OPTIMIZATION OF AXIAL COMPRESSOR STAGE USING NSGA-II TECHNIQUE

G. Chaitanya[1], J. Suresh Kumar[2] and K. Srinivas[1]
[1]Department of Mechanical Engineering, R.V.R. and J.C. College of Engineering, Guntur, A. P., India
[2]Department of Mechanical Engineering, J.N.T.U. Hyderabad, A. P., India
E-Mail: chaitanyagoteti@rediffmail.com

ABSTRACT

Efficiency and Stage Weight [Inlet stage specific area] are two important design issues which demand specific attention in the design of aero space compressors. In this paper these two objectives were optimized using elitist multi objective genetic algorithm, otherwise known as NSGA-II (Non dominated sorted Genetic Algorithm-II) which was developed by Kalyan Moy Deb [2002]. Lingen Chen and Fengrui Sun (2005) implemented optimum design of a subsonic axial flow compressor stage using mean line prediction method and taking 12 design variables and three objective functions. In the present approach two objective functions were formulated taking 5 design variables into account. The results showing optimal front for the two objectives problem is presented and the sensitivity analysis results of influencing design variables are shown.

Keywords: axial flow compressor stage, crowding distance, non-dominated sorting, stage efficiency, stage weight.

Nomenclature

D: Mean diameter of Stage (m)
C_a: Axial Velocity component (m/s)
Ø: Flow coefficient
γ: Process index (Air)
T_{01}: Stage inlet temperature (k)
(P_{03} / P_{01}): Stage pressure ratio
U: Peripheral velocity (m/s)
η: Stage efficiency
C_w: whirl component of velocity (m/s)
ρ_b: Material density of blade (kg/m^3)
n: Number of blades
r_t: Radius of blade at tip (m)
(t/c): thickness to chord ratio of blade
d_j: Crowding distance
k: (r_r / r_t) Radius ratio for blade
N: Rotational speed of shaft (rps)
λ: Work done factor
α_1: Air entry angle to rotor (radians)
C_p: Specific heat at constant pressure (kj/kgk)
$T_{03:}$ Stage exit temperature (k)
m: Mass flow rate (kg/s)
$T_{03}{}^1$: Isentropic temperature at exit (k)
A: Inlet stage specific area (kg/s/m^2)
ρ: Air density (kg /m^3)
h: Blade mean height (m)
r_r: Radius of blade at root (m)
t: Blade thickness (m)
(c/D): Chord length to diameter ratio
β: Blade angles (Radians)
f_1 and f_2: Objective functions

INTRODUCTION

The main goal in aircraft engine design is to improve the specific fuel consumption and the thrust to weight ratio. Conventionally thrust to weight ratio can be improved by reducing the diameter and length of compressor components. As the length of combustion chamber decreases, the gas resident time inside the combustion chamber decreases. Moreover this causes variation in three dimensional flow effects and cascade effects and there by reduces stage efficiency. Alternatively inlet stage specific area can be maximized for improving the thrust. A. Sehra and J. Bettner (1992) implemented design techniques used in air craft compressors to compressors used in industrial engines. I. H. Smith (1970) presented the effect of end wall losses and blade aspect ratios on efficiency of axial flow compressors. A. Massardo and A. Satta (1990) presented the optimum design relations for axial compressors used for industrial applications. S. S. Rao and R. S. Guptha (1980) formulated a non linear mathematical model for minimization of mass and aero dynamic losses in the stage of axial flow compressor. In order to introduce a design system for axial flow compressor stage, some criteria for good design should be stated. The first goal of design system should be to generate a compressor geometry that will produce design point total pressure ratio. The design point pressure ratio, flow rate and rotational speed must be obtained within the aerodynamic and aeromechanical design requirements of the compressor with acceptable level of thermodynamic efficiency and weight of stage.

Generally the minimum acceptable efficiency level for air craft compressors is 0.78. H. Cohen and GFC Rogers (1989) provided information on basic thermodynamic relations and stage design relations for axial flow compressors. In this paper the compressor is assumed to be a fifty percent reaction stage with symmetrical blade angles. Due to symmetrical blade angles the pressure distribution on rotor and stator will be uniform leading to uniform pressure rise in the stage. Also with this type of blading the first stage is usually preceded by inlet guide vanes providing pre-whirl and correct velocity entrance angle of air to rotor. This in turn results in uniform mass flow rate and high blade speeds without exceeding the limiting Mach number. How ever high blade speeds are limited by the centrifugal stresses developed inside the blade material. De Neeve and Dukkipati (1979) optimized the geometry of an axial compressor blade without compromising on aerodynamic design. GU.C and Miao Y. (1987) provided blade design of axial compressors by method of optimal control theory and pontryagin's maximum principles. In the present work the problem is solved for a balanced optimum between efficiency and weight of axial flow compressor stage.

PROBLEM FORMULATION	
Stage efficiency: [Assume $C_{a1} \equiv C_{a2} \equiv C_{a3} = C_a$]	
η = (Isentropic work) / (Actual work)	But $W = mU(C_{w2} - C_{w1})$
Isentropic work / mass = $C_p (T_{03}^1 - T_{01})$	$W = mU(C_a \mathrm{Tan}\alpha_2 - C_a \mathrm{Tan}\alpha_1)$
Actual work / mass = $C_p (T_{03} - T_{01})$	for 50% stage ($\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$)
Therefore $\eta = C_p (T_{03}^1 - T_{01}) / C_p (T_{03} - T_{01})$	Hence $W = mUC_a[\mathrm{Tan}\beta_1 - \mathrm{Tan}\beta_2]$
$\eta = (T_{03}^1 - T_{01}) / (T_{03} - T_{01})$	Actual work / stage is: $\Delta T_{0S} = T_{03} - T_{01}$
$\eta = T_{01} [(T_{03}^1 / T_{01}) - 1] / (T_{03} - T_{01})$	$\Delta T_{0S} = (\lambda U C_a) / C_p[\mathrm{Tan}\beta_1 - \mathrm{Tan}\beta_2]$
Since $(T_{03}^1 / T_{01}) = (P_{03} / P_{01})^m$	From Figure-4 $C_{w2} / C_{a2} = \mathrm{Tan}\alpha_2$
Therefore	And $\mathrm{Tan}\alpha_2 = (U - C_{a2}\mathrm{Tan}\beta_2) / C_{a2}$
$\eta = T_{01} [(P_{03} / P_{01})^m - 1] / [T_{03} - T_{01}]$	Therefore $\mathrm{Tan}\alpha_2 = (U/C_a) - \mathrm{Tan}\alpha_1$ [$\beta_2 = \alpha_1$]
From Figure-4 $U = C_{w1} + (U - C_{w1})$	As $U = \Pi DN$ and $\text{Ø} = (C_a / \Pi DN)$ Therefore
$U = C_{a1} \mathrm{Tan}\alpha_1 + C_{a1} \mathrm{Tan}\beta_1$	$\eta = T_{01} [(P_{03} / P_{01})^m - 1] / [(\lambda \Pi DNC_a / C_p)((1/\text{Ø}) - 2\mathrm{Tan}\alpha_1)]$
$U / C_a = \mathrm{Tan}\alpha_1 + \mathrm{Tan}\beta_1$	Where λ is the work done factor 0.98 for Inlet stage
--	
Inlet stage specific area	But $U_t = \Pi D_t N = \Pi(D+h/2+h/2)N = \Pi(D+h)N$
A_s = Mass flow rate / Area	Therefore $h = (U_t / \Pi N) - D = (D_t - D)$
Area = $(\Pi Dh - nth)$	Hence $As = \rho \Pi r_t^2 C_a[1 - (r_r / r_t)^2] / [D(D_t - D) (0.216)]$
mass flow rate m = ρx(volume/sec)	$A_S = \rho \Pi r_t^2 C_a[1 - (k)^2] / [D(D_t - D) (0.216)]$
$m = \rho \Pi r_t^2[1 - (r_r / r_t)^2]C_a$ Therefore	--
$A_S = \rho \Pi r_t^2[1 - (r_r / r_t)^2]C_a / (\Pi Dh - nth)$	**Centrifugal stress**
$A_S = \rho \Pi r_t^2[1 - (r_r / r_t)^2]C_a / Dh(\Pi - (nt / D))$	$\sigma = (\rho_b /2)(2\Pi N)^2(r_t^2 - r_r^2) = (\rho_b /2)U_t^2[1 - (r_r / r_t)^2]$
From NACA-65 series blade specification	Where $U_t = 2\Pi N r_t$ hence $\sigma = (\rho_b /2) (\Pi D_t N)^2[1-(K)^2]$
$(t / c) = 0.1$, $n=65$ and $(c / D) = 0.45$	$\sigma \leq \sigma_{Max}$ σ_{Max} is limiting stress for 52100 alloy steel
Input data to optimization problem	**Therefore the optimization problem can be stated as:**
Inlet temperature to stage $T_{01} = 300$ k	$F = \mathrm{Max} (f_1 , f_2)$ where
Exit temperature from stage $T_{03} = 346$k	$f_1 = T_{01} [(P_{03} / P_{01})^m - 1] / [(\lambda \Pi DNC_a / C_p) ((1/\text{Ø}) - 2\mathrm{Tan}\alpha_1)]$
Pressure ratio $(P_{03} / P_{01}) = 3.3$	$f_2 = \rho \Pi r_t^2 C_a[1 - (k)^2] / [D(D_t - D) (0.216)]$
Specific heat $C_P = 1.005$ kj /kgk	Subject to $g \equiv (\rho_b /2) (\Pi D_t N)^2 [1-(K)^2]$
Mass flow rate m = 4 kg/s	for $X = \{D, \text{Ø}, N, \alpha_1 , k\}$
Gravity acceleration $g = 9.8$ m/s^2	**Variable bounds**
Process index $\gamma = 1.4$	$0.3 \leq D \leq 0.4$
Tip diameter $D_t = 0.49$ m	$0.2 \leq \text{Ø} \leq 0.6$
Blade density $\rho_b = 7700$ kg/m^3	$0^c \leq \alpha_1 \leq (\Pi / 9)^c$
Air density $\rho = 1.165$ kg/m^3	$350 \leq N \leq 500$
Axial velocity $C_a = 150$ m/s	$0.4 \leq k \leq 0.95$

NSGA-II

Non dominated sorted genetic algorithm is a popular evolutionary technique for multi objective optimization. It is a very effective algorithm but has been generally criticized for its computational complexity and lack of elitism. The danger with lack of elitism is that elite or best fit individual might be eliminated from the selection pool or mating pool. Secondly for accurate working of the algorithm the sharing parameter should be appropriately chosen. These two difficulties have been effectively minimized in the modified version of non dominated sorted genetic algorithm popularly known as NSGA-II. K. Deb (2002) presented a fast multi objective solving technique using NSGA-II taking two non linear mathematical functions as examples. Bingquan Hang and B. Buckley (2010) presented multi objective feature selection technique for telecommunication applications using NSGA-II. S. Farahat and E. Khorasani Nejad (2009) carried out thermodynamic optimization of turbo shaft engine using NSGA-II. Rio G.L D’Souza and K. Chandrasekaran (2010) presented an improved version of NSGA-II based on a novel ranking scheme. Dazhi Sun and Rahim F. Benekohal (2003) applied NSGA-II technique for multi objective traffic signal timing optimization. Carlos A Coello coello (2001) presented a short tutorial on multi objective evolutionary algorithms. For the present problem an initial parent binary population size of 100 with 10 digit binary string with a chromosome length equal to 50 has been chosen. Offspring population is generated with a cross over probability of 0.85 and mutation probability of 0.1 and applying roulette wheel selection operator. The algorithm has been executed for a maximum number of 1000 generations.

PROCEDURE FOR NSGA-II

Initialize the population Pt.
Create the offspring population Qt from the current population Pt.
Perform fitness assignment for creating offspring population
Combine the two populations Qt and Pt to form Rt.
Rt = Pt U Qt
Find the all non-dominated fronts Fi of Rt.
Initiate the new population Pt + 1 ≠ 0 and the counter of front for inclusion i = 1.
While Pt + 1 + Fi ≤ Npop [Population Size], do:
Pt + 1= Pt + 1 U Fi, i = i + 1
Sort the last front Fi using the crowding distance in descending order and choose
The first (Npop - Pt+1) elements of Fi.
Use selection, crossover and mutation operators to create the new offspring
Population Qt+1 size Nobj.

Non-dominated sorting

```
For each (solution p belongs to set P)
  For each (solution q belongs to set P)
    if (p< q)   And then if p dominates q then
      $S_p = S_p \cup \{q\}$   Add q in the set $S_p$
    Else if (q < p)   and then if p dominated by q
      $n_p = n_p + 1$   Increment the dominated counter of P i.e $n_p$
    End
  End
  if ($n_p$= = 0)   Then no solution dominates p
    $F_1 = F_1 \cup \{p\}$   Then p belong to first front i = 1 initialize the front Counter to 1
  End
End
While ($F_i \neq Ø$)
  Q = Ø
  For each (p belongs to $F_i$)   for each p in front $F_i$
    For each (q belongs to $S_p$)   for each q in set Sp
      $n_q = n_q - 1$   Decrement the domination count $n_q$ by 1
      If ($n_q$ = 0)   (then if $n_q$ is zero)
        Q = Q U {q}   Add the q in set Q
      End
    End
  End
  i = i+1   Increment the Front by 1
  $F_i$ = Q   Set Q is next front
End
```

Crowding distance

To provide the diversity in population, it is required to calculate crowding distance. Following algorithm is used for calculate the crowding distance of each point in set I:

```
l = |I| l is the number of solutions in I for each j
  Set I [$d_j$] distance = 0 [initialize the distance to zero for each individual j in Set I]
End
For each objective m
  I = sort (I,m) sort using each objective value m
  I [$d_{j1}$] distance = I[$d_{jl}$] distance = Infinity Assign infinite distance to boundary value for all others
End
For (I = 2 to (l-1))
  I[$d_j$] distance = I[$d_j$] distance + [(I (k+1) m - I (k-1) m) / ($f_m^{max} - f_m^{min}$)]
End
```

I (k).m is the value of the mth objective function of the kth individual in I

Selection

Once the individuals are sorted based on non-domination and with crowding distance assigned, the selection is carried out using a crowded-comparison-operator (>n) and best solution is selected. It assumes that every solution i has two attributes

1. A Non-domination rank (ri) in population
2. A local Crowding distance (I[i]distance)

i >n j Solution i is better than j
if (ri < rj) if rank of ith solution is better than jth or
if (ri = rj) if they have same rank and
(I[i]distance > I[j]distance) but the crowding distance of ith solution is better than jth solution.

DISCUSSIONS

Results shown in Figure-1 illustrate that upto an inlet stage specific area value of 280 sqm efficiency increased proportionately and there after it remains constant and any further increase would reduce the efficiency due to an increase in frontal diameter resulting in increase of mean blade height. It has been observed that at that specific area the optimal value of stage efficiency is 0.895 which is quite acceptable for aero space compressor. It has been observed from sensitivity graphs (Figures 2 and 3) that efficiency is most sensitive to changes in rotor shaft diameter, air inlet angle and flow coefficient where as inlet stage specific area is most sensitive to changes in hub-tip ratio. Table-1 illustrates the initial and optimal values of efficiency and inlet stage specific area.

CONCLUSIONS

In this study the elitist non dominated sorted genetic algorithm (NSGA-II) has been implemented on a two objective non linear multi variable problem of axial flow compressor stage. The Pareto optimal front for the two objectives has been illustrated. The main advantage with this technique is that multi objective problems can be treated successfully while preserving the elitism, which is not possible with conventional multi objective techniques such as weighted sum method and e-constraint method.

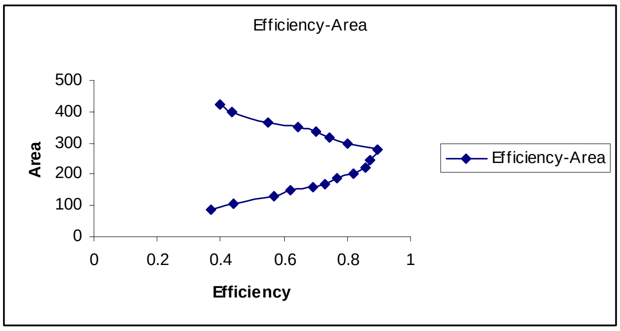

Figure-1. Pareto front for efficiency and specific area.

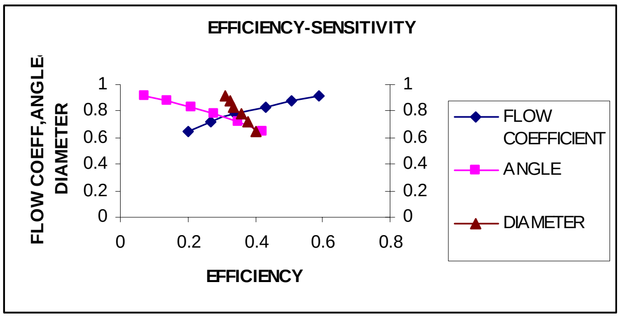

Figure-2. Sensitivity analysis of efficiency.

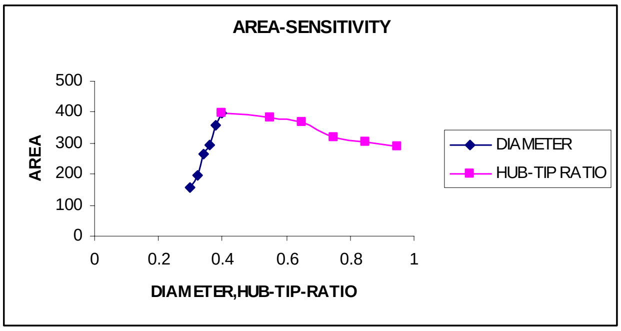

Figure-3. Inlet specific area sensitivity.

Table-1. Initial and optimal values of efficiency and specific area.

Efficiency		**Area** (m^2)	
Initial	Optimal	Initial	Optimal
0.807	0.895	196	280

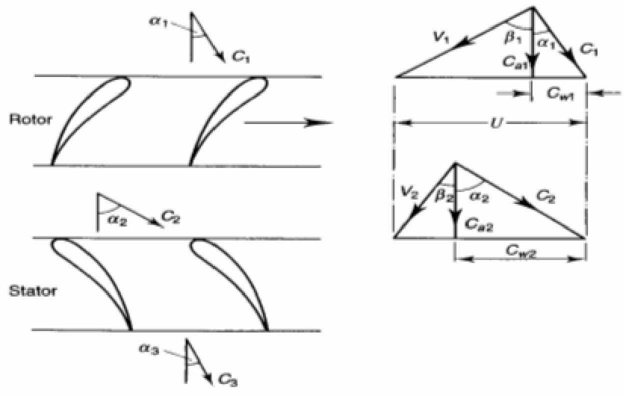

Figure-4. Velocity triangles.

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