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ON YIELD LINE ESTIMATES OF THE PUNCHING STRENGTH OF FULL PANEL UNBONDED POST-TENSIONED FLAT SLABS AT INTERNAL COLUMNS

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ABSTRACT

In this study, the yield line method of analysis is used to calculate the ultimate punching capacity of ten full panel, unbonded post-tensioned slabs, six of which were subject to combined vertical and transfer of moment loading. It is found that the method estimates to within 10% the failure loads for cases involving purely vertical loading. However, the predictions for those slabs subjected to moment transfer are not consistent. An important consideration in this regards was the presence or otherwise of ordinary bonded reinforcement at the critical positive moment regions of the slabs. It is suggested that alternative flexural mechanisms of punching failure should be utilized in lieu of yield line theory to obtain reliable estimates of the punching capacity.

Keywords: slab, column, yield line, punching, flexural.

Notations

c	side dimension for a square column
c_1, c_2	column side dimensions in direction of bending and in the orthogonal direction respectively
e	eccentricity of column load
f_{cu}	characteristic cube compressive strength
f_y	yield stress of reinforcement
h	overall slab depth
L	slab span, centre-line to centre-line of columns
L_n	clear span of slab system
m_1, m_2	negative and positive ultimate moment of resistance per unit width of slab respectively
M_u	ultimate unbalanced bending moment applied
r	radius of yield fans
$(V_o)_{YL}$	predicted failure load for zero moment transfer based on yield line theory
V_T	ultimate experimental punching capacity
V_u	ultimate vertical shear force or column load
V_{YL}	ultimate punching capacity predicted by yield line theory
w	ultimate uniformly distributed load per unit area
ρ	reinforcement ratio
$ø$	angle defining the unknown dimension of yield line pattern when gravity load is small

INTRODUCTION

Limit analysis and design of flat slabs requires the simultaneous consideration of a number of different possible failure modes. The yield line method is frequently used for the limit analysis of such structures subject to gravity or lateral loads [7]. It is an upper bound approach and is based on a rigid-plastic model of slab bending and a requirement for geometric admissibility of the likely collapse modes. The assumption is also made that all regions of the slab possess sufficient rotational capacity to permit the entire yield mechanism to form prior to collapse.

The flexural nature of the punching phenomenon has been noted by several researchers [12, 6, 2]. For reinforced concrete slabs, the slab column junction is a location of both maximum shear and bending moment. Also such slabs are quite thin and flexible (span/depth ratios, $L/h \approx 25$), producing high bending stress to shear stress ratios. In post-tensioned flat slabs, the experimental and theoretical evidence for the relevance of flexural analyses is considerable. For example in tests on post-tensioned lift slabs, the great majority of the specimens developed the full moment capacity at the perimeter of the slab collars [20]. Also the load-deflection curves suggest a pronounced influence of flexure on the punching failure. In another model series, tendon stress increases of about 200% in some instances were observed for tendons passing through the columns [8]. In a different investigation, for one of the models just prior to failure, the initial crack widened considerably instead of new ones forming. For the remaining two specimens of the same series, all the bars passing through the columns yielded well before failure [22]. Also in the two multi-panel slabs tested in which the flexural characteristics and overall behaviour were the main factors under investigation, secondary punching failures were obtained at several of the internal connections after yield lines had developed in the positive moment region causing redistribution of moments to the columns [10]. Finally in moment transfer tests, all the top bonded reinforcement in the direction of moment transfer passing through the column yielded long before failure [9].

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Previous investigators, notably [6] and to a lesser extent [14, 23] have estimated the flexural strength using yield line theory. However these studies were carried out almost exclusively on reinforced concrete slabs and in addition most models were subject to only shear loading. In the present study the main focus is on full panel unbonded post-tensioned slabs under combined shear and moment transfer loadings, although a few models involving purely vertical loadings are examined. The occurrence of moment transfer loading is generally accepted to impose more severe stress conditions on the slab-column junction. Hence a comparison of test data with yield line predictions would help to determine the reliability of the latter in the assessment of the punching capacity of post-tensioned flab slab structures.

MATERIALS AND METHODS

Slabs utilized for present investigation

In recent years several tests of varying significance have been conducted on post-tensioned flat slab models [17, 18, 21, 4, 13, 19]. However, most of these models had span to depth ratios which were unrepresentative of those in prototype structures and did not adequately simulate the boundary conditions existing in such structures. In addition several of the afore-mentioned tests were intended to study the post-punching behaviour of prestressed flat slabs.

The results of six tests [9] are discounted here as they involved several variables including different slab-column configurations, the level of moment transfer, the distribution of tendons and reinforcing steel, the loading arrangements and the testing procedures. The unrealistic span to depth ratios employed, the very high eccentricities obtained for some test models and the probable interaction of the many variables present suggest that the test results should be viewed with considerable caution.

For the yield line analysis the test results employed were those of models extending to mid-span which were therefore capable of developing compressive in-plane forces which normally arise in continuous structures [22, 3]. Also the increases in tendon forces obtained for these models are generally of the same magnitude as those occurring in multi-panel structures. Finally as some of the models are statically indeterminate, load-deflection relationships are meaningful and redistribution of moments can occur as in multi-panel slabs. In addition boundary conditions for both vertical and combined loading situations in real prototype structures can be realistically modelled using such statically indeterminate specimens [3].

Yield line flexural collapse mechanisms

From an examination of crack patterns observed several possible collapse modes have been identified [22, 3]. Similar mechanisms have also been suggested by several investigators as being typical for slab-column connections subject to unbalanced moment transfer [16, 5, 15]. The critical mechanism is partially governed by the level of moment transfer. The yield line mechanisms are illustrated in Figures 1, 2, 3 and 4.

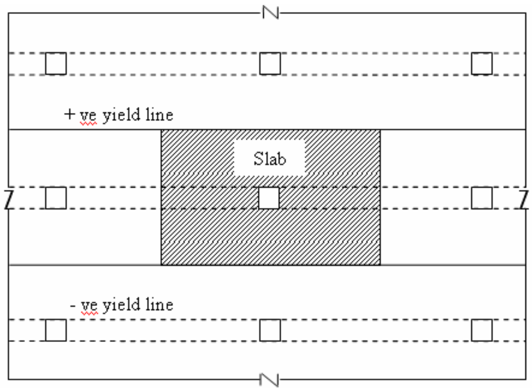

Figure-1. Overall yield line mechanism.

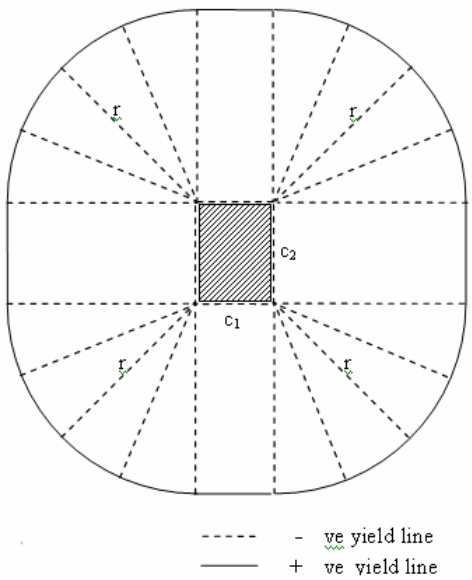

Figure-2. Local yield line mechanism for pure gravity loading.

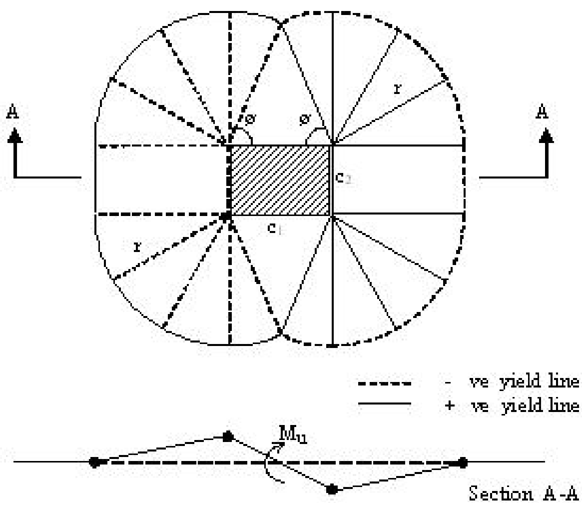

Figure-3. Local yield line mechanism when gravity loading is not significant.

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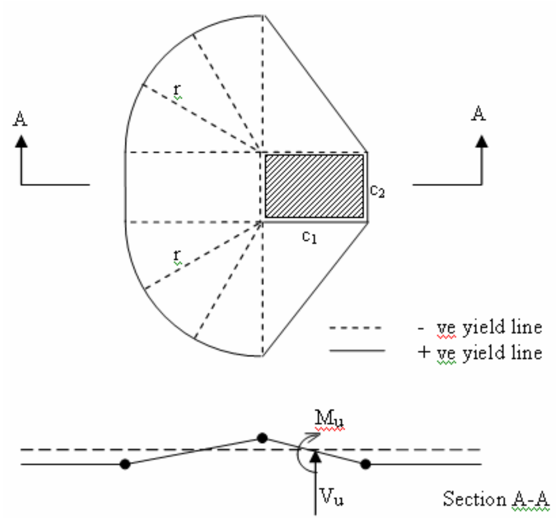

Figure-4. Local yield line mechanism when gravity loading is significant.

(a) Overall flexural collapse

In Figure-1 negative yield lines run across the slab at the column face. For the statically indeterminate models, positive yield lines are also obtained at mid-panel regions. The ultimate load equation is simply given by

$$m_1 + m_2 = wL_n^{\ 2}/8 \tag{1}$$

where m_1 and m_2 are the negative and positive ultimate moments of resistance per unit width of the slab respectively, w is the ultimate uniformly distributed load per unit area, and L_n is the clear span.

(b) Local yield line pattern for pure gravity loading

From Figure-2, for square panels and square columns, $c_1 = c_2 = c$, and

$$\frac{wL^2}{m_1 + m_2} = \frac{2[\pi + (2c/r)]}{1 - (c/L)^2 - (2cr/L^2) - (\pi/3)(r/L)^2} \tag{2}$$

where L is the panel centre-line dimension in directions parallel to the column sides, r is the radius of the yield line fans with centres at the column corners and c is the column side dimension.

(c) Local yield line mechanism for moment transfer when the gravity load is small

Referring to Figure-3, the virtual work equation assuming a square column can be shown to result in

$$M_u = (m_1 + m_2)[2c(\pi - \phi) + c(1 + 2\cos\phi) + c\tan\phi] \tag{3}$$

where M_u is the ultimate unbalanced bending moment applied, and is the angle defining the unknown dimension of the yield line pattern. The minimum M_u can be shown [16] to be

$$M_u = 7.92(m_1 + m_2)c \tag{4}$$

(d) Local yield line pattern for combined loading when the gravity load is significant:
From Figure-4, the minimum value of M_u for square columns can be shown [15] to be

$$M_u = 9.04(m_1 + m_2)c - 0.5V_u c \tag{5}$$

where V_u is the ultimate vertical shear force or column load.

Equations (1), (2), (4) and (5) are applicable in the case of isotropic reinforcement. For orthotropic distributions such as the use of banded tendon arrangement in tests [3], it has been shown that the total yield moment available is the major parameter in determining the flexural punching strength, and not the distribution of the yield moments in the two orthogonal directions. Consequently for such cases, it is possible to use the concept of an equivalent isotropic slab [5]. However calculations not shown here suggest that for pure shear loading, the effect of using a greater concentration of reinforcement in the column strip than in the middle strip is to ensure that the overall collapse mode is the most critical. This conclusion can be verified from a study of the crack patterns recorded in tests [3].

RESULTS AND DISCUSSIONS

The yield line predictions are compared with the test results of Franklin and Long [3] and Smith and Burns [22] in Table-1.

In Table-1 the term e/L is a measure of the eccentricity of loading while $(V_O)_{YL}$ is the calculated flexural capacity assuming no moment transfer. The comparison of yield line predictions with test data is also presented in Figures 5 and 6.

The models of Smith and Burns [22] and models 1B, 2M and 5B of Franklin and Long [3] had similar detailing configurations and from Figure-5 it is obvious that the results for these models are best expressed by a linear trend line. In contrast the results for models 3M, 4M, 6B and 7B of Franklin and Long [3] which were detailed differently do not clearly follow the same trend line as earlier mentioned. From Figure- 6 it is evident that models 1B, 2M and 5B of Franklin and Long [3] failed by the formation of a full yield line mechanism. These models had no bonded reinforcement to control transverse cracking proceeding from the column centre line to the positive moment region at the slab edges. Also models S1, S2 and S3 of Smith and Burns [22] failed in a similar manner, as borne out by the experimental load-deflection curves and from the load-strain relationships for the top reinforcing bars through the columns. This is despite the use of the common 70%: 30% tendon distribution, suggesting that additional bonded reinforcement may be required in the slab middle strips or positive moment regions as well.

Models 3M, 4M, 6B and 7B of Franklin and Long [3] all had extra bonded reinforcement in the middle strips and failed in punching prior to the formation of a full yield line pattern. From Figure-6 it is obvious that the yield line predictions are unsafe and less reliable at higher

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levels of moment transfer. For example, the method overestimates the failure load of model 6B by about 37%. Consequently it is recommended that yield line analysis should not be applied in estimating the flexural strength of such models. It has been noted that an overall yield line mechanism does not form prior to punching [1]. Also it has been concluded that tests on slabs with $\rho f_y/f_{cu}$ values ranging from 0.05 to 0.2 show no evidence of a complete yield line mechanism [11]. Hence it is proposed here that alternative flexural mechanisms of punching failure should be explored in order to assess the ultimate punching strength for combined loading situations.

Table-1. Comparison of yield line predictions with test data.

Investigator	Model	e/L	Test load V_T (kN)	Yield line load V_{YL} (kN)	Yield line load $(V_O)_{YL}$ (kN)	V_T/V_{YL}	$V_T/(V_O)_{YL}$
Franklin and Long [3]	1B	0.039	100.1	96.6	111.1	1.04	0.90
	2M	0.038	102.6	98.0	112.9	1.05	0.91
	3M	0.050	74.5	86.3	140.4	0.86	0.53
	4M	0.070	79.4	97.6	163.0	0.81	0.49
	5B	0.090	57.6	57.6	92.2	1.00	0.62
	6B	0.122	55.2	75.7	141.1	0.73	0.39
	7B	0	127.9	142.0	142.0	0.90	0.90
Smith and Burns [22]	S1	0	112.3	104.1	104.1	1.08	1.08
	S2	0	121.4	117.7	117.7	1.03	1.03
	S3	0	135.3	127.3	127.3	1.06	1.06

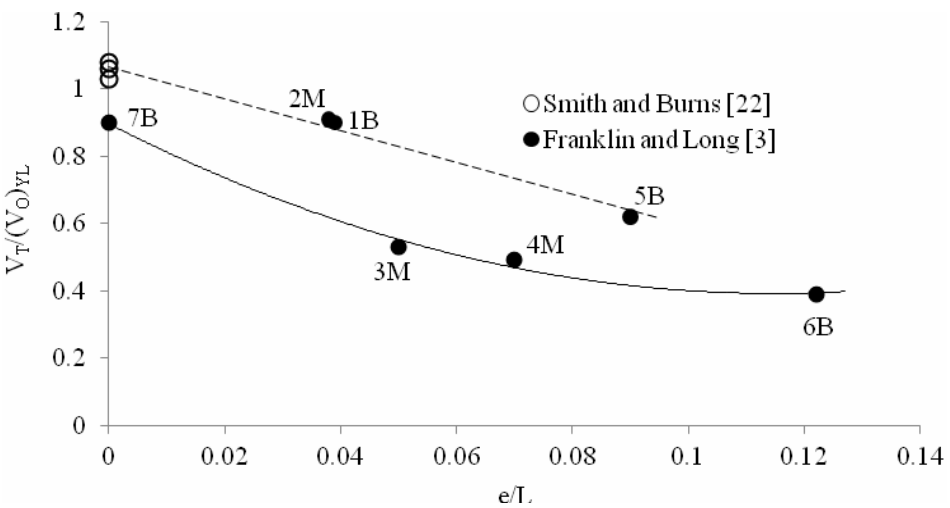

Figure-5. Comparison of yield line predictions with test results.

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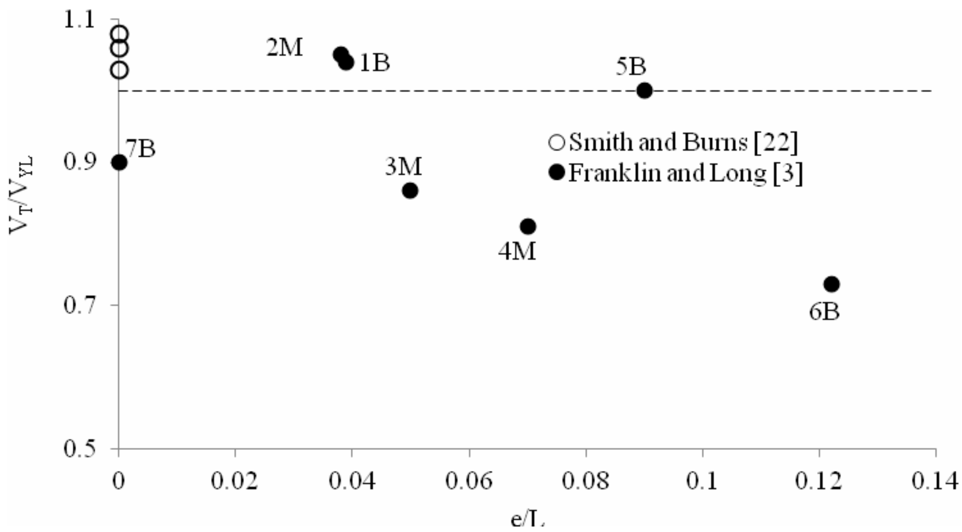

Figure-6. Comparison of yield line estimates of the punching capacity with test results.

CONCLUSIONS

From the preceding study the following conclusions can be drawn:

a) For pure shear loading, yield line predictions are in reasonable agreement with test results;
b) For slabs subject to moment transfer, yield line predictions of the punching capacity are unsafe and less reliable at higher levels of eccentricity. A critical factor is the presence or otherwise of ordinary bonded reinforcement in the positive moment regions; and
c) Alternative flexural mechanisms of punching failure should be used in lieu of yield line theory in assessing the punching capacity for cases involving combined shear and moment transfer.

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