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STABILITY ANALYSIS OF CIRCULAR PRESSURE DAM HYDRODYNAMIC JOURNAL BEARING WITH COUPLE STRESS LUBRICANT

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ABSTRACT

A generalized Reynolds equation has been derived for carrying out the stability analysis of a plain circular pressure dam hydrodynamic bearing operating with couple stress fluids. The Galerkin form of the finite element method has been used to solve the equation obtained. A non-dimensional parameter, '*l*' has been used to indicate the length of the long chain polymer added to the bulk Newtonian fluid. The dynamic characteristics, which have been calculated for a wide range of dam parameters, have been found to be greatly influenced with the variation of the couple stress parameter '*l*'. The critical mass of the journal, obtained as a solution to the linearized equations of motion is used to demonstrate the increased stability of the journal bearing system.

Keywords: journal bearing, couple stress fluid, dynamic analysis, stability

Notation

∇	gradient operator
$(\dot{\ })$	differenttaion with respect to dimensionless time 't'
B	body force vector
c	radial difference between the journal and the lobe
C	body couple vector
$\overline{C}_{mn}$	non dimensional damping coefficients, $\partial\overline{W}_m/\partial\dot{\overline{n}}$, (m,n=x,z)
e	eccentricity
ε	eccentricity ratio e/c
$\overline{h}$	non dimensional film thickness, $\overline{h}=1+\varepsilon\cos\theta$
$\overline{K}_{mn}$	non dimensional stiffness coefficients, $\partial\overline{W}_m/\partial\overline{n}$,(m,n=x,z)
l	couple stress parameter, $l=(\eta/\mu)^{1/2}$
L	length of the bearing
$\overline{M}_j,\overline{M}_c$	non dimensional mass and critical mass of the journal
p	pressure
$\overline{p}$	non dimensional film pressure, $pc^2/\mu\omega R^2$
R	radius of the journal
U,V	tangential and normal velocity component of the journal surface
$\overline{W},\overline{W}_h,\overline{W}_v$	bearing load and bearing load along the horizontal and the vertical directions
$\overline{x},\overline{y},\overline{z}$	non dimensional coordinate, x/R, y/c, z/R
u, v, w	fluid velocity along the x, y and z axis
V	velocity vector
η	material constant responsible for the couple stress property
μ	viscosity coefficient of a Newtonian lubricant
ρ	density of fluid
ϕ	attitude angle
ω	angular velocity
β	slenderness ratio, L/2R=1
10^0	Size of oil supply groove
θ_s	Dam location
L_d	Dam width
h_1-h_2	Dam depth= 3 times clearance, c
L_t	Width of relief track

1. INTRODUCTION

Plain circular hydrodynamic bearing, due to their ease in manufacturing and maintenance, are the most common form of bearing found in extensive use even today. These bearings have however been found to be unstable under light load and high speed conditions. Increasing the stability of these bearings has been a focus of investigation of various researchers and several means have been suggested ranging from minor changes in the geometry of the bearing to alterations in the lubricant. It has been reported by Nicholas [1, 2] that a minor geometrical alteration in the form of a dam and a relief track in the upper and the lower half of the bearing can significantly improve the stability of the plain circular bearing. Among other authors, improvements in stability with dam for two lobe, three lobe, four lobe bearings etc have also been reported by Mehta *et al*., [3-13].

A number of studies have been also carried out using the Stokes microcontinuum theory [14] to investigate the effects of the couple stress parameter, '*l'*, on the stability of different types of fluid film bearings. These studies include work done by Lin [15-17] and Mokhiamer *et al.,* [18], on rolling-element bearings by Sinha and Singh [19] and Das [20]. The characteristics of pure squeeze film bearings has been analyzed by Ramanaiah and Sarkar [21], Ramanaiah [22], Bujurke [23], Lin [24, 25] while slider bearings have been studied by Ramanaiah [26] and Bujurke *et al*., [27].

It has been noticed that literature concerning the stability of plain circular pressure dam bearings is available only for the Newtonian fluids and to the best of the author's knowledge no information is available dealing with the application of couple stress fluids in this area.

The present study is concerned with the stability analysis of a hydrodynamically lubricated plain circular pressure dam bearing. A range of dam variables comprising of its width, depth and length and a recess width have been taken as independent variables. Two values of couple stress parameter '*l*', 0.0 (Newtonian fluid) and 0.2, are taken. All the results have been plotted for an eccentricity ratio of 0.5.

2. ANALYSIS

2.1 Basic equations

From the Stokes micro-continuum theory, for an incompressible fluid with couple stresses

Momentum Equation:

$$\rho \frac{DV}{Dt} = -\nabla p + \rho F + \frac{1}{2}\rho \nabla x C + \mu \nabla^2 V - \eta \nabla^4 V \tag{1}$$

Continuity Equation:

$$\nabla V = 0 \tag{2}$$

Where the vectors V, F and C denote the velocity, body force and body couple per unit mass, respectively; p is the pressure, ρ is the density, μ is the classical viscosity, and η is a new material constant peculiar to fluids with couple stresses.

Figure-1 presents the geometry and co-ordinate system used for analysis of a circular pressure dam bearing with a journal of radius R rotating with angular velocityω. Assume that the fluid film is thin, body forces and body couples are absent, and fluid inertia is small as compared to the viscous shear. Then the field equations governing the motion of the lubricant given in Cartesian coordinates reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} \tag{4}$$

$$\frac{\partial p}{\partial y} = 0 \tag{5}$$

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} \tag{6}$$

Boundary conditions at the surface of bearing (y = h)

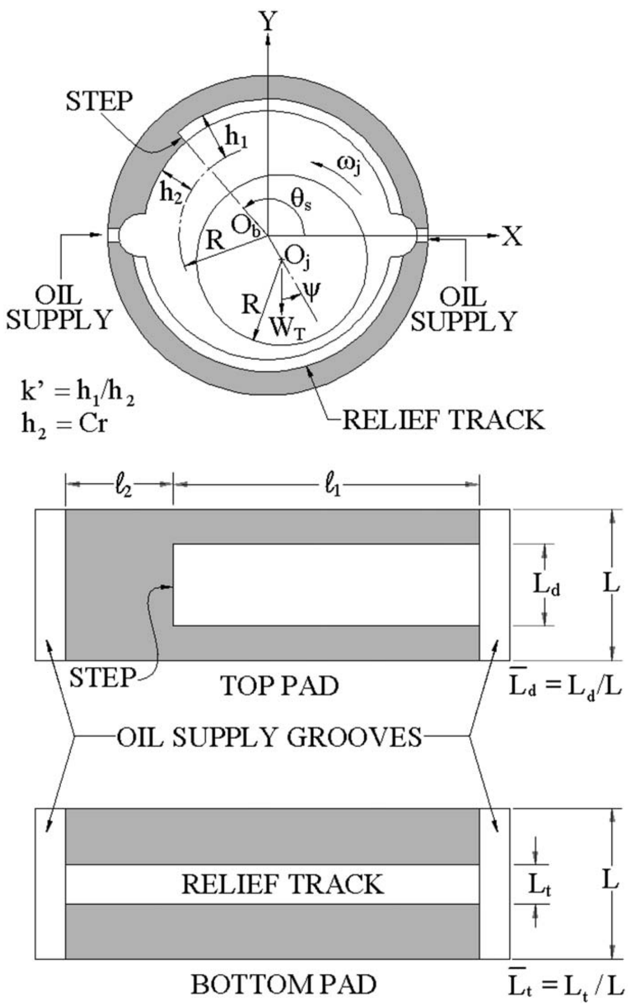

Figure-1. Geometry and coordinate system used.

$$u(x,h,z) = 0\,,\; v(x,h,z) = 0\,,\; w(x,h,z) = 0$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=h} = 0\,, \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=h} = 0 \tag{7}$$

Boundary conditions at the surface of Journal (y = 0)

$$u(x,0,z) = U\,,\; v(x,0,z) = V\,,\; w(x,0,z) = 0$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = 0\,, \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=0} = 0 \tag{8}$$

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Where U and V are tangential and normal velocity components of the journal surface at an angular position, θ.

Integrating equation (4) and integrating equation (6) with respect to 'y' and applying the boundary conditions, we get the modified Reynolds equation as:

$$\frac{\partial}{\partial x}\left(\frac{h^3-12l^2\left[h-2l\tanh\left(\frac{h}{2l}\right)\right]}{6\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3-12l^2\left[h-2l\tanh\left(\frac{h}{2l}\right)\right]}{6\mu}\frac{\partial p}{\partial z}\right)=6U\mu\frac{\partial h}{\partial x}+6\mu h\frac{\partial u}{\partial x}+12\mu V$$

Now introducing non dimensional variables and parameters, dimensionless Reynolds Equation can be written as

$$\frac{\partial}{\partial \bar{x}}\left(f(\bar{h},\bar{l})\frac{\partial \bar{p}}{\partial \bar{x}}\right)+\frac{1}{4\beta^2}\frac{\partial}{\partial \bar{z}}\left(f(\bar{h},\bar{l})\frac{\partial \bar{p}}{\partial \bar{z}}\right)=6\frac{\partial \bar{h}}{\partial \bar{x}}+\frac{12}{\omega}\frac{\partial \bar{h}}{\partial \bar{t}}$$

Where

$$f(\bar{h},\bar{l})=\bar{h}^3-12\bar{l}^2\left[\bar{h}-2\bar{l}\tanh\left(\frac{\bar{h}}{2\bar{l}}\right)\right] \tag{10}$$

The hydrodynamic pressure field in the journal bearing is established by solving equation (10) satisfying the following boundary and symmetry conditions:

$$\left.\frac{\partial \bar{p}}{\partial \bar{x}}\right|_{(\bar{x}=\bar{x}2,\bar{z}=0)}=0\,,\ \left.\frac{\partial \bar{p}}{\partial \bar{z}}\right|_{(\bar{z}=0)}=0\,,\ \bar{p}\left(\bar{x}2,\bar{z}\right)=0\,,$$

$$\bar{p}(0,\bar{z})=0\,,\ \bar{p}\left(\bar{x},\pm\frac{1}{2}\right)=0 \tag{11}$$

2.2 Film thickness

For an aligned bearing the fluid film thickness of the n[th] lobe is expresses as:

$$h^n=1-(X_J-X_L^n)\cos\theta-(Z_J-Z_L^n)\sin\theta$$

Where $(X_J,Z_J)=(\varepsilon\sin\phi,-\varepsilon\cos\phi)$, represents the coordinates of the journal centre and $(X_L{}^n,Z_L{}^n)$ represents the centers of the lobes.

2.3 Bearing performance characteristics

2.3.1. Load carrying capacity

The fluid film reaction components in 'y' and 'x' directions are given by

$$\bar{W}x=\int\bar{p}\cos\theta.d\bar{\theta}.d\bar{y}$$

$$\bar{W}y=\int\bar{p}\sin\theta.d\bar{\theta}.d\bar{y} \tag{12}$$

For vertical load support, the following conditions are satisfied in the journal centre equilibrium position

$$\overline{W_v}=W \text{ And } \overline{W_h}=0 \tag{13}$$

2.3.2 Attitude angle

Angle between the load line and the line joining the bearing and journal centers is defined as the attitude angle. The attitude angle is established by satisfying the condition of vertical load support, Equation (13), using an iterative process for a given load or eccentricity ratio.

2.3.3 Stiffness coefficients

If the journal center is displaced from its static equilibrium position, then in general, the difference in the components of the fluid-film reactions in the disturbed position and in the static equilibrium position gives the out of balance components of the fluid-film force. The non-dimensional fluid-film stiffness coefficients are defined as

$$\begin{bmatrix}\bar{K}xx & \bar{K}xz\\ \bar{K}zx & \bar{K}zz\end{bmatrix}=-\begin{bmatrix}\partial\bar{W}x/\partial\bar{x} & \partial\bar{W}x/\partial\bar{z}\\ \partial\bar{W}_z/\partial\bar{x} & \partial\bar{W}z/\partial\bar{z}\end{bmatrix} \tag{14}$$

The first subscript of the stiffness coefficients denotes the direction of force and the second, the direction of displacement.

2.3.4 Damping coefficients

The damping coefficients are defined as

$$\begin{bmatrix}\bar{C}xx & \bar{C}xz\\ \bar{C}zx & \bar{C}zz\end{bmatrix}=-\begin{bmatrix}\partial\bar{W}x/\partial\bar{u} & \partial\bar{W}x/\partial\bar{v}\\ \partial\bar{W}_z/\partial\bar{u} & \partial\bar{W}z/\partial\bar{v}\end{bmatrix} \tag{15}$$

The first subscript of the damping coefficients denotes the direction of force and the second, the velocity.

2.3.5 Critical mass and whirl ratio

The linearized equation of motion for the journal centre can be written as:

$$M_j^*\ddot{x}^*+C_{xx}^*\dot{x}^*+C_{xy}^*\dot{y}^*+K_{xx}^*x^*+K_{xy}^*y^*=0$$

$$M_j^*\ddot{y}^*+C_{yy}^*\dot{y}^*+C_{yx}^*\dot{x}^*+K_{yy}^*y^*+K_{yx}^*x^*=0 \tag{17}$$

Taking a harmonic equation of the form

$$x^*=x^*e^{\lambda t^*}$$

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$$y^* = y^* e^{\lambda t^*}$$

Here $\lambda = \eta + i\upsilon$ represents a complex frequency. If η<0, the system is stable. On the threshold of frequency, η=0, and x and y are pure harmonic motions with frequency given as $\lambda = i\upsilon$. Equation (16) can now be written as:

$$\begin{bmatrix} K_{xx}^* - M^* \upsilon^2 + i\upsilon C_{xx}^* & K_{xy}^* + i\upsilon C_{xy}^* \\ K_{yx}^* + i\upsilon C_{yx}^* & K_{yy}^* - M^* \upsilon^2 + i\upsilon C_{yy}^* \end{bmatrix} \begin{bmatrix} x^* \\ y^* \end{bmatrix} = 0 \quad (18)$$

For a nontrivial solution the determinant of the equation (18) must vanish. Equating the real and the imaginary parts to zero gives:

$$M^* \gamma^2 = \frac{C_{xx}^* K_{yy}^* + C_{yy}^* K_{xx}^* - C_{xy}^* K_{yx}^* - C_{yx}^* K_{xy}^*}{C_{xx}^* + C_{yy}^*} \quad (19)$$

$$\gamma^2 = \frac{\left(K_{xx}^* - M^* \gamma^2\right)\left(K_{yy}^* - M^* \gamma^2\right) - K_{yx}^* K_{xy}^*}{C_{xx}^* C_{yy}^* - C_{xy}^* C_{yx}^*} \quad (20)$$

Equation (19) and equation (20) are solved to obtain the critical mass of the journal and the whirl ratio.

3. SOLUTION SCHEME

Equation (10) is solved by Finite Element Method using Galerkin's Approach. The domain is discretized into elements and the condition of mesh convergence has been satisfied. The finite element equations contributed by all the elements have been assembled in the global stiffness and the global force matrix which has been modified to meet the boundary conditions mentioned in equation (11).

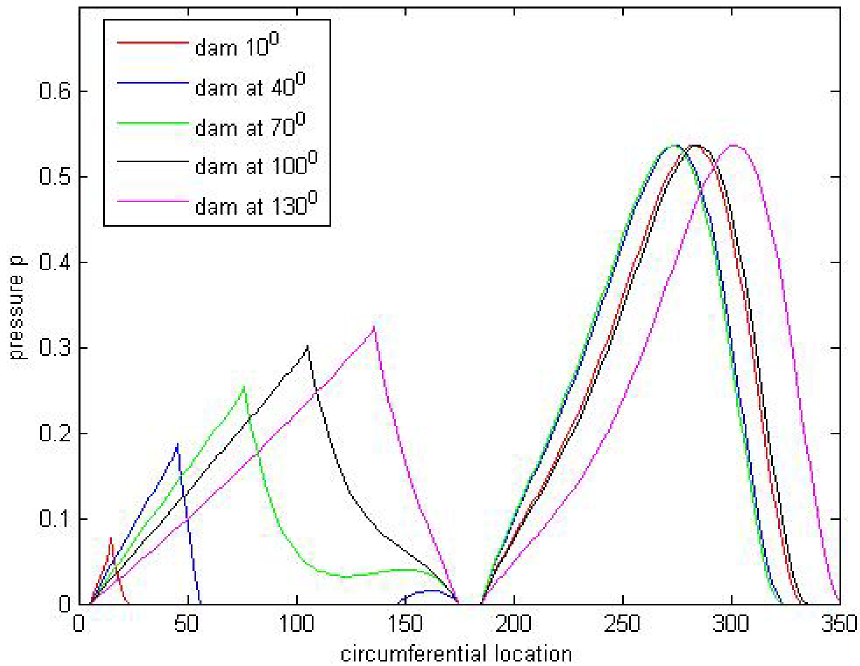

Figure-2(a). Pressure distribution at the dam width of 1.4 for *l*=0.0 (Newtonian fluid).

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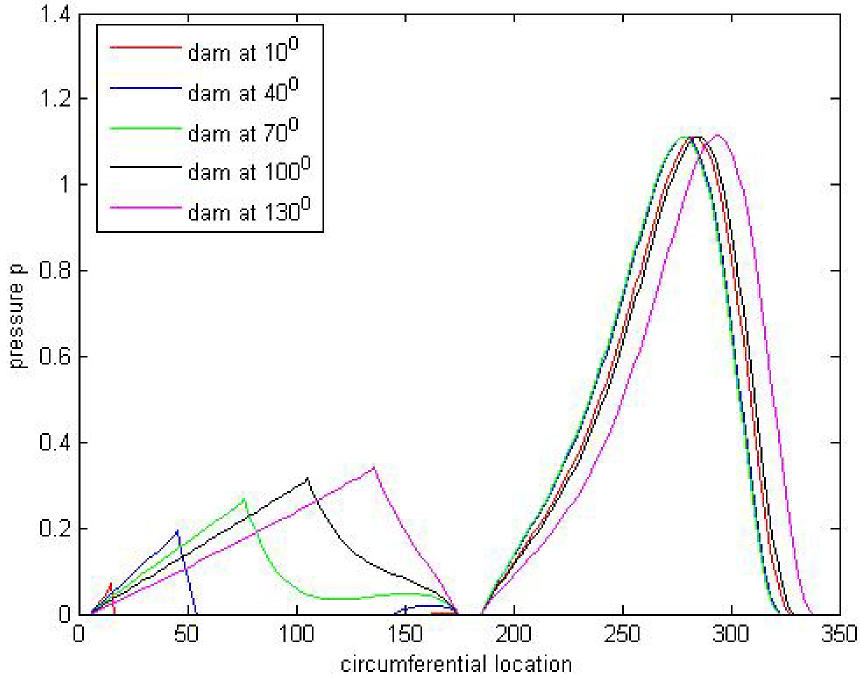

Figure-2(b). Pressure distribution at the dam width of 1.4 for *l*=0.2.

The Reynolds boundary condition has been satisfied by adjusting the trailing edge of the fluid film. For any eccentricity ratio, the above process is repeated to get an attitude angle for vertical load at the equilibrium condition. The journal from this state is perturbed to obtain the stiffness and the damping coefficients (equations 14-15).

The bearing is treated as two independent lobes containing a cavity cut in the upper lobe and a recess created in the lower lobe of the bearing. It is assumed that the refief track is deep enough to divide the lower lobe in two parts, each to be treated independently.

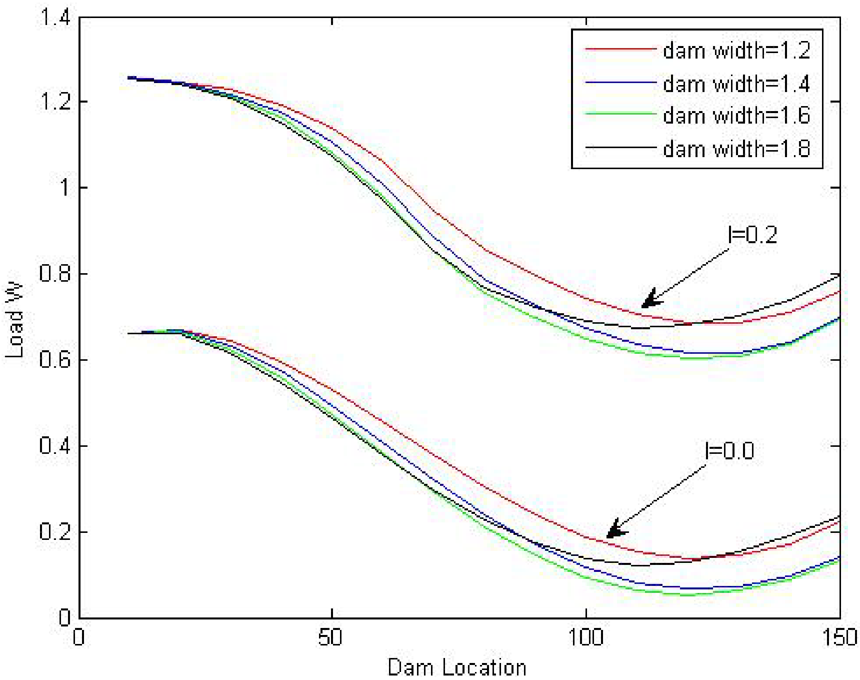

Figure-3. Bearing load for various dam locations at dam width of 1.4 for *l*=0.0 and *l*=0.2.

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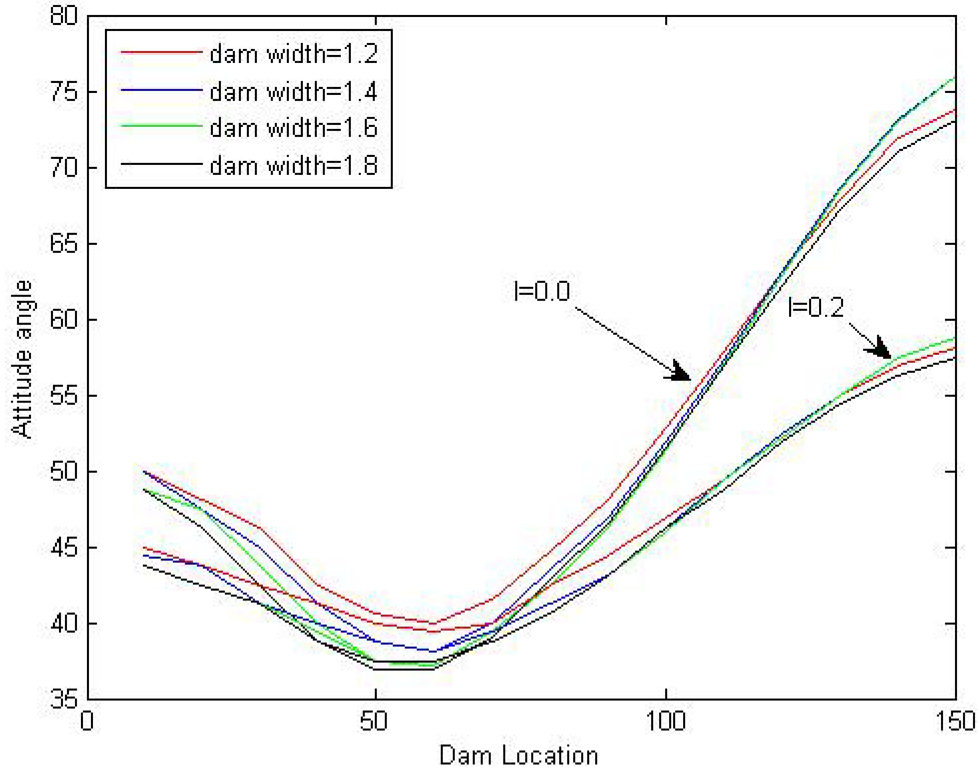

Figure-4. Attitude angle for various dam locations for l=0.0 and l=0.2.

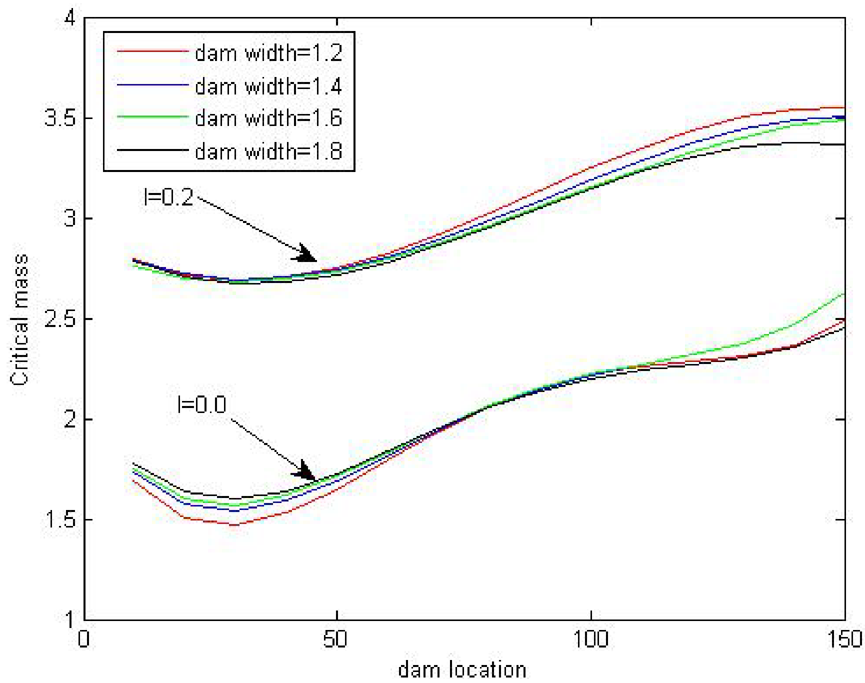

Figure-5. Critical mass for various dam locations for l=0.0 and l=0.2.

4. RESULTS

In the present study the effect of couple stress parameter 'l' on the dynamic performance of plain circular pressure dam bearing is analyzed.

The results obtained for the plain circular pressure dam bearing are plotted for two values of couple stress factor, 0 (Newtonian fluid) and 0.2. Figure-2(a) shows the variation of pressure developed in the upper and the lower pad along the central axis for various locations of the pressure dam for a Newtonian fluid. The presence of the relief track in the lower pad divides it into two identical halves and thus the pressure shown is along the

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central axis of one half only. Figure-2(b) shows the pressure variation at a couple stress value of 0.2. The location of the dam in the upper lobe clearly affects the magnitude of the pressure developed. The maximum pressure developed increases as the dam location shifts from 10^0 to 100^0 but no appreciable change in observed beyond this location. A similar pattern is observed in Figure-2(b) for a couple stress value of 0.2.

It should be noted that for any given dam location, the pressure developed in the upper lobe remains unaffected as the value of couple stress increases from 0.0 to 0.2 while in the lower pad the magnitude of pressure developed is twice for the same values of couple stress parameter *l*. This can be explained as the thickness of fluid film is much more in the upper lobe, three times the clearance, any introduction of the couple stress fluid does not produces any change in the pressure developed. However in the lower lobe, where the film thickness is only about half the clearance (the bearing is operating at an eccentricity of 0.5), the pressure developed is influenced by the couple stress parameter '*l*' to a greater extent. This can also be explained with the help of equation (10) which shows a reduced influence of cpuple stress parameter '*l*'at higher fluid film thickness.

Figure-2(a-b) also shows the shift in location of the peak pressure about the vertical axis with the variation of the dam location. Starting from 285^0 for the case of dam located at 10^0, the peak shifts to the left to 270^0 for dam location of 70^0 and then to 300^0 at the dam location of130^0. A similar variation in the spread of the positive pressure zone can also be noticed. These two factors are primarily responsible in variation of bearing load and attitude angle as shown in Figures 3 and 4.

A shift in the dam location increases the maximum pressure in the upper lobe which acts in the downwards direction (i.e., opposite to the force produced by the lower lobe), and thus reduces the net bearing load. This behavior is observed till a dam angle of almost 100^0 after which the force produces by the lower lobe increases due to the increased pressure spread and thus increases the bearing capacity as seen in Figures 3 and 4.

Figure-5 shows the variation of the critical mass at various dam locations for two values of couple stress parameter, *l*=0.0 and *l*=0.2. These results have been plotted for four values of dam width, i.e., 1.2, 1.4, 1.6 and 1.8. It is noticed that at any particular value of couple stress parameter '*l*', the critical mass shows an increasing trend specifically after a dam location of 50^0. This increase in the critical mass is however found to be less than that which can be obtained by the introduction of couple stress fluid. Variation in the dam width however does not produce appreciable increase in the value of critical mass.

5. DISCUSSIONS

Important inferences can be made about the use of couple stress fluid in plain circular pressure dam bearings and are listed below.

a) The use of couple stress fluid increases the load capacity which means that for any given load the bearing system will be operating at a lower eccentricity ratio.
b) The use of dams in plain circular bearings increases its stability as is evident by the increase in the critical mass.
c) The couple stress fluid can be used for decreasing the value of the attitude angle as well as increasing the stability of the bearing.
d) The use of couple stress fluid is found to increase the stability of the bearing to a larger extent when compared to the effect produced by dams.

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