 \sqrt{f_c'(t_s)}$$

$$f_c'(t_s) = f_c(28) \{t_s / (4 + 0.85t_s)\}$$

$$y_c = c_{tr} / 2; r_c^2 = I_{cr} / A_c; A_c = bc_{tr}$$

$$I_{effective} = (M_{cr} / M_a)^3 I_{gross} + [1 - (M_{cr} / M_a)^3] I_{cr}$$

Creep calculations are as follows:

$$\varphi(t, t_s) = \varphi_0 \beta_c(t, t_s)$$

$$\varphi_0 = \varphi_{RH} \beta(f_{cm}) \beta(t_0)$$

$$\varphi_{RH} = 1 + \left\{ (1 - RH / RH_0) / 0.46 (h_e / h_0)^{1/3} \right\} \quad [9]$$

$$h_e = 2A_g / u$$

$$\beta(f_{cm}) = 5.3 / (f_{cm} / f_{cmo})^{0.5}$$

$$\beta(t_0) = 1 / \{0.1 + (t_0 / t_1)^{0.2}\}$$

$$\beta_c(t, t_s) = \left[ \left\{ (t_s - t_0) / t_1 \right\} / \left\{ \beta_H + (t_s - t_0) / t_1 \right\} \right]^{0.3}$$

$$\beta_H = 150 \left[ 1 + (1.2RH / RH_0)^{18} \right] (h_e / h_0) + 250 \leq 1500$$

Shrinkage calculations are as follows:

$$\varepsilon_{cs}(t, t_s) = \varepsilon_{cso} \beta_s(t, t_s)$$

$$\varepsilon_{cso} = \varepsilon_s(f_{cm}) \beta_{RH}$$

$$\varepsilon_s(f_{cm}) = 1.2 [160 + \beta_{sc} (9 - f_{cm} / f_{cmo})] 10^{-6} \quad [10]$$

$$\beta_{sc} = 50 \text{ for type I cement}$$

$$\beta_{RH} = -1.55 \left[ 1 - (RH / RH_0)^3 \right]$$

Long term deflection at mid-span is provided by:

$$\Delta_l = \psi(t_s) L^2 \times 10^3 / 30 + \ln(t_s) (RH) 0.20 \quad [11]$$

### 3. COMPARISON OF DEFLECTION VALUES

Figure-1 and Table-1 summarize the results of beam B1, while Figure-2 and Table-2 summarize the results of beam B2. To accommodate common humidity ranges in this area, lower and upper values of 50% and 80% were considered. For beam B1, the proposed ratios range from 1.02 to 1.16 as compared to 2.66 to 5.97 for common models. In the case of beam B2, the proposed ratios range from 0.98 to 1.29 as compared to 2.61 to 7.55 range for other models. Figures 1 and 2 show similar conclusions.

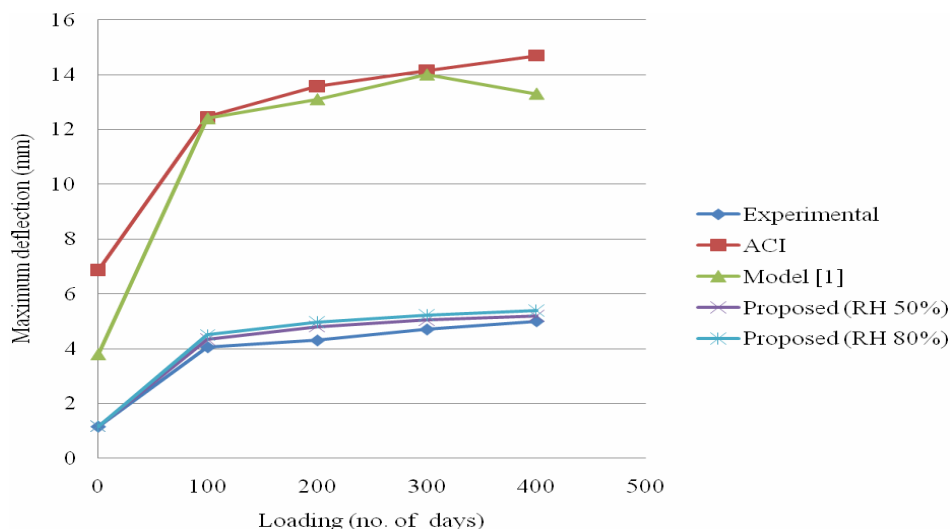
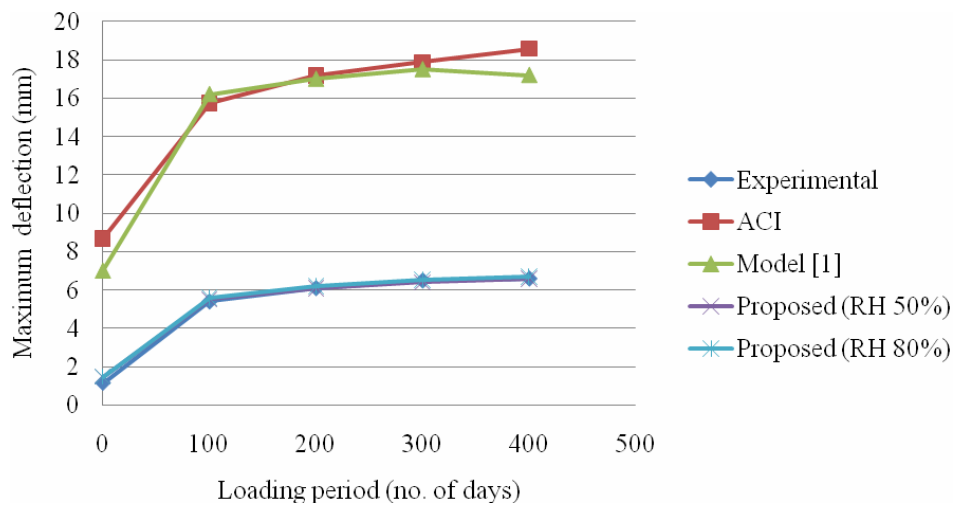


Figure-1. Graphical comparison of deflection values for beam B1.

**Table-1.** Comparison of deflection ratios for beam B1.

Days	$\Delta_{ACI} / \Delta_{Exp}$	$\Delta_{Model [1]} / \Delta_{Exp}$	$\Delta_{Prop [50\%]} / \Delta_{Exp}$	$\Delta_{Prop [80\%]} / \Delta_{Exp}$
0	5.97	5.09	1.02	1.02
100	3.08	3.00	1.07	1.11
200	3.16	3.05	1.12	1.16
300	3.01	2.98	1.07	1.11
400	2.94	2.66	1.04	1.08

**Figure-2.** Graphical comparison of deflection values for beam B2.**Table-2.** Comparison of deflection ratios for beam B2.

Days	$\Delta_{ACI} / \Delta_{Exp}$	$\Delta_{Model [1]} / \Delta_{Exp}$	$\Delta_{Prop [50\%]} / \Delta_{Exp}$	$\Delta_{Prop [80\%]} / \Delta_{Exp}$
0	7.55	6.43	1.29	1.29
100	2.92	3.00	1.02	1.04
200	2.81	2.79	1.00	1.02
300	2.75	2.69	0.98	1.00
400	2.82	2.61	1.00	1.02

#### 4. CONCLUSIONS AND FUTURE WORK

In this paper an improved analytical model to predict the creep and shrinkage based deflections of CFRP reinforced flexural concrete members is presented. Deflection values are assessed with experimental data from literature, a commonly used literature model and ACI procedure. Based on this study, the following conclusions can be drawn:

- Comparison of analytical predictions with the experimental data and other models show that the proposed model correlates better with the experimental values.
- The rate of increase of deflection is higher in the initial loading period and tends to reduce with respect to long-term.

An improved analytical model to predict the creep and shrinkage based deflections of CFRP reinforced concrete members is introduced. Good agreement is observed between analytical predictions and experimental results. This model can be used as a design tool to predict the long term deflections of CFRP reinforced concrete members. However, more experimental and analytical work is needed to further validate this approach. Future research may include:

- Use of ambient atmospheric temperature and shape coefficients for different fiber reinforced polymers in the model.



### Nomenclature

$L$	Length of beam	$\psi(t_s)$	Curvature of the member at time $t_s$
$M_{applied}$	Applied bending moment	$\psi_1$	Instantaneous curvature for a cracked section
$M_{cracked}$	Cracking moment	$\psi_2$	Instantaneous curvature for an uncracked section
$t_0$	28 days (curing period of the CFRP reinforced concrete beams)	$\Delta\psi$	Time dependent change in curvature due to creep and shrinkage
$t_s$	Number of days after the curing period is over	$\Delta\psi_1$	Change in curvature for a cracked section
$t_1$	day 1	$\Delta\psi_2$	Change in curvature for an uncracked section
$E_c$	Modulus of elasticity of concrete	$\varphi(t, t_s)$	Time dependent creep coefficient
$E_{CFRP}$	Modulus of elasticity of CFRP	$\varepsilon_{cs}(t, t_s)$	Time dependent shrinkage coefficient
$E'_c(t, t_s)$	Age adjusted modulus of elasticity of concrete	$\eta'(t, t_s)$	Age adjusted modular ratio
$C_{tr}$	Neutral axis depth of the age adjusted cracked section	$f'_c(t, t_s)$	Age adjusted mean compressive strength of concrete
$y_c$	Half of the neutral axis depth of the age adjusted cracked section	$\varepsilon_{cso}$	Shrinkage strain for a particular concrete strength and relative humidity
$r_c^2$	Radius of gyration of the age adjusted cracked section	$\varepsilon_s(f_{cm})$	Coefficient that accounts for 28 day concrete compressive strength
$I_{gross}$	Gross moment of inertia of the section	$\beta(f_{cm})$	Coefficient that accounts for 28 day concrete compressive strength
$I_{cr}$	Moment of inertia of the age adjusted cracked section	$\beta_s(t, t_s)$	Coefficient that accounts for shrinkage
$I_{effective}$	Effective moment of inertia of the age adjusted cracked section	$\beta_c(t, t_s)$	Coefficient that accounts for creep
$b$	Width of the section	$\beta_H$	Coefficient that takes into account the effect of relative humidity for creep
$d$	Depth of the tension reinforcement from the top of the section	$\beta_{RH}$	Coefficient that takes into account the effect of relative humidity for shrinkage
$d'$	Depth of the compression reinforcement from the top of the section	$\beta_{sc}$	Coefficient that accounts for cement type
$A_g$	Gross area of the cross section	$\beta(t_0)$	Coefficient that accounts for time $t_0$
$u$	Perimeter of the cross section	$f_{cm}$	Mean compressive strength of concrete at 28 days
$A_f$	Area of tension CFRP reinforcement	$f_{cmo}$	10 MPa (constant value)
$A'_f$	Area of compression CFRP reinforcement	$h_0, h_e$	Parameters that takes into account cross-sectional dimension
$A_f^{tr}$	Transformed area of the equivalent tension CFRP reinforcement	$\varphi_0$	Basic creep coefficient
$A_f'^{tr}$	Transformed area of the equivalent compression CFRP reinforcement	$\varphi_{RH}$	Relative humidity coefficient
$A_c$	Area of concrete in compression	$\alpha$	Curvature reduction factor
$h_0$	Parameter that takes into account cross-sectional dimension at $t_0$	$RH$	Relative humidity
$\lambda_1$	Coefficient of instantaneous curvature	$\Delta_i$	Instantaneous deflection
$\lambda_2$	Coefficient of long time curvature	$\Delta_L$	Long term deflection
$\psi$	Instantaneous curvature of bending	$RH_0$	100% relative humidity
$\psi(t_0)$	Mean curvature at time $t_0$		

**REFERENCES**

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