



UNSTEADY TWO-PHASE VISCOUS-IDEAL FLUID FLOW THROUGH A PARALLEL PLATE CHANNEL UNDER A PULSATILE PRESSURE GRADIENT SUBJECTED TO A BODY ACCELERATION

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ABSTRACT

In this paper, we analyzed the flow of combined two-phase motion of viscous-ideal medium through a parallel plate channel under the influence of an imposed pressure gradient and a periodic body acceleration. We elucidate the development of laminar flow in such a medium, starting from the unsteady equations of incompressible two-phase of viscous-ideal fluid. The velocities in both the media and the shear stresses on the boundary plates are analytically evaluated and their behavior with reference to variations in the governing parameters is computationally discussed.

Keywords: Reynolds number, interaction parameter, imposed pressure gradient, viscous-ideal fluid.

1. INTRODUCTION

The study of two-phase fluid flow has gained immense importance in view of its applications in a wide variety of practical and technological problems. Many natural phenomena involve simultaneous flow of several phases of matter [21]. Some occurring in nature such as smog, fog, smoke, rain, dust storm, etc., are examples of two phase flows. A few mechanisms like boiling water, flow of decoction through coffee percolator, preparation of Martin, Bread, Cake, Spaghetti etc, experienced in our daily life, are also examples of two-phase flows. Body fluids like blood and semen are multiphase containing variety of cells. Examples are equally profuse Engineering and Industrial fields. Industrial process such as power generation, refrigeration and distillation depend on evaporation and condensation cycles. The performance of desalination plants is limited by the 'stat of art' in two-phase technology. Steel making, Paper manufacturing, Food processing all containing critical steps which depend on the proper functioning of multiphase devices. The main problem of mathematical modeling of multiphase mixtures consists on setting up the closed system of equations of motion with given physico-chemical properties of each phase and with a given original structure of the mixture [14, 17 and 20]. The mathematical analyses of two-phase flow involves framing the closed system of equation i.e., in finding the equations of simultaneous deformation of the different phased and the equations for interphase and interphase interactions [1, 2, 3, 10-16, 18, 19]. There are two methods of solving the closing problem. In the first, the so called phenomenological method the closing relationships between macro variables are postulated on the basis of macroscopic experiments and intuitive considerations. The second method is the Kinetic one in which the closing relation are derived proceeding from an analyses of micro processes around drops etc., with the use of time, spatial and ensemble averaging.

The pioneering work, in which the closed system of equations for a multiphase mixture of compressible

phases with barotropic properties determining the common pressure in the mixture was suggested by Rakhmtulin [19]. Rakhmatulin scheme neglects the nonlinear dependence of interphase force F on the relative motion of the phases. Infact this approach differs from the others in that the interphase force is linearly proportional to the relative velocity of the phases. Here each phase is considered continuous and its motion is examined as motion in the moving and changing porous medium formed by the other phase. Based on Rakhmatulin [19] approach, Faizullaev [6] developed the theory of interpenetrating motion of multiphase media consisting of incompressible (or barotropic) and viscous (or ideal) fluids and solved several problems related to unidirectional flows in channels or pipes. One of the important common two-phase flow model is the combined viscous ideal flow. For example natural gas with water or petroleum may be considered as viscous ideal two-phase medium [5, 6, 8 and 9]. Applications of gas liquid flow are also found in boilers, condensers, refrigerating and also conditioning equipment. In most of these engineering applications we consider that the liquid and the gas are not of the same substance as is the case between air and water or between oil and natural gas. We assume two substances under certain temperature and pressure one being in its liquid state while the other is in its gaseous state. Also we may consider both the gas and the liquid as continuous media. Because of the complicated flow patterns, most of the treatments of two-phase flow of gas and liquid for engineering applications are semi empirical [2, 4, 7, 10, 16 and 17]. One of the cases of two-phase flow of mixture of gas and liquid where reasonable analytic treatment can be carried is the case of froth flow in which gas to liquid volume ratio is of the order of unity and the gas and the liquid are mixed together homogeneously [7, 8]. In considering the viscous ideal two-phase flows in non uniform channels, we would like to know the flow rate and pressure drop as affected by the fluid properties, channel geometry. Faizullaev [6] studied the combined two-phase motion of a viscous and



an ideal medium in finite and infinite planes. As is known, gases are considered ideal in aerodynamics, many theoretical results and deductions obtained under such an assumption agree with experiment.

In situations like travel in vehicles, aircraft, operating jackhammer and sudden movements of body during sports activities, the human body experiences external body acceleration. Prolonged exposure of a healthy human body to external acceleration may cause serious health problem like headache, increase in pulse rate and loss of vision on account of disturbances in blood flow Majhi and Nair [12]. It has been established that the biological systems in general are greatly affected by the application of external magnetic field. So far, the theoretical studies dealing with the influence of applied magnetic field on blood flow have received very little attention Ramachandra Rao and Deshikachar and Ramachandra Rao [5]. Many researchers have studied blood flow in the artery by considering blood as either Newtonian or non-Newtonian fluids, since blood is a suspension of red cells in plasma; it behaves as a non-Newtonian fluid at low shear rate. Chaturani and Palanisamy [4] studied pulsatile flow of blood through a rigid tube under the influence of body acceleration as a Newtonian fluid.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Consider a two-phase incompressible viscous-ideal two phase fluid flow through a parallel plate channel moving under a pulsatile pressure gradient and subjected to body acceleration.

Choosing Cartesian frame of reference O (x,y, z). Let z=0 and h be the upper and lower plates bounding the two phase fluid. Initially the fluid is rest and at time t = 0, the fluid is subjected to pulsatile pressure gradient and periodic body acceleration given by

$$-\frac{\partial p}{\partial x} = A_1 \cos \omega_1 t \text{ and } PG = a_0 \cos \omega_2 t$$

Where A_1 is the amplitude of the oscillatory part $\omega_1 = 2\pi f_1$ where f_1 is the pulse frequency, a_0 is the amplitude of body acceleration, $\omega_2 = 2\pi f_2$ where f_2 is the body acceleration frequency.

The channel extends to infinity along (x,y) directions and the two phase flow takes place between the parallel channel walls, since the flow is slowly due to axial pressure gradient and the body acceleration parallel to the direction of the flow, the flow is unidirectional and hence the velocity field is $(u, 0, 0)$ and in view of the continuity

$$\text{equation } \frac{\partial u}{\partial x} = 0 \Rightarrow u \neq u(x) \text{ and the motion being two}$$

dimensional $u \neq u(y)$ and hence $u = u(z, t)$.

The two phase mixture being incompressible the density may assume to be constant in each phase. Assuming f_1 and f_2 to be the porosities of the two phases

to be constants. Let ρ_1 and ρ_2 be the reduced densities of the two phases. Let μ_2 be the coefficients of viscosities of the two phase of the medium.

The equations governing the flow of the two phases are

$$\rho_1 \frac{\partial v}{\partial t} = k(u - v) + f_1 A \cos \omega_1 t + \rho_1 a_0 \cos \omega_2 t \dots (1)$$

$$\rho_2 \frac{\partial u}{\partial t} = f_2 \mu_2 \frac{\partial^2 u}{\partial z^2} + k(v - u) + f_2 A \cos \omega_1 t + \rho_2 a_0 \cos \omega_2 t \dots (2)$$

Where u, v are the axial velocities of the two phases? K is the interaction parameter of the two-phases.

The relevant boundary conditions are (dimensional)

$$u = 0 \quad z = 0 \quad \text{and} \quad z = h \dots (3)$$

We introduce the following the following non-dimensional variables.

$$z^* = \frac{z}{h} \quad u^* = \frac{h}{\mu_2} v \quad v^* = \frac{h}{\mu_2} u$$

$$t^* = \omega_1 t \quad A^* = \frac{h^3}{\rho_2 \gamma_2^2} A \dots (4)$$

Substituting these (2.2.4) in equation (2.2.1) to (2.2.2), the governing equations reduces to (on dropping the asterisks)

$$S \frac{\partial v}{\partial t} = K(u - v) + f_1 A \cos t + a_0 \cos \omega t \dots (5)$$

$$R \frac{\partial u}{\partial t} = f_2 \frac{\partial^2 u}{\partial z^2} + K(v - u) + f_2 A \cos t + a_0 \cos \omega t \dots (6)$$

Where

$$R = \frac{w_1 h^2}{\nu_2} \text{ is the Reynolds Number}$$

$$S = \frac{w_1 h^2 \rho_1}{\nu_1 \rho_2} \text{ is the product of Reynolds number and the rate of the densities}$$

$$K = \frac{h^2 k}{\mu_2} \text{ is the interaction parameter of the two-phases}$$

The non-dimensional boundary conditions relevant to the problem are

$$u = 0 \quad z = 0 \quad \text{and} \quad z = 1 \dots (7)$$

Taking Laplace transforms in (2.2.5), we obtain the equation for \bar{v}

$$\bar{v} = \frac{K \bar{u}}{Ss + P} + \frac{f_1 A s}{(Ss + P)(s^2 + 1)} + \frac{a_0 s}{(Ss + K)(s^2 + \omega^2)} \dots (8)$$

Taking transforms in eq. (6) and substituting in eq. (8) for \bar{v} the equation for \bar{u} reduces to



$$\frac{\partial^2 \bar{u}}{\partial z^2} - \frac{(Rs+K)\bar{u}}{f_2} + \frac{K\bar{v}}{f_2} + \frac{As}{(s^2+1)} + \frac{a_0 s}{f_2(s^2+\omega^2)} = 0 \quad \dots (9)$$

The transformed conditions are

$$u = 0 \quad z = 0 \quad \text{and} \quad z = 1$$

Substituting (2.2.8) in (2.2.9) we get

$$\bar{u} = \frac{(Kx+y)\text{Cos}\lambda_1 z}{\lambda_1^2 f_2} + \frac{(Kx+y)\text{Sin}\lambda_1 z}{\lambda_1^2 f_2 \text{Sin}\lambda_1} - \frac{(Px+y)\text{Cos}\lambda_1 h \text{Sin}\lambda_1 z}{\lambda_1^2 f_2 \text{Sin}\lambda_1} - \frac{(Kx+y)}{\lambda_1^2 f_2} \quad \dots (11)$$

Taking laplace inversions

$$\begin{aligned} u = & a_{11}[b_{11}\text{Cos}\alpha_3 e^{-dt} + b_{12}\text{Cos}\alpha_4 e^{it} + b_{13}\text{Cos}\alpha_5 e^{-it} + b_{14}\text{Cos}\alpha_6 e^{-\frac{K}{S}t}] + \\ & + a_{13}[b_{15}\text{Cos}\alpha_3 e^{-dt} + b_{16}\text{Cos}\alpha_7 e^{i\omega t} + b_{17}\text{Cos}\alpha_8 e^{-i\omega t} + b_{18}\text{Cos}\alpha_6 e^{-\frac{K}{S}t}] + \\ & + a_{14}[b_{19}\text{Cos}\alpha_3 e^{-dt} + b_{20}\text{Cos}\alpha_4 e^{it} + b_{21}\text{Cos}\alpha_5 e^{-it}] + a_{15}[b_{22}\text{Cos}\alpha_3 e^{-dt} + b_{23}\text{Cos}\alpha_7 e^{i\omega t} + b_{24}\text{Cos}\alpha_8 e^{-i\omega t}] + \\ & + a_{11}[\frac{b_{12}\text{Sin}\alpha_4 e^{it}}{\text{Sin}\alpha_4} + \frac{b_{13}\text{Sin}\alpha_5 e^{-it}}{\text{Sin}\alpha_5} + \frac{b_{14}\text{Sin}\alpha_6 e^{-\frac{K}{S}t}}{\text{Sin}\alpha_6} + \frac{e^{\alpha t} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+1)(S\alpha+K)(-1)^n h} + \frac{e^{\beta t} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+1)(S\beta+K)(-1)^n h}] + \\ & + a_{13}[\frac{b_{16}\text{Sin}\alpha_7 e^{it}}{\text{Sin}\alpha_7} + \frac{b_{17}\text{Sin}\alpha_8 e^{-it}}{\text{Sin}\alpha_8} + \frac{b_{18}\text{Sin}\alpha_6 e^{-\frac{K}{S}t}}{\text{Sin}\alpha_6} + \frac{e^{\alpha t} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+\omega^2)(S\alpha+K)(-1)^n h} + \frac{e^{\beta t} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+\omega^2)(S\beta+K)(-1)^n h}] + \\ & + a_{14}[\frac{b_{25}\text{Sin}\alpha_4 e^{i\omega t}}{\text{Sin}\alpha_4} + \frac{b_{26}\text{Sin}\alpha_5 e^{-i\omega t}}{\text{Sin}\alpha_5} + \frac{e^{\alpha t} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+1)(S\alpha+K)(-1)^n h} + \frac{e^{\beta t} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+1)(S\beta+K)(-1)^n h}] + \\ & + a_{15}[\frac{b_{27}\text{Sin}\alpha_7 e^{i\omega t}}{\text{Sin}\alpha_7} + \frac{b_{28}\text{Sin}\alpha_8 e^{-i\omega t}}{\text{Sin}\alpha_8} + \frac{e^{\alpha t} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+\omega^2)(S\alpha+K)(-1)^n h} + \frac{e^{\beta t} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+\omega^2)(S\beta+K)(-1)^n h}] - \\ & - a_{11}[b_{11}\alpha_3 e^{-dt} \frac{b_{12}\text{Cos}\alpha_4 \text{Sin}\alpha_4 e^{it}}{\text{Sin}\alpha_4} - \frac{b_{13}\text{Cos}\alpha_5 \text{Sin}\alpha_5 e^{-it}}{\text{Sin}\alpha_5} - \frac{b_{14}\text{Cos}\alpha_6 \text{Sin}\alpha_6 e^{-\frac{K}{S}t}}{\text{Sin}\alpha_6} - \frac{e^{\alpha t} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+1)(S\alpha+K)h} - \frac{e^{\beta t} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+1)(S\beta+K)h}] - \\ & - a_{13}[b_{15}\alpha_3 e^{-dt} \frac{b_{16}\text{Cos}\alpha_7 \text{Sin}\alpha_7 e^{i\omega t}}{\text{Sin}\alpha_7} - \frac{b_{17}\text{Cos}\alpha_8 \text{Sin}\alpha_8 e^{-i\omega t}}{\text{Sin}\alpha_8} - \frac{b_{18}\text{Cos}\alpha_6 \text{Sin}\alpha_6 e^{-\frac{K}{S}t}}{\text{Sin}\alpha_6} - \frac{e^{\alpha t} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+\omega^2)(S\alpha+K)h} - \frac{e^{\beta t} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+\omega^2)(S\beta+K)h}] - \\ & - a_{14}[b_{19}\alpha_3 e^{-dt} \frac{b_{20}\text{Cos}\alpha_4 \text{Sin}\alpha_4 e^{i\omega t}}{\text{Sin}\alpha_4} - \frac{b_{21}\text{Cos}\alpha_5 \text{Sin}\alpha_5 e^{-i\omega t}}{\text{Sin}\alpha_5} - \frac{e^{\alpha t} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+1)(S\alpha+K)h} - \frac{e^{\beta t} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+1)(S\beta+K)h}] - \\ & - a_{15}[b_{22}\alpha_3 e^{-dt} \frac{b_{23}\text{Cos}\alpha_7 \text{Sin}\alpha_7 e^{i\omega t}}{\text{Sin}\alpha_7} - \frac{b_{24}\text{Cos}\alpha_8 \text{Sin}\alpha_8 e^{-i\omega t}}{\text{Sin}\alpha_8} - \frac{e^{\alpha t} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+\omega^2)(S\alpha+K)h} - \frac{e^{\beta t} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+\omega^2)(S\beta+K)h}] - \\ & - a_{11}[b_{29} e^{-\frac{K}{S}t} - b_{30} e^{it} - b_{31} e^{-it} - b_{32} e^{-dt}] - a_{13}[b_{29} e^{-\frac{K}{S}t} - b_{32} e^{it} - b_{33} e^{-it} - b_{34} e^{-dt}] - \\ & - a_{14}[b_{36} e^{it} - b_{37} e^{-it} - b_{38} e^{-dt}] - a_{15}[b_{39} e^{i\omega t} - b_{40} e^{-i\omega t} - b_{41} e^{-dt}] \\ v = & a_{41}[b_{11}\text{Cos}\alpha_3 e^{-dt} + b_{12}\text{Cos}\alpha_4 e^{it} + b_{13}\text{Cos}\alpha_5 e^{-it} + b_{14}\text{Cos}\alpha_6 e^{-\frac{K}{S}t}] + \end{aligned}$$



$$\begin{aligned}
& + a_{42} [b_{15} \text{Cos} \alpha_3 e^{-dt} + b_{16} \text{Cos} \alpha_7 e^{i\omega t} + b_{17} \text{Cos} \alpha_8 e^{-i\omega t} + b_{18} \text{Cos} \alpha_6 e^{-\left(\frac{K}{S}\right)t}] + \\
& + a_{41} \left[\frac{b_2 \text{Sin} \alpha_4 e^{it}}{\text{Sin} \alpha_4} + \frac{b_3 \text{Sin} \alpha_5 e^{-it}}{\text{Sin} \alpha_5} + \frac{b_4 \text{Sin} \alpha_6 e^{-\left(\frac{K}{S}\right)t}}{\text{Sin} \alpha_6} + \frac{e^{\alpha} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+1)(S\alpha+K)(-1)^n h} + \frac{e^{\beta} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+1)(S\beta+K)(-1)^n h} \right] + \\
& + a_{42} \left[\frac{b_{16} \text{Sin} \alpha_7 e^{it}}{\text{Sin} \alpha_7} + \frac{b_{17} \text{Sin} \alpha_8 e^{-it}}{\text{Sin} \alpha_8} + \frac{b_{18} \text{Sin} \alpha_6 e^{-\left(\frac{K}{S}\right)t}}{\text{Sin} \alpha_6} + \frac{e^{\alpha} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+\omega^2)(S\alpha+K)(-1)^n h} + \frac{e^{\beta} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+\omega^2)(S\beta+K)(-1)^n h} \right] - \\
& - a_{41} \left[b_1 \alpha_5 e^{-dt} - \frac{b_2 \text{Cos} \alpha_4 \text{Sin} \alpha_4 e^{it}}{\text{Sin} \alpha_4} - \frac{b_3 \text{Cos} \alpha_5 \text{Sin} \alpha_5 e^{-it}}{\text{Sin} \alpha_5} - \frac{b_4 \text{Cos} \alpha_6 \text{Sin} \alpha_6 e^{-\left(\frac{K}{S}\right)t}}{\text{Sin} \alpha_6} - \frac{e^{\alpha} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+1)(S\alpha+K)h} - \frac{e^{\beta} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+1)(S\beta+K)h} \right] - \\
& - a_{42} \left[b_5 \alpha_3 e^{-dt} - \frac{b_6 \text{Cos} \alpha_7 \text{Sin} \alpha_7 e^{i\omega t}}{\text{Sin} \alpha_7} - \frac{b_7 \text{Cos} \alpha_8 \text{Sin} \alpha_8 e^{-i\omega t}}{\text{Sin} \alpha_8} - \frac{b_8 \text{Cos} \alpha_6 \text{Sin} \alpha_6 e^{-\left(\frac{K}{S}\right)t}}{\text{Sin} \alpha_6} - \frac{e^{\alpha} \text{Sin} \frac{n\pi}{h} z}{(\alpha+d)(\alpha^2+\omega^2)(S\alpha+K)h} - \frac{e^{\beta} \text{Sin} \frac{n\pi}{h} z}{(\beta+d)(\beta^2+\omega^2)(S\beta+K)h} \right] - \\
& - a_{41} [b_{29} e^{-\left(\frac{K}{S}\right)t} - b_{30} e^{it} - b_{31} e^{-it} - b_{32} e^{-dt}] - a_{42} [b_{35} e^{-\left(\frac{K}{S}\right)t} - b_{32} e^{i\omega t} - b_{33} e^{-i\omega t} - b_{34} e^{-dt}] + \\
& + a_{43} [b_{42} e^{it} + b_{43} e^{-it} + b_{45} e^{-\left(\frac{K}{S}\right)t}] + a_{44} [b_{45} e^{it} + b_{46} e^{-it} + b_{47} e^{-\left(\frac{K}{S}\right)t}] - a_{45} [b_{48} e^{-it} - b_{49} e^{it} - b_{50} e^{-dt}]
\end{aligned}$$

Solve the equation, we get a_{ij} , b_{ij} .

The shear stresses on the lower plate have been calculated using

$$(\tau)_{lower} = f_1 \frac{\partial v}{\partial z}$$

Solution of the shear stresses on the lower plate is

$$(\tau)_{lower} = f_1 (G_1 - G_2 - G_3 - G_4 - G_5 + G_6 + G_7 + G_8)$$

The shear stresses on the upper plate have been calculated using

$$(\tau)_{upper} = f_2 \frac{\partial u}{\partial z}$$

Solution of the shear stresses on the upper plate is

$$(\tau)_{upper} = f_2 (-G_9 - G_{10} - G_{11} - G_{12} - G_{13} + G_{14} + G_{15} + G_{16})$$

Solve the above equations, we get $G_1, G_2, G_3, \dots, G_{16}$

3. DISCUSSION OF THE RESULTS

We now discuss the behavior of two-phase flow (viscous ideal flow) in a parallel plate channel whose boundaries are at rest. The flow is taking place under the influence of imposed pressure gradient and periodic body acceleration. The velocity of the two-phases as well as the shear stresses are evaluated numerically for different values of the governing parameters S , R , K , fixing A, a_0, f_1 and f_2 . It is to be noted that one of the phases being an inviscid phase, the axial velocity v of the inviscid phases does not satisfy the any slip condition on the boundary, and infact attains its maximum on it. The non dimensional variables are so chosen that the non dimensional parameters S associated with the inviscid phase is always less than the viscous Reynolds number R associated with the viscous phase.

The pressure gradient is computationally chosen as negative, so that the flow due to the pressure gradient is

from right to left along the channel. That is the axial velocity u triggered due to the imposed pressure gradient is negative. However, the body acceleration is chosen positive so that the flow caused by the body acceleration is in the positive direction along the channel Figures 1 and 2 correspond to the behavior of the axial velocity of the viscous phase with reference to variation in K the interaction parameter.

In general u rises from zero on the lower plate to attain its maximum near the upper plate ($z = 0.8$), before reducing to rest on the upper plate. Also the magnitude of u reduces with increase in K for fixed R and S (Figures 1 and 2). From Figure-3 fixing S and K , we notice that u reduces with increase in R . However we observe that u enhances with increase in S fixing R and K (Figure-4). Figures 5 and 6 correspond to the behavior of v w.r.t variation in K fixing S and R . In general v enhances from its minimum on the lower plate to its maximum on the upper plate. The magnitude of v reduces with increase in K (Figures 5 and 6). Fixing S and K , v reduces with increase in R in the entire flow region (Figure-7). From Figure-8 we find that the velocity of the inviscid phases reduces with increase in S fixing R and K . The shear stresses on the lower plate and upper plate have been evaluated for variations in K , S and R and tabulated in Tables 1 and 2, respectively. On the upper plate the shear stress reduces with R for all K fixing S . Also the stresses reduce rapidly with increase in K fixing R (Table-2). On the lower plate the shear stress shows a declination for increase in K (≤ 0.5) but latter enhances for K (> 0.5). When K and S are fixed we notice the shear stress reduces with increase in R (Table-1).

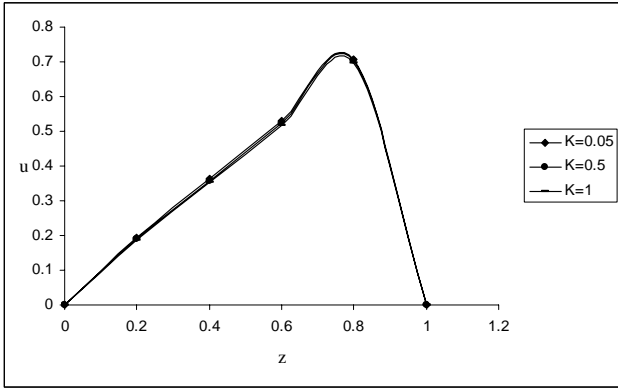


Figure-1. The velocity profile u with K
 $A = 1, f_1 = 0.4, f_2 = 0.6, R = 150, S = 100, a_0 = 1$

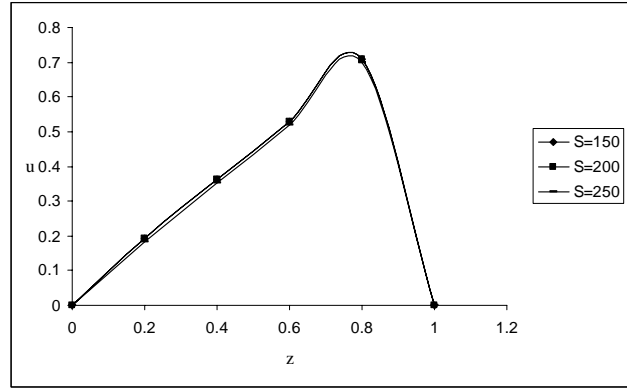


Figure-4. the velocity profile u with S
 $A = 1, f_1 = 0.4, f_2 = 0.6, K = 0.05, R = 250, a_0 = 1$

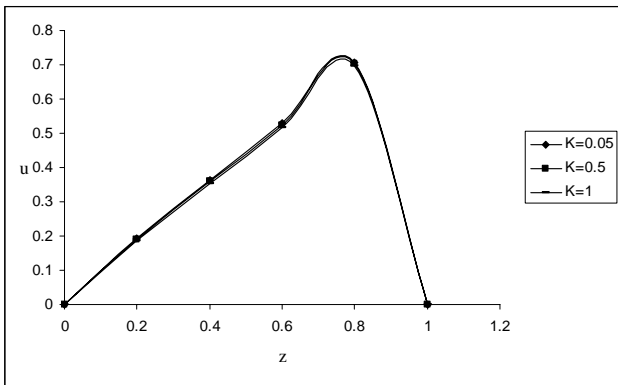


Figure-2. The velocity profile u with K
 $A = 1, f_1 = 0.4, f_2 = 0.6, R = 200, S = 100, a_0 = 1$

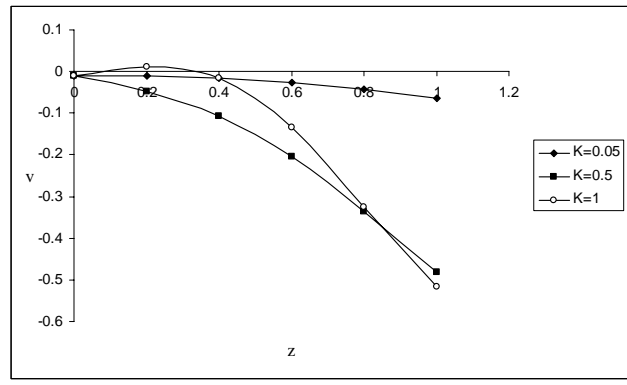


Figure-5. The velocity profile v with K
 $A = 1, f_1 = 0.4, f_2 = 0.6, S = 100, R = 100, a_0 = 1$

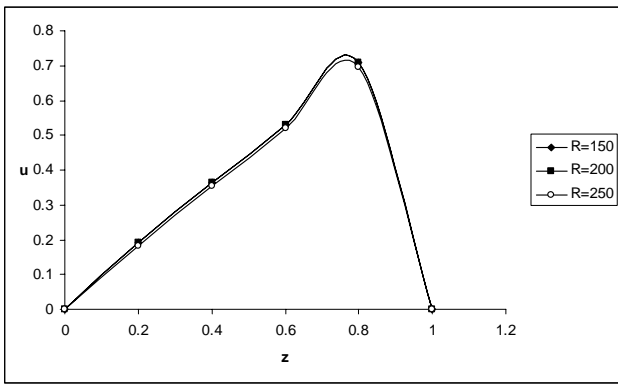


Figure-3. The velocity profile u with R
 $A = 1, f_1 = 0.4, f_2 = 0.6, K = 0.05, S = 150, a_0 = 1$

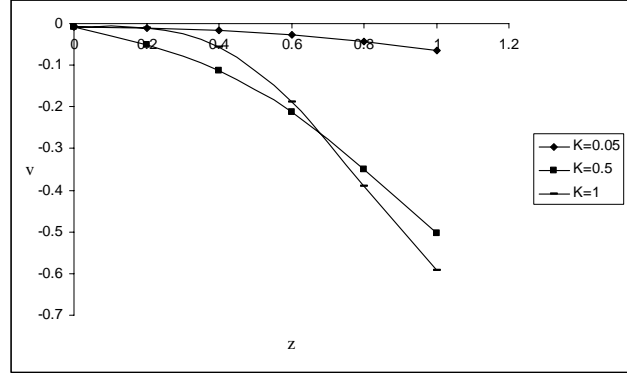


Figure-6. The velocity profile v with K
 $A = 1, f_1 = 0.4, f_2 = 0.6, S = 100, R = 200, a_0 = 1$

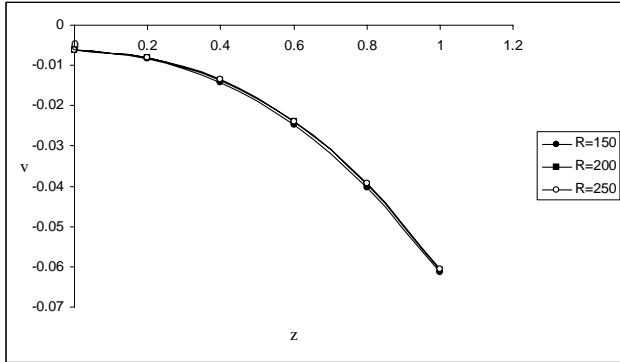


Figure-7. The velocity profile v with R
 $A = 1, f_1 = 0.4, f_2 = 0.6, S = 100, K = 0.05, a_0 = 1$.

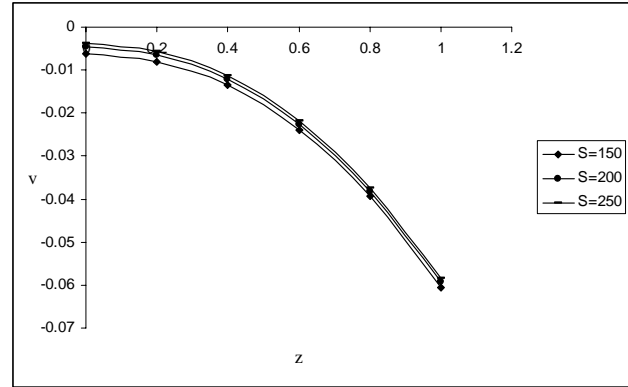


Figure-8. The velocity profile v with S
 $A = 1, f_1 = 0.4, f_2 = 0.6, R = 250, K = 0.05, a_0 = 1$.

Table-1.

| I | II | III | IV | V |
|---------|----------|---------|----------|----------|
| 0.63713 | 0.538864 | 0.64189 | 0.631057 | 0.535013 |

| | I | II | III | IV | V |
|---|------|-----|-----|------|-----|
| K | 0.05 | 0.5 | 1 | 0.05 | 1 |
| S | 100 | 100 | 100 | 100 | 100 |
| R | 150 | 150 | 150 | 200 | 200 |

The shear stresses on the lower plate at $z = 0$ level.

$$A = 1, f_1 = 0.4, f_2 = 0.6, R = 100, a_0 = 1, t = 0.9, h = 1, \omega = \frac{\pi}{2}, z = 0$$

Table-2.

| I | II | III | IV | V |
|----------|---------|---------|---------|----------|
| 0.618351 | 0.40133 | 0.39923 | 0.61796 | 0.364202 |

| | I | II | III | IV | V |
|---|------|-----|-----|------|-----|
| K | 0.05 | 0.5 | 1 | 0.05 | 1 |
| S | 100 | 100 | 100 | 100 | 100 |
| R | 150 | 150 | 150 | 200 | 200 |

The shear stresses on the lower plate at $z = 1$ level.

$$A = 1, f_1 = 0.4, f_2 = 0.6, R = 100, a_0 = 1, t = 0.9, h = 1, \omega = \frac{\pi}{2}$$



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