A RECURSIVE PROCEDURE FOR COMMENSAL- HOST ECOLOGICAL MODEL WITH REPLENISHMENT RATE FOR BOTH THE SPECIES - A NUMERICAL APPROACH

N. Phani Kumar¹ C.Srinivasa kumar² and N.Ch. Pattabhi Ramacharyulu³
¹Faculty of Mathematics, Malla Reddy Engineering College, Hyderabad, A.P. India
²Principal, Gopal Reddy College of Engineering & Technology, A.P, India,
³Mathematics (Retd) National Institute of Technology, Warangal, A.P. India
E-Mail: phanikumar_nandanavanam@yahoo.com

ABSTRACT
In this paper a four stage recursive procedure is adopted to give an approximate solution to the mathematical model equations of a commensal-host ecological model with replenishment rate for both species. Numerical examples are discussed to explain the trajectories of the solutions and the results are illustrated.

Keywords: commensal, host, replenishment rate, recursive procedure.

INTRODUCTION

In the present investigation we present a four stage recursive procedure to give an approximate solution for a first order non linear coupled differential equations and same is applied to solve the commensal-host ecological model with replenishment rate for both the species. A similar algorithm was earlier developed by Langlosis and Rivlin [4] to obtain approximate solution for the flows of slightly visco- elastic fluids.

Srinivas N.C. [7] and Bhaskara Rama Sarma [1] adopted this algorithm for a general two species eco system. This paper is divided into two sections. In section - I, the general four stage recursive procedure is presented for solving a first order non - linear differential equations. In section - II the recursive procedure is applied to a commensal- host ecological model with replenishment rate for both species. Some numerical illustrations have been carried out for a wide range of the system characterizing parameters $a_1, a_2, a_{11}, a_{22}, a_{12}$ for this matlab has been used and the results are illustrated.

Section-I
The general recursive procedure with four stage approximation for first order non-linear coupled differential equations:

Consider the system of non-linear coupled differential equations is of the form

$$\dot{\bar{N}}(t) = \Lambda \bar{N}(t) + f(\bar{N}, t) + \bar{h}(t)$$

Where

$$\bar{N}(t) = \frac{d \bar{N}}{dt}$$

with initial conditions $\bar{N}(0) = \bar{N}_0 \geq 0 \ (2)$

$\Lambda \bar{N} (t)$ : Linear dependence term of the system.

$f(\bar{N}, t)$ : Non Linear dependence term of the system.

$\bar{h} (t)$ : Replenishment / renewal rates of the system.

The recursive procedure for solving the system (1) is presented in the following four stages.

Stage -1: Consider the linear system of (1)

$$\dot{\bar{N}}(t) = \Lambda \bar{N}(t)$$

With the initial condition $\bar{N}(0) = \bar{N}_0$

The above system is obtained by suppressing the non-linear part and in the absence of replenishment te on the right hard side of (1)

Let the solution of (3) together with (2) be

$$\bar{N}^{(1)}(t) = N_0 e^{\Lambda t}$$

(4)

Stage-2: Compute the replenishment rate that could sustain the above solution $\bar{N}^{(1)}(t)$ in the non-Linear part of the system (1)

$$\bar{h}(t) = \bar{N}(t) - \Lambda \bar{N}^{(1)}(t) - f(\bar{N}^{(1)}, t)$$

(5)

From (3), equation (5) becomes

$$\bar{h}(t) = -f(\bar{N}^{(1)}, t)$$

(6)

Stage-3: Consider the system

$$\dot{\bar{N}}(t) = \Lambda \bar{N}(t) - f(\bar{N}^{(1)}, t)$$

(7)
With the homogeneous initial conditions \( N(0) = 0 \) \( (8) \)

Let \( \overline{N}^{(2)}(t) \) be the solution of the system (7) together with (8)

**Stage-4:** The approximate solution of non-linear system (1) with (2) is given by

\[
\overline{N}(t) = \overline{N}^{(1)}(t) - \overline{N}^{(2)}(t)
\]

**Stage-3:** Using (16) along with homogeneous initial conditions we setup the linear system

\[
\frac{dN_1}{dt} = a_1 N_1 + h_1 \text{ (1)}
\]

\[
\frac{dN_2}{dt} = a_2 N_2 + h_2 \text{ (2)}
\]

The solution of the system (17) along with initial conditions (18) are

\[
N_1^{(2)} = N_{10} e^{a_1 t}
\]

\[
N_2^{(2)} = N_{20} e^{a_2 t}
\]

**Stage-2:** By using (14) and (15) the replenishment rates \( h_1 \) and \( h_2 \) of (10) are calculated as

\[
h_1(t) = a_{11} N_{10} e^{a_1 t} + a_{12} N_0 e^{a_2 t}
\]

\[
h_2(t) = a_{22} N_{20} e^{a_2 t}
\]

**Stage-4:** An approximate solution for the system 10 is given by

\[
N_1 = N_{10} e^{a_1 t}
\]

\[
N_2 = N_{20} e^{a_2 t}
\]

A numerical solution of the basic non-linear coupled differential equations: The variation of \( N_1 \) and \( N_2 \) verses time ‘t’;

The variation of \( N_1 \) and \( N_2 \) verses ‘t’ in the internal (0, 1) are computed numerically by employing recursive procedure with four stage approximation. Technique for a wide range of the system characterizing parameters \( a_1, a_2, a_{11}, a_{22}, a_{12}, a_{20} \) for this matlab has been used and results are illustrated.
Table

<table>
<thead>
<tr>
<th></th>
<th>$N_{10}=1$</th>
<th>$N_2=0.5$</th>
<th>$a_{12}=1$</th>
<th>$a_1=1$</th>
<th>$a_2=0.5$</th>
<th>$a_{11}=1$</th>
<th>$a_{22}: 0.5,1,1.5,2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$N_{10}=1$</td>
<td>$N_2=0.5$</td>
<td>$a_{12}=1$</td>
<td>$a_1=1$</td>
<td>$a_2=1$</td>
<td>$a_{11}=1$</td>
<td>$a_{22}: 0.5,1,1.5,2$</td>
</tr>
<tr>
<td>2</td>
<td>$N_{10}=1$</td>
<td>$N_2=0.5$</td>
<td>$a_{12}=1$</td>
<td>$a_1=1$</td>
<td>$a_2=1$</td>
<td>$a_{11}=1$</td>
<td>$a_{22}: 0.5,1,1.5,2$</td>
</tr>
<tr>
<td>3</td>
<td>$N_{10}=1$</td>
<td>$N_2=0.5$</td>
<td>$a_{12}=1$</td>
<td>$a_1=1$</td>
<td>$a_2=1.5$</td>
<td>$a_{11}=1$</td>
<td>$a_{22}: 0.5,1,1.5,2$</td>
</tr>
<tr>
<td>4</td>
<td>$N_{10}=1$</td>
<td>$N_2=0.5$</td>
<td>$a_{12}=1$</td>
<td>$a_1=1$</td>
<td>$a_2=0.5$</td>
<td>$a_{11}=1$</td>
<td>$a_{22}: 0.5,1,1.5,2$</td>
</tr>
<tr>
<td>5</td>
<td>$N_{10}=1$</td>
<td>$N_2=0.5$</td>
<td>$a_{12}=0.5$</td>
<td>$a_1=1$</td>
<td>$a_2=0.5$</td>
<td>$a_{11}=1$</td>
<td>$a_{22}: 0.5,1,1.5,2$</td>
</tr>
<tr>
<td>6</td>
<td>$N_{10}=1$</td>
<td>$N_2=0.5$</td>
<td>$a_{12}=1$</td>
<td>$a_1=0.5$</td>
<td>$a_2=0.5$</td>
<td>$a_{11}=1$</td>
<td>$a_{22}: 0.5,1,1.5,2$</td>
</tr>
<tr>
<td>7</td>
<td>$N_{10}=1$</td>
<td>$N_2=0.5$</td>
<td>$a_{12}=1$</td>
<td>$a_1=2$</td>
<td>$a_2=1$</td>
<td>$a_{11}=1$</td>
<td>$a_{22}: 0.5,1,1.5,2$</td>
</tr>
<tr>
<td>8</td>
<td>$N_{10}=1$</td>
<td>$N_2=0.5$</td>
<td>$a_{12}=1.5$</td>
<td>$a_1=2$</td>
<td>$a_2=1.5$</td>
<td>$a_{11}=1$</td>
<td>$a_{22}: 0.5,1,1.5,2$</td>
</tr>
</tbody>
</table>

Figure-1. $N_{10}=1$, $N_2=0.5$, $a_{12}=1$, $a_1=1$, $a_2=0.5$, $a_{11}=1$ Variation of $N_1$ and $N_2$ verses time $t$. 
Figure-2. $N_{10}=1$, $N_{20} = 0.5$, $a_{12}=1$, $a_{1}=1$, $a_{2}=1$, $a_{11}=1$ Variation of $N_1$ and $N_2$ versus time $t$.

Figure-3. $N_{10}=1$, $N_{20} = 0.5$, $a_{12}=1$, $a_{1}=1$, $a_{2}=1.5$, $a_{11}=1$ Variation of $N_1$ and $N_2$ versus time $t$.

Figure-4. $N_{10}=1$, $N_{20} = 0.5$, $a_{12}=0.5$, $a_{1}=1$, $a_{2}=0.5$, $a_{11}=1$ Variation of $N_1$ and $N_2$ versus time $t$. 
Figure-5. $N_{10}=1$, $N_{20}=0.5$, $a_{12}=1$, $a_1=1$, $a_2=0.5$, $a_{11}=1$ Variation of $N_1$ and $N_2$ versus time $t$.

Figure-6. $N_{10}=1$, $N_{20}=0.5$, $a_{12}=1$, $a_1=0.5$, $a_2=0.5$, $a_{11}=1$ Variation of $N_1$ and $N_2$ versus time $t$. 
CONCLUSIONS

Following conclusions are derived from the graphs (Figure-1 to Figure-8)

a) If growth rate of host increases the $N_2$ curves are falling with increasing steepness.
b) Both $N_1$ and $N_2$ are decreases with time ‘t’ because of utilization of energy.
c) For $a_{12} < 1$ weak comensalism of host over comensal, $N_2$ falls with slower rate than $N_1$. However with increasing growth rate of host steepness increases for the $N_2$ curves and falling rate is decreased comparison with $N_1$. With increasing $a_{22}$ steepness decreases.
d) For $a_{12} = 1$ the steepness of $N_2$ increases with $a_2$ however falling of $N_2$ is faster along with increasing $a_{22}$ than $N_1$. 

Figure-7. $N_{10}=1$, $N_{20} = 0.5$, $a_{12}= 1$, $a_1 = 2$, $a_2 = 1$ Variation of $N_1$ and $N_2$ verses time ‘t’.

Figure-8. $N_{10}=1$, $N_{20} = 0.5$, $a_{12} = 1.5$, $a_1 = 2$, $a_2 = 1.5$ $a_{11}=1$ Variation of $N_1$ and $N_2$ verses time ‘t’.
e) For \( a_{12} > 1 \) strong commensalism of host over commensal it is observed that \( N_1 \) increasing to some extent and then start falling. This tendency is observed at slow rate in case of \( a_{12} = 1 \).

f) Because of growth rate of \( S_1 \), \( N_1 \) falls much faster than \( N_2 \). However this fall becomes slower in case of \( a_{12} > 1 \)

<table>
<thead>
<tr>
<th>Trend index</th>
<th>( N_1 )</th>
<th>( N_2 )</th>
<th>Steepness of ( N_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_2 )</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
</tr>
<tr>
<td>( a_{22} )</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>( a_{12} )</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
</tr>
</tbody>
</table>

REFERENCES


