ABSTRACT

The objectives of the present study are to investigate the radiation effects on unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation. The method of solution can be applied for Finite element technique. Numerical results for the velocity, the temperature, and the concentration are shown graphically. The expressions for the skin-friction, Nusselt number and Sherwood number are obtained. The results show that increased cooling (Gr>0) of the plate and the Eckert number leads to a rise in the velocity. Also, an increase in the Eckert number leads to an increase in the temperature, whereas increase in radiation parameter lead to a decrease in the temperature distribution when the plate is being cooled.

Keywords: heat transfer, viscous dissipation, radiation, chemical reaction, finite element technique.

INTRODUCTION

For some industrial applications such as glass production and furnace design in space technology applications, cosimal flight aerodynamics rocket, propulsion systems, plasma physics which operate at higher temperatures, radiation effects can be significant. Soundalgekar and Takhar [1] considered the radiative free convection flow of an optically thin grey-gas past a semi-infinite vertical plate. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hussain and Takhar [2]. Raptis and Perdikis [3] have studied the effects of thermal radiation and free convection flow past a moving vertical plate.


In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., Polymer production, manufacturing of ceramics or glassware and food procession. Das et al. [7] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction.

In all these investigations, the viscous dissipation is neglected. The viscous dissipation heat in the natural convective flow is important, when the flow field is of extreme size or at low temperature or in high gravitational field. Gebhar [16] showed the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux in the plate. Soundalgekar [17] analyzed the effect of viscous dissipative heat on the two dimensional unsteady, free convective flow past a vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Israel Cocokey et al [18] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in porous medium with time dependent suction.

The objective of the present study is to analyze the radiation and mass transfer effects on an unsteady two-dimensional laminar convective boundary layer flow of a viscous, incompressible, chemically reacting fluid along a semi-infinite vertical plate with suction, by taking into account the effects of viscous dissipation. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field, are solved by using finite element technique. The behavior of the velocity, temperature, concentration has been discussed for variations in the governing parameters.

MATHEMATICAL ANALYSIS

An unsteady two-dimensional laminar boundary layer flow of a viscous, incompressible, radiating fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects is considered, by taking the effect of viscous dissipation into account. The \(x'\)-axis is taken along the vertical infinite plate in the upward direction and the \(y'\)-axis normal to the plate. The level of concentration of foreign mass is assumed to be low, So that the Soret and Dufour effects are negligible. Now under Boussinesq’s approximation, the flow field is governed by the following equations:

\[
\frac{\partial u'}{\partial t} + v' \frac{\partial u'}{\partial y'} = \nu' \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_e) + g\beta_s(C - C_s) \tag{2}
\]
\[ \frac{\partial T}{\partial t'} + \nu \frac{\partial T}{\partial y'} = \alpha \frac{(\partial T)^2}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} + \frac{\mu}{\rho c_p} \left( \frac{\partial u'}{\partial y'} \right)^2 \]  

(3)

\[ \frac{\partial C}{\partial t'} + \nu \frac{\partial C}{\partial y'} = D \frac{(\partial C)^2}{\partial y'^2} - k_r^2 \left( C - C_\infty \right) \]  

(4)

Where \( u', v' \) are the velocity components in \( x', y' \) directions respectively? \( t' \) - the time, \( \rho \) - the fluid density, \( \nu \) - the kinematic viscosity, \( c_p \) - the specific heat at constant pressure, \( g \) - the acceleration due to gravity, \( \beta \) and \( \beta^* \) - the thermal and concentration expansion coefficient respectively, \( T \) - the dimensional temperature, \( C \) - the dimensional concentration, \( \alpha \) - the fluid thermal diffusivity, \( \mu \) - the coefficient of viscosity, \( D \) - the mass diffusivity, \( k_r \) - the chemical reaction parameter.

The boundary conditions for the velocity, temperature and concentration fields are

\[ u' = U_0, \quad T = T_w + e(T_w - T_\infty) e^\nu, \quad C = C_w + e(C_w - C_\infty) e^\nu \text{ at } y' = 0 \]

\[ u' \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } y' \rightarrow \infty \]  

(5)

Where \( U_0 \) is the scale of free stream velocity, \( T_w \) and \( C_w \) are the wall dimensional temperature and concentration respectively, \( T_\infty \) and \( C_\infty \) are the free stream dimensional temperature and concentration respectively, \( n' \) - the constant.

By using Rosseland approximation, the radiative heat flux is given by

\[ q_r = \frac{4 \sigma_e \phi^{T^4}}{3 K_e} \]  

(6)

Where \( \sigma_e \) - the Stefan-Boltzmann constant and \( K_e \) - the mean absorption coefficient. It should be noted that by using Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficient, small, then equation (6) can be linearised by expanding \( T^4 \) in the Taylor series about \( T_\infty \), which after neglecting higher order terms take the form

\[ T^4 \approx 4 T_\infty^4 T - 3 T_\infty^4 \]  

(7)

In view of equations (6) and (7), equation (3) reduces to

\[ \frac{\partial T}{\partial t'} + \nu \frac{\partial T}{\partial y'} = \frac{16 \sigma_e \alpha T_\infty^4}{3 \rho c_p K_e} \frac{(\partial T)^2}{\partial y'^2} + \frac{\mu}{\rho c_p} \left( \frac{\partial u'}{\partial y'} \right)^2 \]  

(8)

From the continuity equation (1), it is clear that suction velocity normal to the plate is either a constant or function of time. Hence, it is assumed in the form

\[ v' = -V_0 \left( 1 + \varepsilon A e^{\nu y} \right) \]  

(9)

Where \( A \) is a real positive constant, \( \varepsilon \) and \( \varepsilon A \) are small values less than unity and \( V_0 \) is scale of suction velocity at the plate surface.

In order to write the governing equations and the boundary condition in dimension less form, the following non-dimensional quantities are introduced:

\[ u = \frac{u'}{U_0}, \quad y = \frac{y e}{v}, \quad t = \frac{t e}{v}, \quad n = \frac{w e}{v}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_\infty - C_w} \]

\[ Pr = \frac{\nu \rho C_w}{\alpha}, \quad Sc = \frac{v}{\alpha}, \quad Gr = \frac{g \beta (T_w - T_\infty)}{U_0^2 T_w}, \quad Gm = \frac{g \beta^* (C_w - C_\infty)}{U_0^2 C_w}, \quad Ec = \frac{U_0^2}{c_p (T_w - T_\infty)}, \quad k_r^2 = \frac{k^2}{V_0^2}, \quad R = \frac{16 \sigma_e T_\infty^4}{3 K_e} \]  

(10)

In view of the equations (6) - (10), Equations (2) - (4) reduce to the following dimensionless form.

\[ \frac{\partial u}{\partial t} - (1 + \varepsilon A e^{\nu y}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y'^2} + Gr \theta + Gm \phi \]  

(11)

\[ \frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{\nu y}) \frac{\partial \theta}{\partial y} = \left( \frac{1 + R}{Pr} \right) \frac{\partial^2 \theta}{\partial y'^2} + Ec \left( \frac{\partial u}{\partial y} \right)^2 \]  

(12)

\[ \frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{\nu y}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y'^2} - k_r \phi \]  

(13)

Where \( Gr, Gm, Pr, R, Ec, Sc \) and \( k_r \) are the thermal Grashof number, solutal Grashof number, Prandtl number, radiation parameter, Eckert number, Schmidt number and chemical reaction parameter respectively.

The corresponding boundary conditions are

\[ u = 0.5, \quad \theta = 1 + \varepsilon e^{\nu y}, \quad \phi = 1 + \varepsilon e^{\nu y} \text{ at } y = 0 \]

\[ u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \text{ as } y \rightarrow \infty \]  

(14)

**SOLUTION OF THE PROBLEM**

The Galerkin equation for the differential equation (11) becomes

\[ \int_{y_j}^{y_i} N^{(e)} \left[ \frac{\partial^2 u^{(e)}}{\partial y'^2} + P \frac{\partial u^{(e)}}{\partial t} - \frac{\partial u^{(e)}}{\partial t} + R \right] dy = 0 \]  

(15)

Where \( P = 1 + \varepsilon A e^{\nu y}, \quad R = Gr \theta + Gm \phi \)

Let the linear piecewise approximation solution

\[ u^{(e)} = N_j(y) u_j(t) + N_k(y) u_k(t) = N_j u_j + N_k u_k \]
Where \( N_j = \frac{y_k - y_j}{y_k - y_j} \), \( N_k = \frac{y - y_j}{y_k - y_j} \)

\[
N^{(\gamma)}_j \left( \frac{\partial u^{(\gamma)}}{\partial y} \right)_{y_j} - \int_{y_j} y \left( \frac{\partial N^{(\gamma)}_j}{\partial y} \frac{\partial u^{(\gamma)}}{\partial y} - N^{(\gamma)}_j \left( \frac{p}{\partial u^{(\gamma)}} \frac{\partial u^{(\gamma)}}{\partial t} - R \right) \right) dy = 0 \tag{16}
\]

Neglecting the first term in Equation (16) we get

\[
\frac{1}{l^2} \left[ \begin{array}{ccc}
1 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 1
\end{array} \right] \frac{u^{(i)}}{u_i} + \frac{1}{6} \left[ \begin{array}{ccc}
2 & 1 & 0 \\
1 & 4 & 1 \\
0 & 1 & 2
\end{array} \right] u^{(i)} = R^{(\gamma)} \frac{1}{2}
\]

Where \( l^{(\gamma)} = y_j - y_j = h \) and dot denotes the differentiation with respect to \( t \).

We write the element equations for the elements \( y_{i-1} \leq y \leq y_i \) and \( y_i \leq y \leq y_{i+1} \) as three element equations, we obtain

\[
\frac{1}{l^2} \left[ \begin{array}{ccc}
1 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 1
\end{array} \right] \frac{u^{(i)}}{u_i} + \frac{1}{6} \left[ \begin{array}{ccc}
2 & 1 & 0 \\
1 & 4 & 1 \\
0 & 1 & 2
\end{array} \right] u^{(i)} = \frac{R^{(\gamma)}}{2}
\]

Now put row corresponding to the node \( i \) to zero, from Equation (16) the difference schemes is

\[
\frac{1}{l^2} \left[ \begin{array}{ccc}
1 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 1
\end{array} \right] \frac{u^{(i)}}{u_i} + \frac{1}{6} \left[ \begin{array}{ccc}
2 & 1 & 0 \\
1 & 4 & 1 \\
0 & 1 & 2
\end{array} \right] u^{(i)} = \frac{R^{(\gamma)}}{2}
\]

Applying Crank-Nicholson method to the above equation (10), then we get

\[
A_{i+1} u^{(i)} + A_{i} u^{(i+1)} + A_{i} u^{(i)} = A_{i} u^{(i-1)} + A_{i} u^{(i)} + A_{i} u^{(i+1)} + P^{*} \tag{17}
\]

Where

\[
A_{i} = 2 - 6r + 3Pfrh, A_{i} = 8 + 12r, A_{i} = 2 - 6r - 3Pfrh
\]

\[
A_{i} = 2 + 6r - 3Pfrh, A_{i} = 8 - 12r, A_{i} = 2 + 6r + 3Pfrh
\]

\[
P^{*} = 12(Gr) k \theta^{(i)} + 12(Gm) k \phi^{(i)}
\]

Applying similar procedure to equation (11) and (12) then we get

\[
B_{i} \theta^{(i)} + B_{i} \theta^{(i)} + B_{i} \theta^{(i)} = B_{i} \theta^{(i)} + B_{i} \theta^{(i)} + B_{i} \theta^{(i)} + P^{**} \tag{18}
\]

\[
C_{i} \phi^{(i)} + C_{i} \phi^{(i)} + C_{i} \phi^{(i)} = C_{i} \phi^{(i)} + C_{i} \phi^{(i)} + C_{i} \phi^{(i)} \tag{19}
\]

Where

\[
B_{i} = 2P_{1} - 6r + 3PP_{1} r h, B_{i} = 8P_{1} + 12r
\]

\[
B_{3} = 2P_{1} - 6r - 3PP_{1} r h\]

\[
B_{4} = 2P_{1} + 6r - 3PP_{1} r h, B_{5} = 8P_{1} - 12r
\]

\[
B_{6} = 2P_{1} + 6r + 3PP_{1} r h
\]

\[
P^{**} = 12PrEc(u[i + 1] - u[i])^2
\]

Where

\[
C_{1} = 2Sc - 6r + 3PSc r h + Qk, C_{2} = 8Sc + 12r + 4Qk, C_{1} = 2Sc - 6r - 3PSc r h + Qk
\]

\[
C_{4} = 2Sc + 6r - 3PSc r h - Qk, C_{5} = 8Sc - 12r - 4Qk
\]

\[
C_{6} = 2Sc + 6r + 3PSc r h - Qk
\]

Here \( Q = ScK^{2} \), \( P_{1} = \frac{Pr}{1 + R} \), \( r = \frac{k h}{h^{2}} \) and \( h,k \) are the mesh sizes along \( y \)-direction and time \( t \)-direction respectively. Index \( i \) refers to the space and \( j \) refers to the time. In Equations (17)-(19), taking \( i = 1(1) n \) and using initial and boundary conditions (13), the following system of equations are obtained:

\[
A_{i} X_{i} = B_{i} \quad i = 1(1) 3
\]

Where \( A_{i} \)’s are matrices of order \( n \) and \( X_{i} \)’s column matrices having \( n \)-components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C-program. In order to prove the convergence and stability of Galerkin finite element method, the same C-program was run with slightly changed values of \( h \) and \( k \) and no significant change was observed in the values of \( u, \theta \) and \( \phi \). Hence, the Galerkin finite element method is stable and convergent.

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow.

The skin-friction at the plate, which in the non-dimensional form is given by

\[
C_{f} = \frac{\tau_{w}}{\rho U_{0} V_{0}} = \left( \frac{\partial u}{\partial y} \right)_{y=0} \tag{21}
\]

The rate of heat transfer coefficient, which in the non-dimensional form in terms of the Nusselt number is given by

\[
Nu = \left( \frac{\partial T}{\partial y} \right)_{y=0} \Rightarrow Nu Re x^{-1} = \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \tag{22}
\]
The rate of heat transfer coefficient, which in the non-dimensional form in terms of the Sherwood number, is given by

\[ Sh = -x \frac{\partial C}{\partial y} \bigg|_{y=0} \Rightarrow Sh Re_x^{-1} = -\left( \frac{\partial \phi}{\partial y} \right) \bigg|_{y=0} \quad (23) \]

Where \( Re_x = \frac{V_0 x}{\nu} \) is the local Reynolds number.

**RESULTS AND DISCUSSIONS**

In the preceding sections, the problem of an unsteady free convective flow of a viscous, incompressible, radiating and dissipating fluid past a semi-infinite plate with chemically reacting was formulated and solved by finite element technique. The expressions for the velocity, temperature and concentration were obtained. To illustrate the behavior of these physical quantities, numeric values were computed with respect to the variations in the governing parameters viz., the thermal Grashof number \( Gr \), solutal Grashof number \( Gm \), Eckert number \( Ec \), radiation parameter \( R \), Prandtl number \( Pr \), Schmidt number \( Sc \) and chemical reaction parameter \( k_r \).

The velocity profiles for different values of the thermal Grashof number \( Gr \) are described in Figure-1. It is observed that an increase in \( Gr \), leads to a rise in the values of velocity. Hence the positive values of \( Gr \) corresponds to cooling of the plate. In addition, it is observed that the velocity increases rapidly near the wall of the plate as Grashof number incrases and then decays to the free stream velocity.

For the case different values of the solutal Grashof number \( Gm \), the velocity profiles in the boundary layer are shown in Figure-2. It is noticed that an increase in \( Gm \), leads to a rise in the values of velocity.

Figures 3(a) and 3(b) shows the velocity and temperature profiles for different values of the Radiation parameter \( R \), it is noticed that an increase in the radiation parameter results decrease in the velocity and temperature with in boundary layer, as wellas decreased the thickness of the velocity and temperature boundary layers.

The effects of the viscous dissipation parameter i.e., Eckert number on the velocity and temperature are shown in Figure-4(a) and Figure-4(b). Greater viscous
dissipative heat causes a rise in the temperature as well as the velocity.

Figure 4(a) shows the velocity profiles for different values of Ec. As Ec increases, the velocity increases.

Figure 4(b) illustrates the temperature profiles for different values of Ec. As Ec increases, the temperature decreases.

The effect of the Prandtl number on the velocity and temperature are shown in Figures 5(a) and 5(b). As the Prandtl number increases, the velocity and temperature decreases. This causes the buoyancy effects to decrease, yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

Figure 5(b) shows the temperature profiles for different values of Pr. As Pr increases, the temperature decreases.

The effect of the Schmidt number on the velocity and concentration are shown in Figures 6(a) and 6(b). As the Schmidt number increases, the velocity and concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

Figure 6(a) illustrates the velocity profiles for different values of Sc. As Sc increases, the velocity decreases.

Figure 6(b) shows the concentration profiles for different values of Sc. As Sc increases, the concentration decreases.

Figures 7(a) and 7(b) illustrates the behavior velocity and concentration for different values of chemical
reaction parameter $k_r$. It is observed that an increase in $k_r$ leads to a decrease in both the values of velocity and concentration.

Tables 1-5 present the effects of the thermal Grashof number, solutal Grashof number, radiation parameter, Schmidt number and Eckert number on the skin-friction coefficient, Nusselt number and Sherwood number. From Tables 1 and 2, it is observed that as $Gr$ or $Gm$ increases, the skin-friction coefficient increases. However, from Table-3, it can be seen that as the radiation parameter increases, the skin-friction coefficient increases and Nusselt number decreases. From Table-4, it is noticed that an increase in the Schmidt number reduces the skin-friction coefficient and increases the Sherwood number. Finally, it is observed from Table-5 that as Eckert number increases, the skin-friction coefficient increases and the Nusselt number decreases.

### Table-1. Effect of $Gr$ on $C_f$ Reference values as in Figure-1.

<table>
<thead>
<tr>
<th>$Gr$</th>
<th>$C_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.8343</td>
</tr>
<tr>
<td>1.0</td>
<td>1.6445</td>
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<tr>
<td>2.0</td>
<td>2.4548</td>
</tr>
<tr>
<td>3.0</td>
<td>3.2652</td>
</tr>
</tbody>
</table>

### Table-2. Effect of $Gm$ on $C_f$ Reference values as in Figure-1.

<table>
<thead>
<tr>
<th>$Gm$</th>
<th>$C_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0816</td>
</tr>
<tr>
<td>1.0</td>
<td>1.7682</td>
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<tr>
<td>2.0</td>
<td>2.4548</td>
</tr>
<tr>
<td>3.0</td>
<td>3.1414</td>
</tr>
</tbody>
</table>

### Table-3. Effect of $R$ on $C_f$ and $Nu$ Reference values as in Figure-3(a).

<table>
<thead>
<tr>
<th>$R$</th>
<th>$C_f$</th>
<th>$Nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2.1664</td>
<td>0.8365</td>
</tr>
<tr>
<td>0.5</td>
<td>2.4548</td>
<td>0.6139</td>
</tr>
<tr>
<td>1.0</td>
<td>2.6536</td>
<td>0.5032</td>
</tr>
<tr>
<td>2.0</td>
<td>2.9037</td>
<td>0.4010</td>
</tr>
</tbody>
</table>

### Table-4. Effect of $Sc$ on $C_f$ and $Sh$ Reference values as in Figure-3(a).

<table>
<thead>
<tr>
<th>$Sc$</th>
<th>$C_f$</th>
<th>$Sh$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22</td>
<td>3.1068</td>
<td>0.4515</td>
</tr>
<tr>
<td>0.60</td>
<td>2.4548</td>
<td>0.8431</td>
</tr>
<tr>
<td>0.78</td>
<td>2.2767</td>
<td>1.0214</td>
</tr>
<tr>
<td>0.94</td>
<td>2.1540</td>
<td>1.1745</td>
</tr>
</tbody>
</table>

### Table-5. Effect of Ec on $C_f$ and $Nu$ Reference values as in Figure-3(a).

<table>
<thead>
<tr>
<th>$Ec$</th>
<th>$C_f$</th>
<th>$Nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2.4546</td>
<td>0.6143</td>
</tr>
<tr>
<td>0.25</td>
<td>2.5010</td>
<td>0.5130</td>
</tr>
<tr>
<td>0.50</td>
<td>2.5489</td>
<td>0.4039</td>
</tr>
<tr>
<td>0.75</td>
<td>2.5985</td>
<td>0.2863</td>
</tr>
</tbody>
</table>

### CONCLUSIONS
We have formulated and solved approximately the problem of two-dimensional fluid flow in the presence of radiative heat transfer, viscous dissipation and chemical reaction parameter. A finite element technique is employed to solve the resulting coupled partial differential equations. The conclusions of the study are as follows:

a) The velocity increases with the increase in thermal Grashof number and solutal Grashof number;
b) An increase in the Eckert number increases the velocity and temperature;
c) An increase in the Prandtl number decreases the velocity and temperature;
d) An increase in the radiation parameter leads to increase in the velocity and temperature;
e) The velocity as well as concentration decreases with an increase in the Schmidt number; and
f) The velocity as well as concentration decreases with an increase in the chemical reaction parameter.

REFERENCES


