AXISYMMETRIC STEADY TWO-PHASE LIQUID-GAS FLOW IN A COAXIAL VARYING CYLINDRICAL SPACE BOUNDED INTERNALLY BY A RIGID PIPE

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ABSTRACT

In this paper, we discuss the two phase viscous-ideal flow taking place in the space between any two cylindrical pipes which approximate to an annular region bounded internally by a rigid pipe. The external boundary is coaxial non-uniform gap with no axial flow across the boundary in conformity with the symmetry. The governing nondimensional equations are solved using perturbation method with the slope of the non-uniform outer boundary very small. The velocity components both axial and radial in both the phases are evaluated and their behavior is discussed for variations in the governing parameters.

Keywords: Gas-liquid flow, cylindrical pipes, space, rigid pipe, velocity.

1. INTRODUCTION

Flows of two immiscible liquids are encountered in a diverse range of processes and equipment. In particular in the petroleum industry, where mixtures of oil and water are transported in pipes over long distances. Accurate prediction of oil-water flow characteristics, such as flow pattern, water holdup and pressure gradient is important in many engineering applications. However, despite of their importance, liquid-liquid flows have not been explored to the same extent as gas-liquid flows. In fact, gas-liquid systems represent a very particular extreme of two-fluid systems characterized by low-density ratio and low viscosity ratio. in recent years; Paras and Karabelas [1] have reported local velocity data inside the liquid layer; Paras *et al*., [2] have presented detailed measurements of liquid layer thickness, including its wave characteristics; Vlachos *et al*., [3] have measured the liquid/wall shear stress distribution; Paras *et al*., [2] have reported local velocity profiles inside the gas phase. Similar measurements and observations in the gas phase have recently been presented by Dykhno *et al*., [4] and Flores *et al*., [5]. During the last few decades, there has been considerable growth of interest among researchers in Fluid Mechanics to understand the two phase flow involving gas-liquid or liquid-liquid mixture which has several applications in petroleum and Reservoir engineering. The effect of the presence of a second phase in any flow phenomenon has been discussed by several authors notably Govier. G.W, Griffith. P, Rakhmathulin Kh. A and Faizullae D.F *et al*., [6,7,8,9,10,11, 12,13,14,15,16].

Gas-liquid flow is encountered in oil and gas wells, in chemical processing plants and in Nuclear reaction system. In the petroleum industry the production of gas oil through wells almost invariably involves the flow of mixed fluid phases. In the case of will producing gas, frequently at least small amounts of water in the liquid phase or light liquid hydrocarbons are produced simultaneously and the flow mixture is one of two or even three phases the gas-liquid ratio is high but the presence of even a small amount of liquid has significant effect on the flow. In many oil wells gas is produced simultaneously and often water is present resulting in a mixture of two or three phases. Two phase transportation in oil and gas fields in addition to offering economies in pipeline construction permits the centralization of gas processing and crude oil usually resulting in both improved processing economies and improved conservation. In the production of oil from deep reservoirs in order to avoid solidification of petroleum long small pipes are inserted in uniformly which are heated to suitable temperatures such that the flow to be place smoothly in the space between these rods. Each of such space approximates to annular space bounded internally by a rigid cylinder. The flow is identical in all such annular spaces in a reservoir.

2. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the steady axisymmetric flow of a two-phase liquid -gas flow in an annular space bounded internally by a rigid pipe. The outer boundary is coaxial variable gap cylindrical shape and in view of identical condition with other consecutive spacing there is no radial flow across as well as the radial variation of axial velocity is zero. The cylindrical polar system (r, z) is chosen with z-axis along the axis of the pipe. The variable outer boundary is assumed to be $r = af(\frac{\delta z}{a})$ where a is its mean depth and δ the slope assumed to be small. F is an arbitrary function twice differentiable. The governing equations of motion for the steady axisymmetric two-phase liquid-gas flow are:

$$\rho_1(U_1\frac{\partial W_1}{\partial r}+W_1\frac{\partial W_1}{\partial z}) = -f_1\frac{\partial p}{\partial z}+f_1\mu_1(\frac{\partial^2 W_1}{\partial z^2}+\frac{1}{r}\frac{\partial W_1}{\partial r}+\frac{\partial^2 W_1}{\partial r^2})+k(W_2-W_1) \quad (2.1)$$

VOL. 6, NO. 1, JANUARY 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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$$\rho_1(U_1\frac{\partial U_1}{\partial r}+W_1\frac{\partial U_1}{\partial z})=-f_1\frac{\partial p}{\partial r}+f_1\mu_1(\frac{\partial^2 U_1}{\partial z^2}+\frac{1}{r}\frac{\partial W_1}{\partial r}+\frac{\partial^2 U_1}{\partial r^2})+k(U_2-U_1) \quad (2.2)$$

$$\rho_2(U_2\frac{\partial W_2}{\partial r}+W_2\frac{\partial W_2}{\partial z})=-f_2\frac{\partial p}{\partial z}+k(W_1-W_2) \quad (2.3)$$

$$\rho_2(U_2\frac{\partial U_2}{\partial r}+W_2\frac{\partial U_2}{\partial z})=-f_2\frac{\partial p}{\partial r}+k(U_1-U_2) \quad (2.4)$$

The equation of continuity are

$$\frac{\partial U_1}{\partial r}+\frac{1}{r}U_1+\frac{\partial W_1}{\partial z}=0 \quad (2.5)$$

$$\frac{\partial U_2}{\partial r}+\frac{1}{r}U_2+\frac{\partial W_2}{\partial z}=0 \quad (2.6)$$

Where (U_1, W_1) & (U_2, W_2) are velocity components of liquid and gas phases along (r, z) direction respectively. ρ_1, ρ_2 are the densities of liquid and gas, respectively. P is the pressure; K is the interaction coefficient of the two phases.

The boundary conditions of the relevant problem are

$$U_1=0 \quad W_1=0 \text{ on } r=a \quad (2.7)$$

$$U_1=0 \quad \frac{\partial W_1}{\partial r}=0 \text{ on } r=af(z)$$

We introduce the following the non-dimensional variables are

$$W_1=\frac{\mu_1}{\rho_1 a}w^*,\ \frac{\partial p}{\partial z}=P_0=\frac{\mu_1^2P^*}{\rho_1 a^2},W_2=\frac{\mu_1}{\rho_1 a}w^*\ U_2=\frac{\mu_1}{\rho_1 a}u^*\quad r=ar^*,$$

$$z=\frac{az^*}{\delta},U_1=\frac{\mu_1}{\rho_1 a}u^*,\ \frac{\partial p}{\partial z}=\frac{\mu_1^2P^*}{\rho_1 a^2} \quad (2.8)$$

Substituting these (2.8) in equations (2.1) to (2.6) the governing non-dimensional equations reduces to (on dropping the asterisks)

$$(\delta w\frac{\partial w}{\partial z}+u\frac{\partial w}{\partial r})=-f_1P_0+f_1(\delta^2\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial r^2})+K(w'-w) \quad (2.9)$$

$$(u\frac{\partial u}{\partial r}+\delta w\frac{\partial u}{\partial z})=-f_1P_1+f_1(\delta^2\frac{\partial^2 u}{\partial z^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}-\frac{u}{r^2})+K(u'-u) \quad (2.10)$$

$$(u'\frac{\partial w'}{\partial r}+\delta w'\frac{\partial w'}{\partial z})=-f_2(\frac{\rho_1}{\rho_2})\delta\frac{\partial p}{\partial z}+(\frac{\rho_1}{\rho_2})K(w-w') \quad (2.11)$$

$$(u'\frac{\partial u'}{\partial r}+\delta w'\frac{\partial u'}{\partial z})=-f_2(\frac{\rho_1}{\rho_2})\frac{\partial p}{\partial r}+(\frac{\rho_1}{\rho_2})K(u'-u) \quad (2.12)$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\delta\frac{\partial w}{\partial z}=0 \quad (2.13)$$

$$\frac{\partial u'}{\partial r}+\frac{u'}{r}+\delta\frac{\partial w'}{\partial z}=0 \quad (2.14)$$

Where $K=\frac{ka^2}{\mu_1}$ is the interaction parameter?

The non-dimensional boundary conditions are

$$u=0 \text{ and } w=0 \quad \text{on} \quad r=1 \quad (2.15)$$

$$u=0 \text{ and } \frac{\partial w}{\partial r}=0 \text{ on } r=f(z) \quad (2.16)$$

Making use of regular perturbation techniques with δ(<<1), the slope of the variable boundary as much small, and expanding u, u', w, w' in terms of δ

$$w=w_0+\delta w_1+\delta^2w_2 \quad u=\delta u_1+\delta^2u_2$$

$$w'=w_0'+\delta w_1'+\delta^2w_2' \quad u'=\delta u_1'+\delta^2u_2'$$

The equations corresponding to the zeroth order are

$$-f_1P_0+\frac{f_1}{r}\frac{\partial w_0}{\partial r}+f_1\frac{\partial^2 w_0}{\partial r^2}+K(w_0'-w_0) \quad (2.17)$$

$$\frac{f_1}{r}\frac{\partial u_1}{\partial r}+f_1\frac{\partial^2 u_1}{\partial r^2}-f_1\frac{u_1}{r^2}+K(u_1'-u_1) \quad (2.18)$$

$$(\frac{\rho_1}{\rho_2})K_0(w_0-w_0')=f_2(\frac{\rho_1}{\rho_2})P_0 \quad (2.19)$$

$$K(u_1-u_1')-f_2P_1=0 \quad (2.20)$$

Corresponding boundary conditions are

$$w_0=0 \ r=1 \quad (2.21)$$

$$\frac{\partial w_0}{\partial r}=0 \quad r=f(z) \quad (2.22)$$

Adding (2.17) and (2.18) we obtain

$$\left.\begin{aligned}&\frac{\partial}{\partial r}(r\frac{\partial w_0}{\partial r})=(\frac{f_2}{f_1}+1)P_0r\\&\frac{\partial w_0}{\partial r}=\frac{(\frac{f_2}{f_1}+1)P_0r}{2}+\frac{C_1}{r}\end{aligned}\right\} \quad (2.23)$$

Integrating (4.2.20) we obtain

$$w_0=\frac{(\frac{f_2}{f_1}+1)P_0r^2}{4}+C_1\log r+C_2 \quad (2.24)$$

Where C_1 and C_2 are the arbitrary constants to be determined using the conditions (2.21) and (2.22) on solving for C_1 and C_2 we obtain

$$w_0=\frac{(\frac{f_2}{f_1}+1)P_0}{2}+\frac{(\frac{f_2}{f_1}+1)P_0s^2}{2}\log r+\frac{(\frac{f_2}{f_1}+1)P_0}{4} \quad (2.25)$$

Substituting for (2.25) in (2.19) we get

$$w_0'=-\frac{f_2}{K}P_0+\frac{(\frac{f_2}{f_1}+1)P_0r^2}{4}-\frac{(\frac{f_2}{f_1}+1)P_0s(z)^2}{2}\log r-\frac{(\frac{f_2}{f_1}+1)P_0}{4} \quad (2.26)$$

Adding (2.18) and (2.19) we obtain

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$$r^2\frac{\partial^2 u_1}{\partial z^2} + r\frac{\partial u_1}{\partial r} - u_1 = (\frac{f_2}{f_1})P_1 r \qquad (2.27)$$

Corresponding boundary conditions are

$$u_1 = 0, \quad r = 1, \; r = f(z) \qquad (2.28)$$

Integrating (4.2.26) we obtain

$$u_1 = C_1 r + \frac{C_2}{r} + (\frac{f_2}{f_1})P_1(\frac{r}{2}\log r - \frac{r}{4}) \qquad (2.29)$$

Where C_1 and C_2 are the arbitrary constants to be determined using the conditions (2.28)
We obtain,

$$u = -(\frac{f_2}{f_1})\frac{P_1}{8}(1+s(z)^2)r + (\frac{f_2}{f_1})\frac{P_1}{8r}(s(z))^2 - (\frac{f_2}{f_1})\frac{P_1 r^3}{8} \qquad (2.30)$$

Substituting for u_1 in (4.2.20) we get

$$u' = -\frac{f_2}{K}P_1 + (\frac{f_2}{f_1})\frac{P_1}{8}(1+s(z)^2)r - (\frac{f_2}{f_1})\frac{P_1}{8r}(s(z))^2 - (\frac{f_2}{f_1})\frac{P_1 r^3}{8} \qquad (2.31)$$

The equations corresponding to the first order are

$$w_0\frac{\partial w_0}{\partial z} + u_1\frac{\partial w_0}{\partial r} = \frac{f_1}{r}\frac{\partial w_1}{\partial r} + f\frac{\partial^2 w_1}{\partial r^2} + K(w_1' - w_1) \qquad (2.32)$$

$$u_1'\frac{\partial w_0'}{\partial r} + w_0'\frac{\partial w_0'}{\partial z} = (\frac{\rho_2}{\rho_1})K(w_1 - w_1') \qquad (2.33)$$

Adding (4.2.32) and (4.2.33) we obtain

$$\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r^2}\frac{\partial w_1}{\partial r} = -\{(\frac{\rho_2}{\rho_1})(u_1'\frac{\partial w_0'}{\partial r} + w_0'\frac{\partial w_0'}{\partial z} - w_0\frac{\partial w_0}{\partial z} - u_1\frac{\partial w_0}{\partial r})\} \qquad (2.34)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} & w_1 = 0, \quad r = 1 \\ & \frac{\partial w_1}{\partial r} = 0, \; r = f(z) \end{aligned}\right\} \qquad (2.35)$$

Solving (4.2.34) subject to (4.2.35) we get

$$w_1 = rI + C_1\log r + C_2 \qquad (2.36)$$

Where C_1 and C_2 are the arbitrary constants to be determined using the conditions (2.15) on solving for C_1 and C_2 we obtain

$$C_1 = -\frac{(\frac{f_2}{f_1}+1)P_0((1+\beta e^{-z^2})^2)Log[r]}{2} \qquad C_2 = -\frac{(\frac{f_2}{f_1}+1)P_0}{4}$$

$$w_1 = rI + -\frac{(\frac{f_2}{f_1}+1)P_0((1+\beta e^{-z^2})^2)Log[r]}{2} - \frac{(\frac{f_2}{f_1}+1)P_0}{4}$$

$$w = G_{25} + G_{26} + G_{27} - G_{28} + G_{29} - G_{30} + G_{31} + G_{32} - G_{33} + G_{34}$$

The expressions for $G_{25}, G_{26}, G_{27} .....G_{34}$ are mentioned in the appendix.

Substituting (2.36) in (2.33) we get

$$w_1' = G_{35} + G_{36} + G_{37} - G_{38} + G_{39} + G_{40} - G_{41} + G_{42} + G_{43} + G_{44} + G_{45} - G_{46} + G_{47}$$

The expressions for $G_{35}, G_{36}, G_{37} .....G_{47}$ are mentioned in the appendix.

3. DISCUSSION OF THE RESULTS

The axial and radial velocity components related to the viscous and ideal fluid phases in the axisymmetric region between the rigid pipe and the variable cylindrical gap enclosing the rigid pipe have been evaluated at different axial positions and for different amplitudes of the variable boundary. It is to be noted that the external non uniform boundary is non rigid although we assume that there is neither radial velocity, nor radial variation of the axial velocity of the viscous fluid phase across the boundary. This is in par with the mechanism of petroleum extraction through Oil reservoirs, wherein the rigid pipes are inserted into the reservoirs which are used as heaters to avoid the solidification of the petroleum product.

For computational purpose, the variable gap is assumed to $f(z) = 1 + \beta e^{-z^2}$ Figures (1 to 12) correspond to the axial and radial velocity profiles for variation in K at different amplitudes β .When β is small (β = 0.5), the axial velocity different axial positions 0 and $\frac{\pi}{4}$ is negative and hence is in the downward direction. Whereas when β is (≥ 1) it is positive and hence in the upward direction (Figures 1 to 6). At the axial position z = 0 the radial gap is 1.5, 2, and 2.5 (Figures 1 to 3) while in the axial position $z = \frac{\pi}{4}$ it is 1.26, 2.85 and 3.78 according as β is 0.5, 1 and 1.5 (Figures 4 to 6). In general, the axial velocity gradually rises from 0 on the inner rigid boundary to a maximum on the outer boundary. At any given axial position the magnitude of the velocity reduces with increase in K at all radial positions. We also observe that the magnitude of W at $z = 0$ is sufficiently larger than its corresponding magnitude at $z = \frac{\pi}{4}$ at all radial positions and this is true even for β =1 or 1.5. The radial velocity profiles are drawn in Figures (7 to 12) with reference to variation in K at difference β . We find that irrespective of β a radial velocity is always towards the inner cylinder and in view of the symmetrical conditions the radial velocity of viscous phase is zero on either of the internal and external boundaries and hence in general u rises from zero on the inner cylinder to attain maximum near the outer boundary before reducing to rest on the outer boundary (Figures 7 to 12).

VOL. 6, NO. 1, JANUARY 2011 ISSN 1819-6608
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The magnitude of u enhances with K at all radial positions. This is true at axial position $z = \frac{\pi}{4}$ (Figures 10 to 12). We may also observe that, this behavior of u with reference to K persist for all values of β, amplitude of the variable gap (Figures 7 to 12). The behavior of the axial velocity of the in viscid phase w' with reference to K, β may be observed from are plotted in Figures 13-18. Taking zeroth and first order approximations in equations (2.33) governing w' we obtain the equations (2.19) relating the zeroth order axial velocity of the viscous phase w_0 with the zeroth order axial velocity of the in viscid phase w_0' and equations (2.31) relating the first order axial velocity of the viscous phase w_1 with the first orders axial velocity of the in viscid phases w_1'. Combining (2.19 and 2.31) together gives the relation between w and w' to the first order. This relationship decides the behavior of w', since the behavior of w for different sets of parameters is known fully. We observe that the behavior of w' is very much similar to that of w for variations is the governing parameters. However in view of the fact that the w satisfies the no slip conditions on the boundary and in view of the relation (2.19) the maximum w' is attained in the vicinity of the outer cylinder and attains almost fixed values on the inner and outer cylinders, and enhancements in K fixing the other values reduces the magnitude of w' at all radial positions under different axial positions along the cylindrical gap (Figures 13 to 18). Also the magnitude of w' at axial position $z = 0$ is sufficiently large in comparison to corresponding magnitude at $z = \frac{\pi}{4}$. The radial velocity of the in viscid phase is governed by (2.12) and at the first order u_1' is related to the first order radial velocity of the viscous phase u_1 through the equation (2.20). Thus at this order radial velocities differ by a constant. Hence the behavior of u' is similar to that of u_1 except that on the boundaries it takes a constant values which differs with K. We also know that the radial velocity enhances with K and its magnitude at $z = 0$ is larger them corresponding magnitude at $z = \frac{\pi}{4}$ at all radial positions.

We now discuss the resultant flow pattern of the in viscid phase for different β and K. In general, we conclude that at the boundaries, the in viscid phases move towards the mid region for all β and K. With in the region both the ideal and viscous phases moves upwards for β > 1 and downwards for β <1

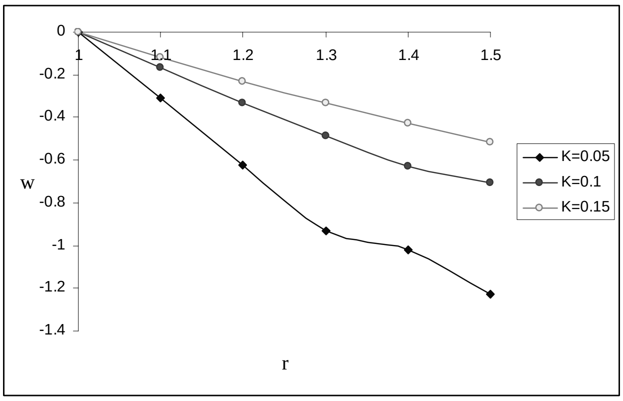

Figure-1. The velocity profile w with K
$z = 0, f_1 = 0.4, f_2 = 0.6, \rho_1 = 1.5, \rho_2 = 1, P_0 = 1, P_1 = 0.01, \beta = 0.5$

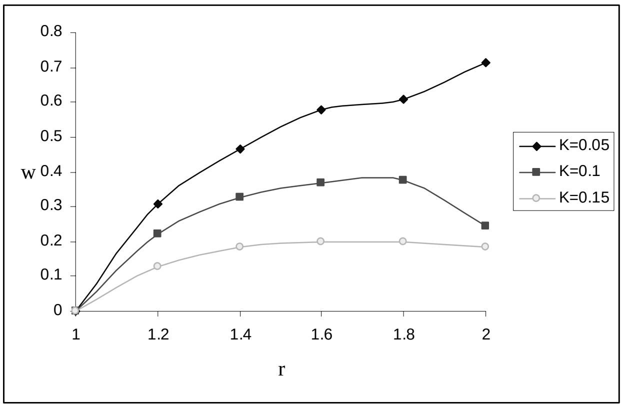

Figure-2. The velocity profile w with K
$z = 0, f_1 = 0.4, f_2 = 0.6, \rho_1 = 1.5, \rho_2 = 1, P_0 = 1, P_1 = 0.01, \beta = 1$

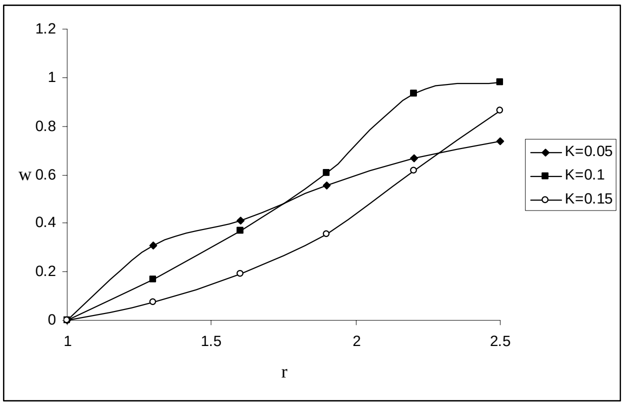

Figure-3. The velocity profile w with K
$z = 0, f_1 = 0.4, f_2 = 0.6, \rho_1 = 1.5, \rho_2 = 1, P_0 = 1, P_1 = 0.01, \beta = 1.5$

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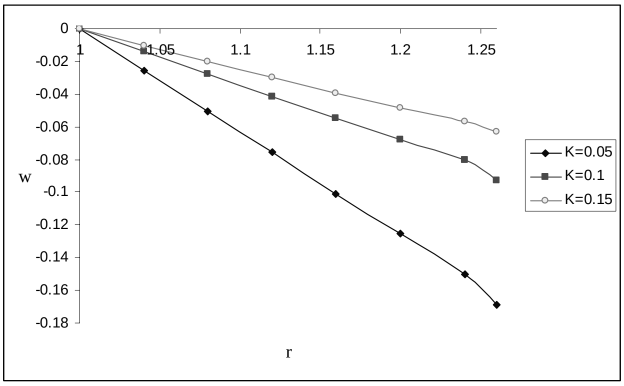

Figure-4. The velocity profile w with K
$z = \frac{\pi}{4}, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=0.5$

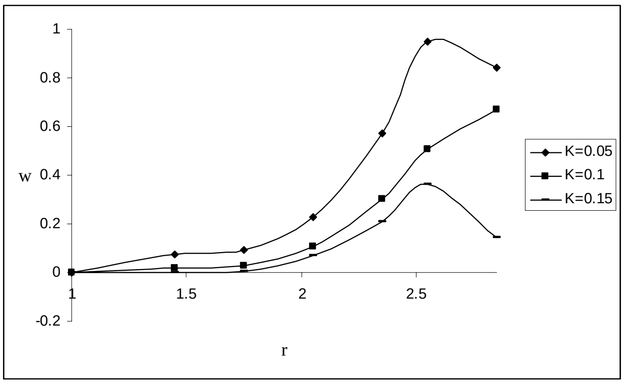

Figure-5. The velocity profile w with K
$z = \frac{\pi}{4}, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=1$

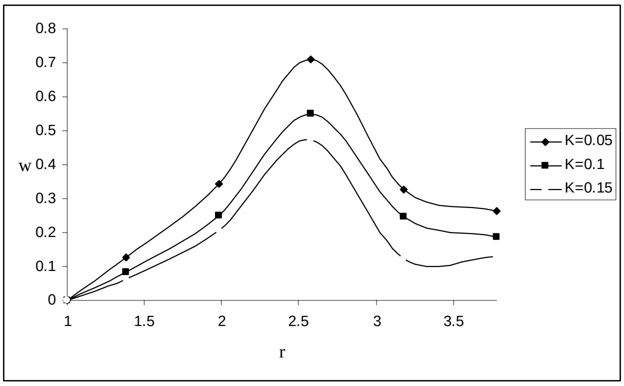

Figure-6. The velocity profile w with K
$z = \frac{\pi}{4}, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=1.5$

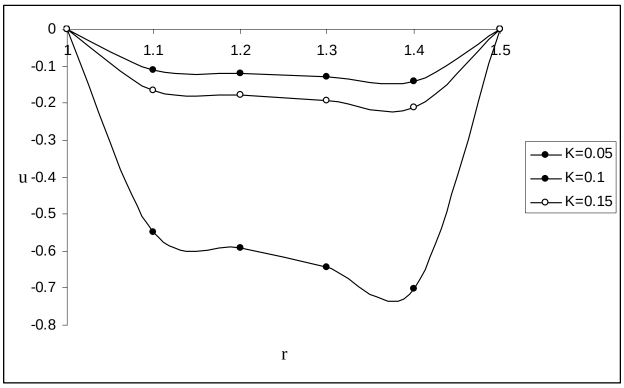

Figure-7. The velocity profile u with K
$z = 0, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=0.5$

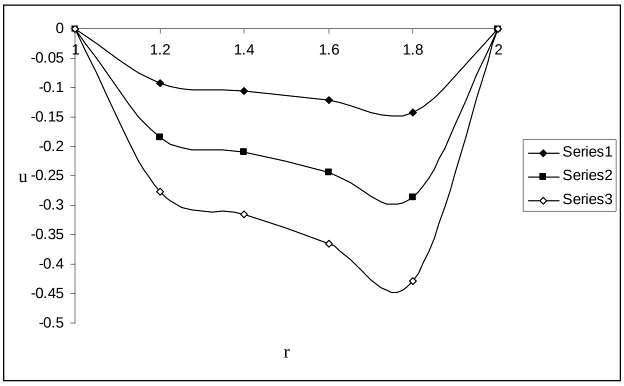

Figure-8. The velocity profile u with K
$z = 0, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=1$

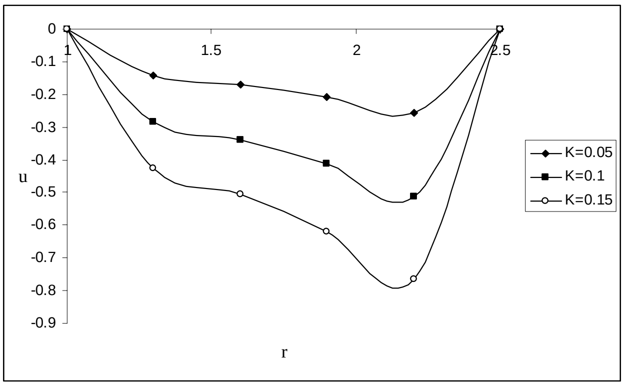

Figure-9. The velocity profile u with K
$z = 0, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=1.5$

VOL. 6, NO. 1, JANUARY 2011 ISSN 1819-6608
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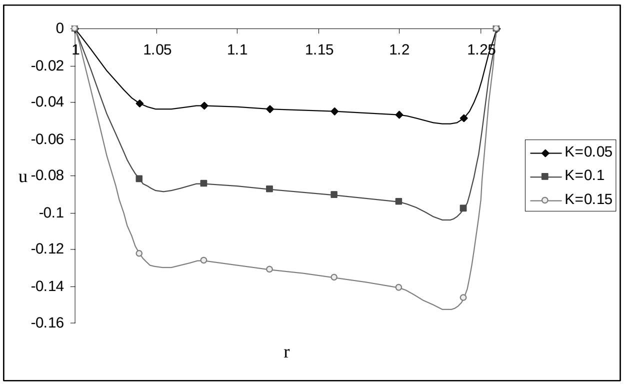

Figure-10. The velocity profile u with K

z = $\frac{\pi}{4}$,f_1=0.4,f_2=0.6,ρ_1=1.5,ρ_2=1, P_0=1,P_1=0.01,β=0.5

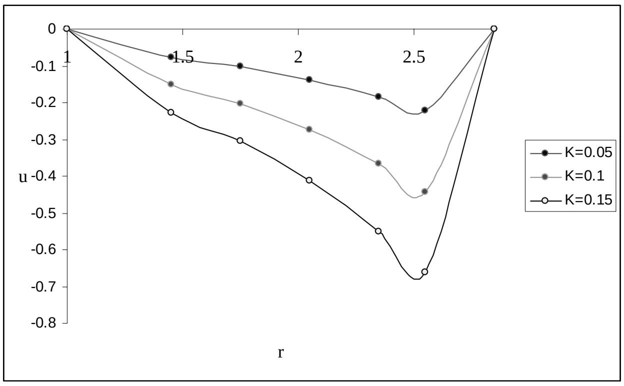

Figure-11. The velocity profile u with K

z = $\frac{\pi}{4}$,f_1=0.4,f_2=0.6,ρ_1=1.5,ρ_2=1, P_0=1,P_1=0.01,β=1

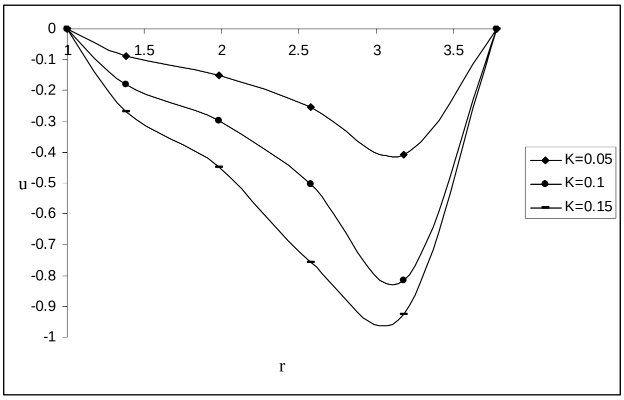

Figure-12. The velocity profile u with K

z = $\frac{\pi}{4}$,f_1=0.4,f_2=0.6,ρ_1=1.5,ρ_2=1, P_0=1,P_1=0.001,β=1.5

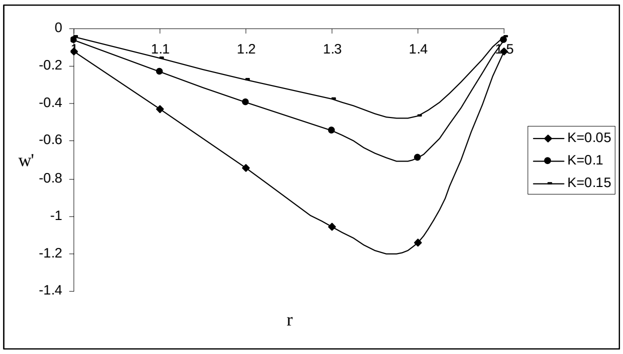

Figure-13. The velocity profile $w^{'}$ with K

z = 0,f_1=0.4,f_2=0.6,ρ_1=1.5,ρ_2=1, P_0=1,P_1=0.01,β=0.5

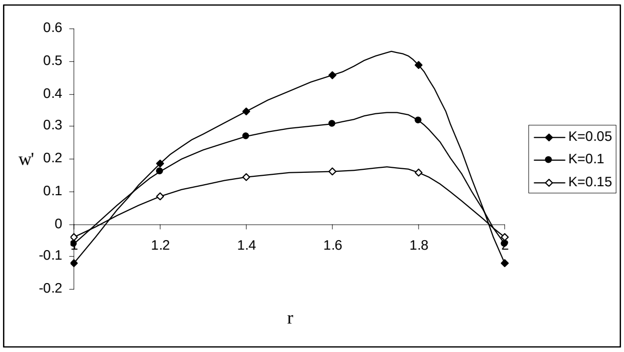

Figure-14. The velocity profile $w^{'}$ with K

z = 0,f_1=0.4,f_2=0.6,ρ_1=1.5,ρ_2=1, P_0=1,P_1=0.01,β=1

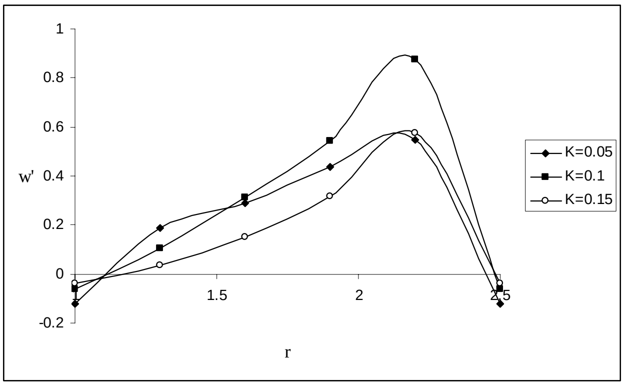

Figure-15. The velocity profile $w^{'}$ with K

z = 0,f_1=0.4,f_2=0.6,ρ_1=1.5,ρ_2=1, P_0=1,P_1=0.01,β=1.5

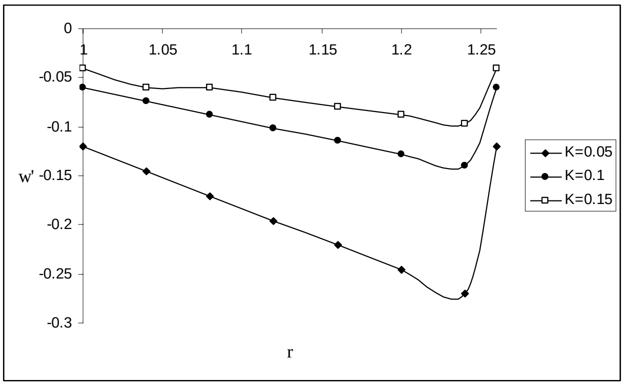

Figure-16. The velocity profile w' with K

$z = \frac{\pi}{4}, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=0.5$

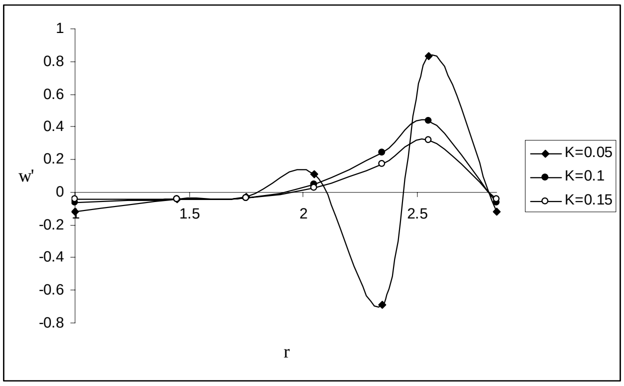

Figure-17. The velocity profile w' with K

$z = \frac{\pi}{4}, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=1$

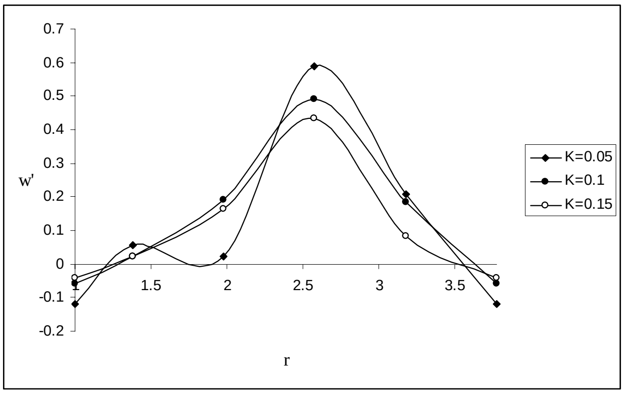

Figure-18. The velocity profile w' with K

$z = \frac{\pi}{4}, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=1.5$

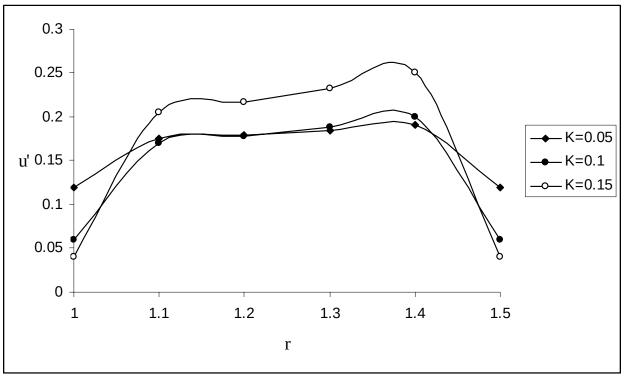

Figure-19. The velocity profile u' with K

$z = 0, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=0.5$

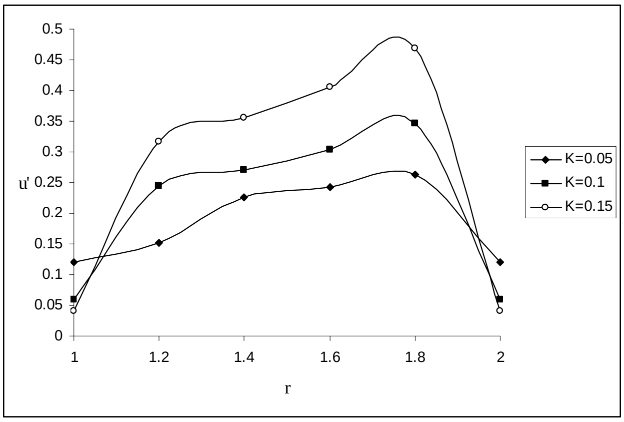

Figure-20. The velocity profile u' with K

$z = 0, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=1$

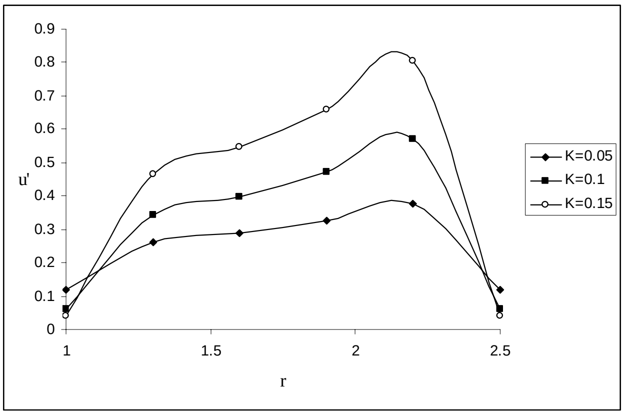

Figure-21. The velocity profile u' with K

$z = 0, f_1=0.4, f_2=0.6, \rho_1=1.5, \rho_2=1, P_0=1, P_1=0.01, \beta=1.5$

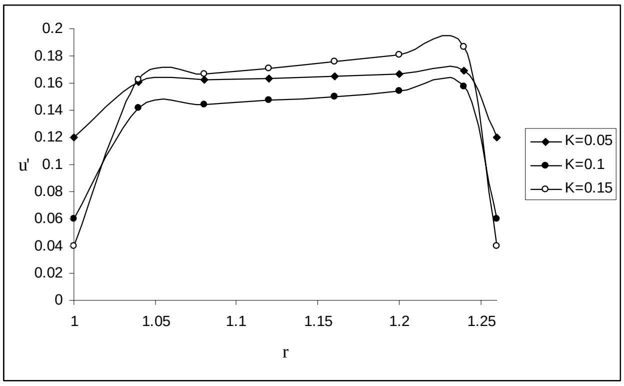

Figure-22. The velocity profile u' with K

z = $\frac{\pi}{4}$, f_1=0.4, f_2=0.6, ρ_1=1.5, ρ_2=1, P_0=1, P_1=0.01, β=0.5

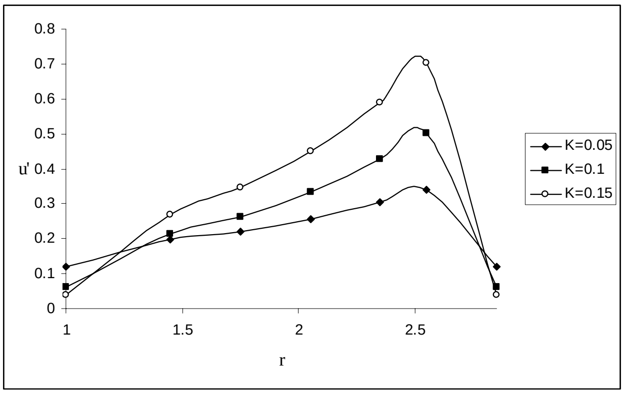

Figure-23. The velocity profile u' with K

z = $\frac{\pi}{4}$, f_1=0.4, f_2=0.6, ρ_1=1.5, ρ_2=1, P_0=1, P_1=0.01, β=1

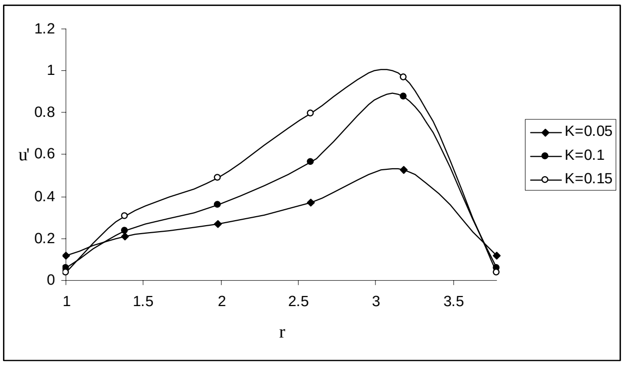

Figure-24. The velocity profile u' with K

z = $\frac{\pi}{4}$, f_1=0.4, f_2=0.6, ρ_1=1.5, ρ_2=1, P_0=1, P_1=0.01, β=1.5

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APPENDIX

$$G_{25}=\left((\frac{f_2}{f_1}+1)P_0\frac{r^2}{4}-\frac{(\frac{f_2}{f_1}+1)P_0}{4}-\frac{(\frac{f_2}{f_1}+1)P_0((1+\beta e^{-z^2})^2)Log[r]}{2}\right)$$

$$G_{26}=\frac{1}{72f_1^2K\rho_1}\left(e^{-4z^2}\beta e^{-2z^2}(-72e^{4z^2}\beta e^{-2z^2}Log[r]\frac{1}{12f_1^2\rho_1}((1+\beta e^{-z^2})^2(f_1+f_2)P_0(6e^{-z^2}z\beta(1+e^{-z^2}\beta)\right)$$
$$(2e^{-z^2}\beta+e^{-2z^2}\beta^2-\beta e^{-2z^2}(1+2\beta e^{z^2}))Log[1+\beta e^{-z^2}]^2(f_1+f_2)P_0$$

$$G_{27}=\frac{1}{K}(3e^{-4z^2}z\beta(\beta+e^{z^2})\beta e^{-2z^2}((2e^{z^2}\beta\beta e^{z^2}+\beta^2\beta e^{2z^2}-e^{2z^2}(1+2\beta e^{z^2}))f_2K+f_1(-2e^{2z^2}\beta e^{2z^2}f_2$$

$$G_{28}=-(-2e^{z^2}\beta\beta e^{2z^2}-\beta^2\beta e^{2z^2}+e^{2z^2}(1+2\beta e^{z^2}))K))P_0)+\frac{1}{K}(6e^{-4z^2}z\beta(e^{z^2}+\beta)\beta e^{-2z^2}Log[1+\beta e^{-z^2}]$$
$$((-2\beta\beta e^{2z^2}e^{z^2}-\beta^2\beta e^{2z^2}+e^{2z^2}(1+2\beta e^{z^2})f_2K$$

$$G_{29}=f_1(2\beta e^{2z^2}e^{z^2}f_2(-2\beta\beta e^{2z^2}e^{z^2}-\beta^2\beta e^{2z^2}+e^{2z^2}(1+2\beta e^{z^2}))K))P_0)$$

$$G_{30}=-\frac{4(1+\beta e^{-z^2})f_1f_2P_1}{K_0})\rho_2)f_1^2K_0\rho_1+r(f_1+f_2)P_0(9rz\beta(e^{z^2}+\beta)((2e^{z^2}\beta\beta e^{z^2}+\beta^2\beta e^{2z^2}-e^{2z^2}(1+2\beta e^{z^2}))(3-4Log[r]$$

$$G_{31}=2Log[r]^2)KP_0+f_1(9rz\beta(e^{z^2}+\beta)(2e^{z^2}\beta\beta e^{2z^2}+\beta^2\beta e^{2z^2}-e^{2z^2}(1+2\beta e^{z^2}))(3-4Log[r]+2Log[r]^2)KP_0$$

$$G_{32}=4e^{2z^2}f_2(9rz\beta(e^{z^2}+\beta)\beta e^{z^2}(-1+Log[r])P_0+e^{2z^2}(-9-18\beta e^{z^2}+(-9+r^2)\beta e^{z^2})P_1)))\rho_2))$$

$$G_{33}=\frac{1}{72f_1^2K\rho_1}(e^{-4z^2}\beta e^{-2z^2}(f_1+f_2)P_0(27z\beta(\beta+e^{z^2})(2e^{z^2}\beta\beta e^{2z^2}+\beta^2\beta e^{2z^2}-e^{2z^2}(1+2\beta e^{z^2}))f_2KP_0+f_1(27z\beta(\beta+e^{z^2})$$

$$(2e^{z^2}\beta\beta e^{2z^2}+\beta^2\beta e^{2z^2}-e^{2z^2}(1+2\beta e^{z^2}))KP_0$$

$$G_{34}=4e^{2z^2}f_2(-9z\beta(e^{z^2}+\beta)\beta e^{2z^2}P_0-e^{2z^2}(9+18\beta e^{2z^2}+8\beta e^{2z^2})P_1)))\rho_2)$$

$$G_{35}=(\frac{f_2}{f_1}+1)P_0\frac{r^2}{4}-\frac{(\frac{f_2}{f_1}+1)P_0}{4}-\frac{(\frac{f_2}{f_1}+1)P_0((1+\beta e^{-z^2})^2)Log[r]}{2}$$

$$G_{36}=\frac{1}{72f_1^2K\rho_1}\left(e^{-4z^2}\beta e^{-2z^2}(-72e^{4z^2}\beta e^{-2z^2}Log[r]\frac{1}{12f_1^2\rho_1}((1+\beta e^{-z^2})^2(f_1+f_2)P_0(6e^{-z^2}z\beta(1+e^{-z^2}\beta)\right)$$
$$(2e^{-z^2}\beta+e^{-2z^2}\beta^2-\beta e^{-2z^2}(1+2\beta e^{z^2}))Log[1+\beta e^{-z^2}]^2(f_1+f_2)P_0$$

$$G_{37}=\frac{1}{K}(3e^{-4z^2}z\beta(\beta+e^{z^2})\beta e^{-2z^2}((2e^{z^2}\beta\beta e^{z^2}+\beta^2\beta e^{2z^2}-e^{2z^2}(1+2\beta e^{z^2}))f_2K_0+f_1(-2e^{2z^2}\beta e^{2z^2}f_2$$

$$G_{38}=(-2e^{z^2}\beta\beta e^{2z^2}-\beta^2\beta e^{2z^2}+e^{2z^2}(1+2\beta e^{z^2}))K))P_0)$$

$$G_{39} = \frac{1}{K}(6e^{-4z^2} z\beta(e^{z^2}+\beta)\beta e^{-2z^2} Log[1+\beta e^{-z^2}]((-2\beta\beta e^{2z^2} e^{z^2} - \beta^2\beta e^{2z^2} + e^{2z^2}(1+2\beta e^{z^2}) f_2 K$$

$$G_{40} = f_1(2\beta e^{2z^2} e^{z^2} f_2 + (-2\beta\beta e^{2z^2} e^{z^2} - \beta^2\beta e^{2z^2} + e^{2z^2}(1+2\beta e^{z^2}))K))P_0)$$

$$G_{41} = \frac{4(1+\beta e^{-z^2}) f_1 f_2 P_1}{K})\rho_2) f_1^2 K\rho_1 + r(f_1+f_2)P_0(9rz\beta(e^{z^2}+\beta)$$

$$((2e^{z^2}\beta\beta e^{z^2} + \beta^2\beta e^{2z^2} - e^{2z^2}(1+2\beta e^{z^2}))(3-4Log[r]+2Log[r]^2)KP_0$$

$$G_{42} = f_1(9rz\beta(e^{z^2}+\beta)(2e^{z^2}\beta\beta e^{2z^2} + \beta^2\beta e^{2z^2} - e^{2z^2}(1+2\beta e^{z^2}))(3-4Log[r]+2Log[r]^2)KP_0$$

$$G_{43} = 4e^{2z^2} f_2(9rz\beta(e^{z^2}+\beta)\beta e^{z^2}(-1+Log[r])P_0 + e^{2z^2}(-9-18\beta e^{z^2} + (-9+r^2)\beta e^{z^2})P_1)))\rho_2))$$

$$G_{43} = \frac{1}{72 f_1^2 K\rho_1}(e^{-4z^2}\beta e^{-2z^2}(f_1+f_2)P_0(27z\beta(\beta+e^{z^2})(2e^{z^2}\beta\beta e^{2z^2} + \beta^2\beta e^{2z^2} - e^{2z^2}(1+2\beta e^{z^2})) f_2 KP_0$$

$$G_{44} = f_1(27z\beta(\beta+e^{z^2})(2e^{z^2}\beta\beta e^{2z^2} + \beta^2\beta e^{2z^2} - e^{2z^2}(1+2\beta e^{z^2}))KP_0$$

$$G_{45} = 4e^{2z^2} f_2(-9z\beta(e^{z^2}+\beta)\beta e^{2z^2} P_0 - e^{2z^2}(9+18\beta e^{2z^2} + 8\beta e^{2z^2})P_1)))\rho_2)$$

$$G_{46} = \left(\frac{\rho_2}{\rho_1}\right)(\frac{1}{16r^2 f_1^2}(-1+r^2)e^{-4z^2}\beta(1+2\beta e^{z^2} - (-1+r^2)\beta e^{z^2})^2 f_2(f_1+f_2)P_0P_1)$$

$$G_{47} = \frac{1}{2} z\beta e^{-z^2}(1+\beta e^{-z^2})Log[r](1+\frac{f_2}{f_1})(-1-\frac{f_2}{f_1}+r^2(1+\frac{f_2}{f_1}) - 2(1+\beta e^{-z^2})Log[r](1+\frac{f_2}{f_1}) - \frac{4f_2}{K})P_0^2)$$

$$u = \delta(-(\frac{f_2}{f_1})\frac{P_1}{8}(1+s(z)^2)r + (\frac{f_2}{f_1})\frac{P_1}{8r}(s(z))^2 - (\frac{f_2}{f_1})\frac{P_1 r^3}{8})$$

$$u' = \delta(-\frac{f_2}{K}P_1 + (\frac{f_2}{f_1})\frac{P_1}{8}(1+s(z)^2)r - (\frac{f_2}{f_1})\frac{P_1}{8r}(s(z))^2 - (\frac{f_2}{f_1})\frac{P_1 r^3}{8})$$

$$G_{44} = f_1(27z\beta(\beta+e^{z^2})(2e^{z^2}\beta\beta e^{2z^2} + \beta^2\beta e^{2z^2} - e^{2z^2}(1+2\beta e^{z^2}))KP_0$$

$$G_{45} = 4e^{2z^2} f_2(-9z\beta(e^{z^2}+\beta)\beta e^{2z^2} P_0 - e^{2z^2}(9+18\beta e^{2z^2} + 8\beta e^{2z^2})P_1)))\rho_2)$$

$$G_{46} = -\left(\frac{\rho_2}{\rho_1}\right)(\frac{1}{16r^2 f_1^2}(-1+r^2)e^{-4z^2}\beta(1+2\beta e^{z^2} - (-1+r^2)\beta e^{z^2})^2 f_2(f_1+f_2)P_0P_1)$$

$$G_{47} = +\frac{1}{2} z\beta e^{-z^2}(1+\beta e^{-z^2})Log[r](1+\frac{f_2}{f_1})(-1-\frac{f_2}{f_1}+r^2(1+\frac{f_2}{f_1}) - 2(1+\beta e^{-z^2})Log[r](1+\frac{f_2}{f_1}) - \frac{4f_2}{K})P_0^2)$$

$$u = \delta(-(\frac{f_2}{f_1})\frac{P_1}{8}(1+s(z)^2)r + (\frac{f_2}{f_1})\frac{P_1}{8r}(s(z))^2 - (\frac{f_2}{f_1})\frac{P_1 r^3}{8})$$

$$u' = \delta(-\frac{f_2}{K}P_1 + (\frac{f_2}{f_1})\frac{P_1}{8}(1+s(z)^2)r - (\frac{f_2}{f_1})\frac{P_1}{8r}(s(z))^2 - (\frac{f_2}{f_1})\frac{P_1 r^3}{8})$$