VOL. 5, NO. 10, OCTOBER 2010 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

A RECURSIVE PROCEDURE FOR COMMENSAL- HOST ECOLOGICAL MODEL WITH REPLENISHMENT RATE FOR BOTH THE SPECIES - A NUMERICAL APPROACH

N. Phani Kumar[1] C.Srinivasa kumar[2]and N.Ch. Pattabhi Ramacharyulu[3]
[1]Faculty of Mathematics, Malla Reddy Engineering College, Hyderabad, A.P. India
[2]Principal, Gopal Reddy College of Engineering & Technology, A.P, India,
[3]Mathematics (Retd) National Institute of Technology, Warangal, A.P. India
E-Mail: phanikumar_nandanavanam@yahoo.com

ABSTRACT

In this paper a four stage recursive procedure is adopted to give an approximate solution to the mathematical model equations of a commensal-host ecological model with replenishment rate for both species. Numerical examples are discussed to explain the trajectories of the solutions and the results are illustrated.

Keywords: commensal, host, replenishment rate, recursive procedure.

INTRODUCTION

The analytical solutions for the first order non-linear coupled differential equations are not possible directly because of interactability of non-linear terms. Techniques like perturbation to linearizing the non-linear coupled differential equations to give qualitative nature of solutions in treatises of Kapoor [3], Meyer [5] Cushing [2], Phanikumar N, Pattabhi Ramacharyulu N.Ch [6] *et al.*

In the present investigation we present a four stage recursive procedure to give an approximate solution for a first order non linear coupled differential equations and same is applied to solve the commensal-host ecological model with replenishment rate for both the species. A similar algorithm was earlier developed by Langlosis and Rivlin [4] to obtain approximate solution for the flows of slightly visco- elastic fluids.

Srinivas N.C. [7] and Bhaskara Rama Sarma [1] adopted this algorithm for a general two species eco system. This paper is divided into two sections. In section - I, the general four stage recursive procedure is presented for solving a first order non - linear differential equations. In section - II the recursive procedure is applied to a commensal- host ecological model with replenishment rate for both species. Some numerical illustrations have been carried out for a wide range of the system characterizing parameters a_1, a_2, a_{11}, a_{22}, a_{12} for this matlab has been used and the results are illustrated.

Section-I

The general recursive procedure with four stage approximation for first order non-linear coupled differential equations:

Consider the system of non-linear coupled differential equations is of the form

$$\dot{\overline{N}}(t) = A\,\overline{N}(t) + f(\overline{N}, t) + \overline{h}(t) \qquad (1)$$

Where

$$\dot{\overline{N}}(t) = \frac{d\overline{N}}{dt} \text{ with initial conditions } \overline{N}(0) = \overline{N_0} \geq 0 \qquad (2)$$

$A\overline{N}(t)$: Linear dependence term of the system.

$f(\overline{N}, t)$: Non Linear dependence term of the system.

$\overline{h}(t)$: Replenishment / renewal rates of the system.

The recursive procedure for solving the system (1) is presented in the following four stages.

Stage -1: Consider the linear system of (1)

$$\dot{\overline{N}}(t) = A\,\overline{N}(t) \qquad (3)$$

With the initial condition $\overline{N}(0) = \overline{N_0}$

The above system is obtained by suppressing the non-linear part and in the absence of replenishment te on the right hard side of (1)

Let the solution of (3) together with (2) be

$$\overline{N^{(1)}(t)} = N_O e^{AT} \qquad (4)$$

Stage-2: Compute the replenishment rate that could sustain the above solution $\overline{N^{(1)}(t)}$ in the non-Linear part of the system (1)

$$\overline{h}(t) = \dot{\overline{N}}(t) - A\,\overline{N^{(1)}(t)} - f(\overline{N^{(1)}}, t) \qquad (5)$$

From (3), equation (5) becomes

$$\overline{h}(t) = -f(\overline{N^{(1)}}, t) \qquad (6)$$

Stage-3: Consider the system

$$\dot{\overline{N}}(t) = A\,\overline{N}(t) - f(\overline{N^{(1)}}, t) \qquad (7)$$

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

With the homogeneous initial conditions $\overline{N}(0)$ =0 (8)

Let $\overline{N^{(2)}}(t)$ be the solution of the system (7) together with (8)

Stage-4: The approximate solution of non-linear system (1) with (2) is given by

$$\overline{N}(t) = \overline{N^{(1)}}(t) - \overline{N^{(2)}}(t) \quad (9)$$

Where $\overline{N^{(1)}}(t)$ and $\overline{N^{(2)}}(t)$ are the solutions obtained by stage-1 and stage-2.

Section- II

In this section we discussed the approximate solution of commensal - host ecological model with replenishment rate for both the species S_1 and S_2. Here both the species S_1 and S_2 with limited resources are considered.

Notation Adopted:

N_i : Population of Species S_i, 1 = 1, 2 at time t

a_i : Natural growth rates of species S_i, i = 1, 2

a_{ii} : Rates of decrease of species S_i, i = 1, 2 due to limited resources

a_{12} : inhibition coefficient

h_i : Replenishment rates of S_i, I = 1,2

All these Co-efficients a_1, a_2, a_{11}, a_{22}, h_1, h_2 > 0

Consider the system

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^{\,2} + a_{12}N_1N_2 + h_1$$

$$\frac{dN_2}{dt} = a_2N_2 - a_{22}N_2^{\,2} + h_2 \quad (10)$$

With initial conditions

$N_i(0) = N_{i0}$; i = 1, 2 (11)

Now the recursive procedure explained in section - I is carried out in the foregoing analysis of the system (10)

Stage-1: Taking the system (10) with linear terms along with initial conditions (11) we get

$$\frac{dN_1}{dt} = a_1 N_1$$

$$\frac{dN_2}{dt} = a_2 N_2 \quad (12)$$

and $N_1(0) = N_{10}$; $N_2(0) = N_{20}$ (13)

Now the solution of (12) with (13) is given by

$N_1^{(1)} = N_{10}\, e^{a_1 t}$ (14)

$N_2^{(1)} = N_{20}\, e^{a_2 t}$ (15)

Stage-2: By using (14) and (15) the replenishment rates h_1 and h_2 of (10) are calculated as

$h_1(t) = a_{11} N_{10}^2\, e^{2 a_1 t} -\!- a_{12} N_{10} N_{20}\, e^{(a_1 + a_2) t}$

$h_2(t) = a_{22} N_{20}^2\, e^{2 a_2 t}$ (16)

Stage-3: Using (16) along with homogeneous initial conditions we setup the linear system

$$\frac{dN_1}{dt} = a_1 N_1 + h_1(t)$$

$$\frac{dN_2}{dt} = a_2 N_2 + h_2(t) \quad (17)$$

with initial conditions $N_1(0) = 0$; $N_2(0) = 0$ (18)

The solution of the system (17) along with initial conditions (18) are

$$N_1^{(2)} = N_{10}\, e^{a_1 t} \left\{ \frac{a_{11}N_{10}}{a_1}(e^{a_1 t} - 1) + \frac{a_{12}N_{20}}{a_2}(1 - e^{a_2 t}) \right\}$$

$$N_2^{(2)} = N_{20}\, e^{a_2 t} \left\{ \frac{a_{22}N_{20}}{a_2}(e^{a_2 t} - 1) \right\} \quad (19)$$

Stage-4: An approximate solution for the system 10 is given by

$N_1 = N_1^{(1)} - N_1^{(2)}$

$N_2 = N_2^{(1)} - N_2^{(2)}$ then (20)

$$N_1 = N_{10}\, e^{a_1 t} \left\{ 1 - \left[\frac{a_{11}N_{10}}{a_1}(e^{a_1 t} - 1) + \frac{a_{12}N_{20}}{a_2}(1 - e^{a_2 t}) \right] \right\}$$

$$N_2 = N_{20}\, e^{a_2 t} \left\{ 1 - \frac{a_{22}N_{20}}{a_2}(e^{a_2 t} - 1) \right\} \quad (21)$$

A numerical solution of the basic non-linear coupled differential equations: The variation of N_1 and N_2 verses time‘t’:

The variation of N_1 and N_2 verses‘t’ in the internal (0, 1) are computed numerically by employing recursive procedure with four stage approximation. Technique for a wide range of the system characterizing parameters a_1, a_2; a_{11}, a_{22}; a_{12}, a_{20} for this matlab has been used and results are illustrated.

VOL. 5, NO. 10, OCTOBER 2010 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Table

1	**N_{10}=1**	**N_2=0.5**	**a_{12}=1**	**a_1=1**	**a_2=0.5**	**a_2=0.5**	**a_{11}=1**	**a_{22}: 0.5,1,1.5,2**
2	N_{10}=1	N_2=0.5	a_{12}=1	a_1=1	a_2=1	a_2=1	a_{11}=1	a_{22}:0.5,1,1.5,2
3	N_{10}=1	N_2=0.5	a_{12}=1	a_1=1	a_2=1.5	a_2=1.5	a_{11}=1	a_{22}:0.5,1,1.5,2
4	N_{10}=1	N_2=0.5	a_{12}=1	a_1=1	a_2=0.5	a_2=0.5	a_{11}=1	a_{22}:0.5,1,1.5,2
5	N_{10}=1	N_2=0.5	a_{12}=0.5	a_1=1	a_2=0.5	a_2=0.5	a_{11}=1	a_{22}:0.5,1,1.5,2
6	N_{10}=1	N_2=0.5	a_{12}=1	a_1=0.5	a_2=0.5	a_2=0.5	a_{11}=1	a_{22}:0.5,1,1.5,2
7	N_{10}=1	N_2=0.5	a_{12}=1	a_1=2	a_2=1	a_2=1	a_{11}=1	a_{22}:0.5,1,1.5,2
8	N_{10}=1	N_2=0.5	a_{12}=1.5	a_1=2	a_2=1.5	a_2=1.5	a_{11}=1	a_{22}:0.5,1,1.5,2

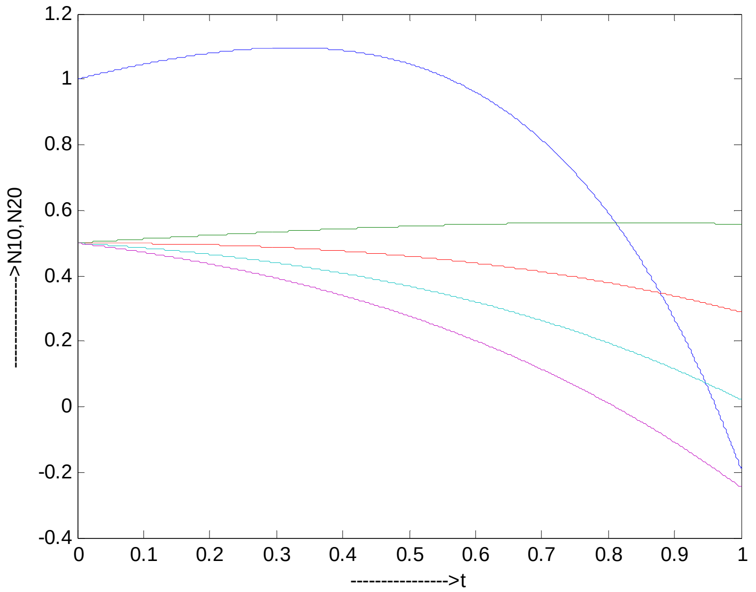
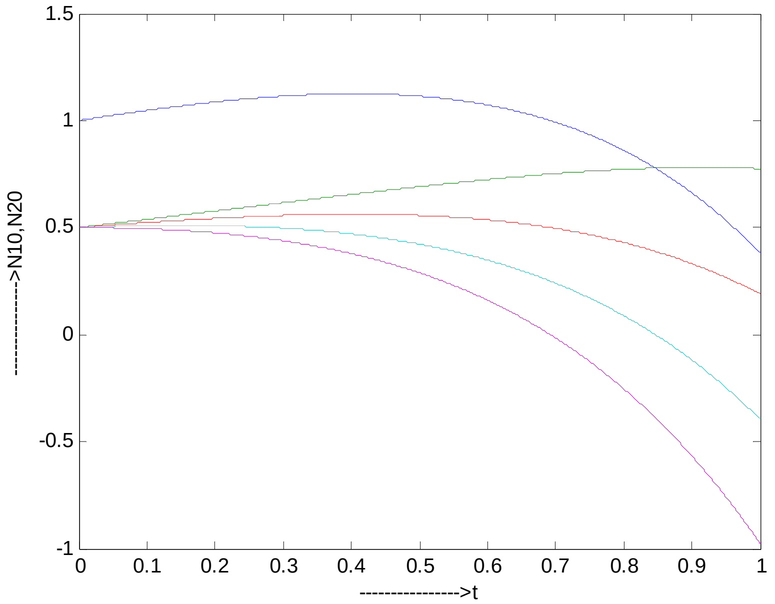


Figure-1. N_{10}=1, N_20 = 0.5, a_{12}=1, a_1=1, a_2=0.5 a_{11}=1 Variation of N_1 and N_2 verses time t.



 ISSN 1819-6608

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Figure-2. N10=1, N20 = 0.5, a12=1, a1=1, a2=1 a11=1 Variation of N1 and N2 verses time t.

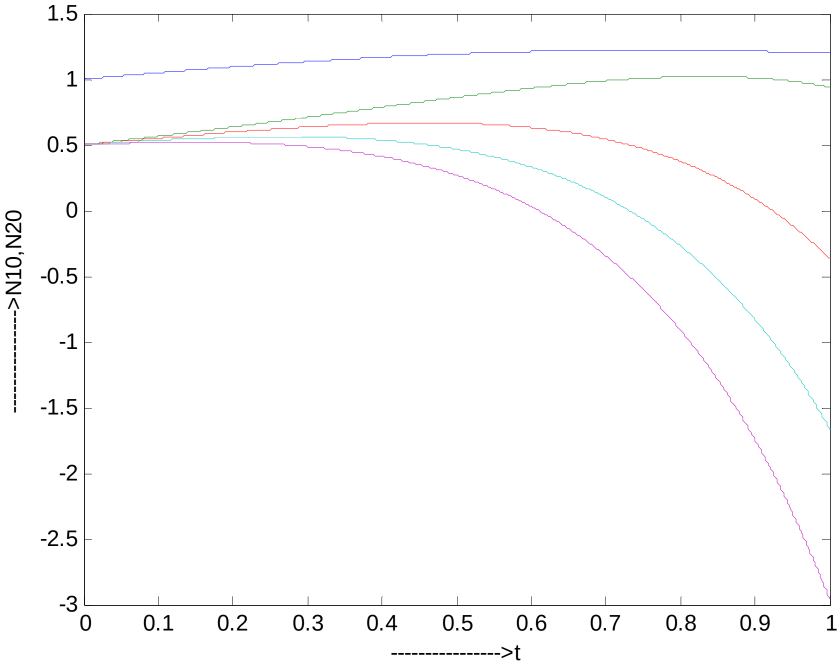


Figure-3. N_{10}=1, N_{20} = 0.5, a_{12}=1, a_1=1, a_2=1.5 a_{11}=1 Variation of N_1 and N_2 verses time t.

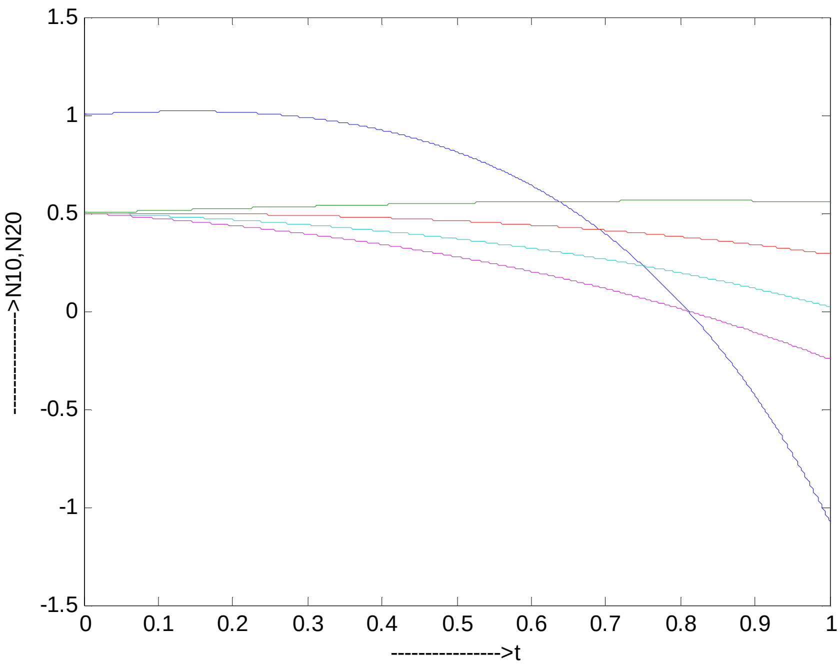


Figure-4. N_{10}=1, N_{20} = 0.5, a_{12}=0.5, a_1=1, a_2=0.5 a_{11}=1 Variation of N_1 and N_2 verses time t.

ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

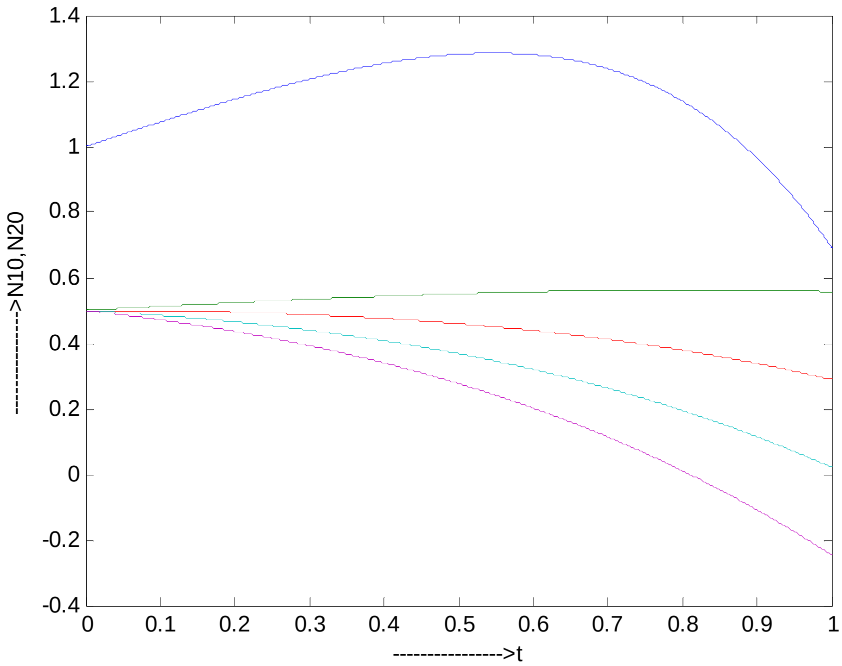


Figure-5. N_{10}=1, N_{20} = 0.5, a_{12}= 1, a_1=1, a_2=0.5 a_{11}=1 Variation of N_1 and N_2 verses time t.

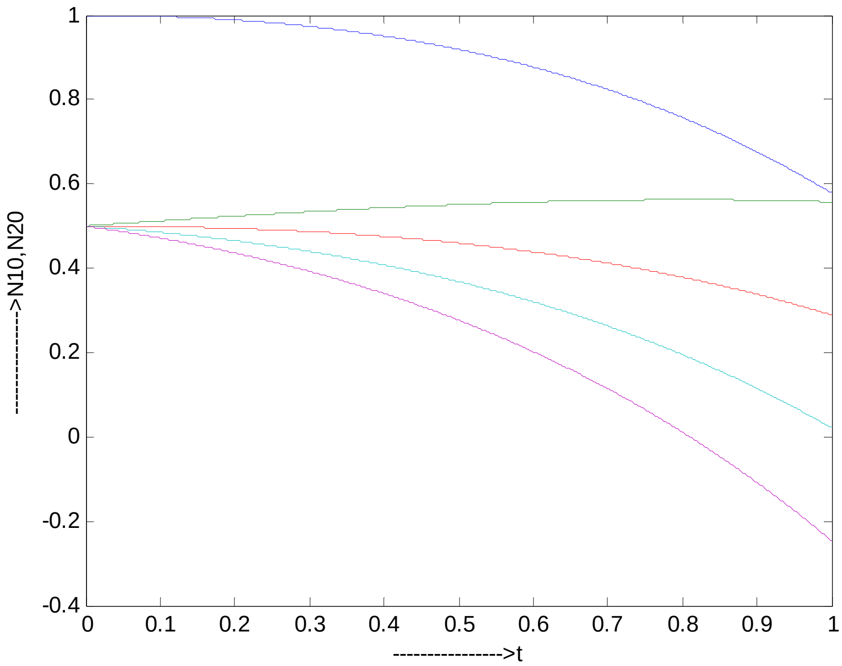


Figure-6. N_{10}=1, N_{20} = 0.5, a_{12}= 1, a_1=0.5, a_2=0.5 a_{11}=1 Variation of N_1 and N_2 verses time t.

VOL. 5, NO. 10, OCTOBER 2010 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

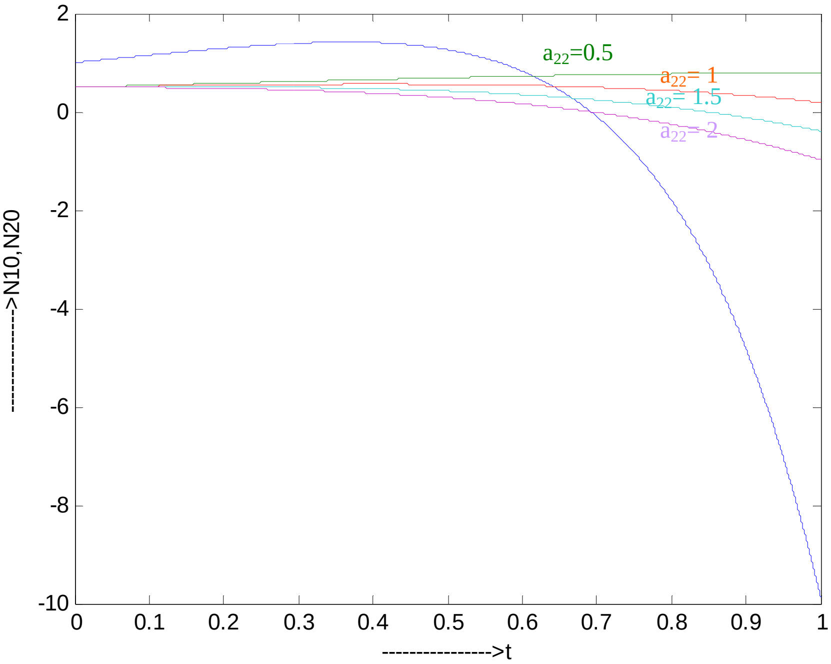


Figure-7. N_{10}=1, N_{20} = 0.5, a_{12}= 1, a_1=2, a_2=1 a_{11}=1 Variation of N_1 and N_2 verses time t.

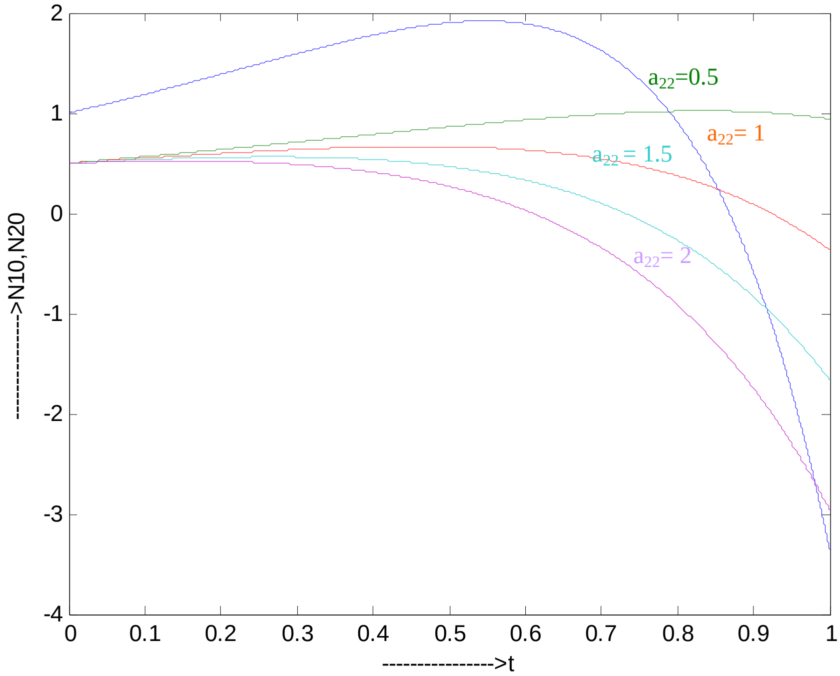


Figure-8. N_{10}=1, N_{20} = 0.5, a_{12}= 1.5, a_1=2, a_2=1.5 a_{11}=1 Variation of N_1 and N_2 verses time t.

CONCLUSIONS

Following conclusions are derived from the graphs (Figure-1 to Figure-8)

a) If growth rate of host increases the N_2 curves are falling with increasing steepness.
b) Both N_1 and N_2 are decreases with time‘t’ because of utilization of energy.
c) For a_{12} < 1 weak comensalism of host over comensal, N_2 falls with slower rate than N_1. However with increasing growth rate of host steepness increases for the N_2 curves and falling rate is decreased comparison with N_1. With increasing a_{22} steepness decreases.
d) For a_{12} =1 the steepness of N_2 increases with a_2 however falling of N_2 is faster along with increasing a_{22} than N_1

ARPN Journal of Engineering and Applied Sciences

e) For a_{12}>1 strong commenslism of host over commensal it is observed that N_1 increasing to some extent and then start falling. This tendency is observed at slow rate in case of $a_{12} = 1$.
f) Because of growth rate of S_1, N_1 falls much faster than N_2. However this fall becomes slower in case of $a_{12} > 1$

Trend index	N_1	N_2	Steepness of N_2
a_2 ↑	↓	↓	↑
a_{22} ↑	↓	↓	↓
a_{12} ↑	↓	↓	↑

REFERENCES

[1] Bhaskara Rama Sarma. B, Pattabhi Ramacharyulu N.Ch and Lalitha. S. V. N. 2009. A Recursive procedure for two species competing Eco system with decay and replenishment for one Species. Acta ciencia Indica. Xxxv M. (2): 487-496.

[2] Cushing J.M. 1977. Integro - differential equations and delay models in population Dynamics. Lect. Notes in Biomathematics. Vol. 20, springer - reflag, Heidelbegh 9.

[3] Kapur J.N. 1985. Mathematical models in Biology and Medicine. Affiliated East-West 9.

[4] Langlois W.E and Rivlin R.S. 1957. Steady flow of slightly visco-elastic fluids. (Doctoral thesis of W-Elanglois), Brown universality.

[5] Meyer W.J. 1985. Concepts of Mathematical modeling. Mc-Grawhill.

[6] Phanikumar N and Pattabhi Ramacharyulu N. Ch. On the stability of a commesal-host harvested species pair with limited resources. Communicated to International journal of computational cognition.

[7] Srinvias N.C. 1991. Some mathematical aspects of modeling in Bio-medical sciences. Ph.D. thesis, submitted to Kakatiya University.