ARPN Journal of Engineering and Applied Sciences


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# PERISTALTIC PUMPING OF A JEFFREY FLUID IN A POROUS TUBE

S. V. H. N. Krishna Kumari P.[1], M. V. Ramana Murthy, Y. V. K. Ravi Kumar[2] and S. Sreenadh[3]
[1]Department of Mathematics, Osmania University, Hyderabad, India
[2]Department of Mathematics, Stanley College of Engineering and Technology for Women, Hyderabad, India
[3]Department of Mathematics, Sri Venkateswara University, Tirupati, India
E-Mail: yvkravi@rediffmail.com

## ABSTRACT

Peristaltic pumping of a Jeffrey fluid in a porous tube is studied under long wavelength and low Reynolds number assumptions. Solutions are obtained by using Beavers- Joseph and Saffman boundary conditions. The effect of various parameters on the pumping characteristics is studied and discussed through graphs. Comparison of various wave forms (namely sinusoidal, triangular and trapezoidal) on the flow is discussed.

**Keywords:** peristaltic pumping, Jeffrey fluid, pump characteristics, porous tube, boundary conditions, wave forms.

## 1. INTRODUCTION

A peristaltic pump is a device for pumping fluids generally from a region of lower to higher pressure by means of a contraction wave traveling along a tube like structure. This travelling - wave phenomenon is referred to as peristaltic Pumping. It is a mechanism of pumping fluids in ducts when a progressive wave of area contraction or expansion propagates along the length of a distensible tube containing fluid. In general it induces propulsive and mixing movements and pumps the fluids against pressure rise. Peristalsis is used by a living body to propel or to mix the contents of the tube such as, in transport of urine from the kidney through the ureter to the bladder, food through the digestive tract, bile from the gall- bladder into the duodenum, movement of ovum in the fallopian tube etc.

Porous tube wall and deformable porous layer have been observed in many physiological applications such as the gastrointestinal tract, intra- pleural membranes, capillary walls etc. The gastrointestinal tract is surrounded by a number of heavily innervated muscle layers having smooth muscle. These muscle layers consist of many folds and there are pores through the tight junctions of them (Keener and Sneyd [1]).

It has now been accepted that most of the physiological fluids behave like a non-Newtonian fluids. This approach provides a satisfactory understanding of the peristaltic mechanism involved in small blood vessels, lymphatic vessels, intestine, ductus efferentes of the male reproductive tract and in transport of spermatozoa in the cervical canal.

The flow of non-Newtonian fluids is widely observed in industry and physiology, e.g. enhanced oil recovery, chemical processes such as in distillation towers and fixed bed reactors and, in the applications of pumping fluids such as synthetic lubricants, colloidal fluids, liquid crystals and biofluids ( e.g. animal and human blood).

Most of the theoretical investigations have been carried out by assuming that blood and most of the physiological fluids behave like non- Newtonian fluids. Peristaltic transport of blood in small vessels was investigated using the viscoelastic, power- law, Casson, micropolar fluids by Bohme and Friedrich [2], Radhakrishnamacharya [3], Srivastava and Srivastava [4], Srinivasacharya and Rao [5], respectively. Srivastava and Srivastava [6] studied the peristaltic transport of a power-law fluid with an application to ductus effernetus of the male of the reproductive tract. The non-Newtonian effects of Maxwell fluid on the peristaltic transport have been studied by Tsikdauri and Bresnev [7]. El Naby and El Misiery [8] have investigated the peristaltic transport of Carreu fluid in a tube, while Johnson- Segalman fluid has been used for studies by Hayat *et al.*, [9].

Beavers and Joseph [10] were the first to investigate the fluid flow at the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary conditions at the interface. The theoretical justification of the boundary conditions of Beavers - Joseph was given by Saffman [11] and proposed an improved boundary condition.

In this paper our concern is to investigate the peristaltic transport of a non- Newtonian fluid in a porous tube. A simplest linear non- Newtonian model namely the Jeffrey fluid model is used in this paper. The Jeffrey type model is relatively simpler linear model using time derivatives instead of convicted derivatives for example Oldroyd - B model does.

## 2. MATHEMATICAL FORMULATION

Consider the peristaltic transport of an incompressible Jeffrey fluid in an axisymmetric tube as shown in Figure-1.

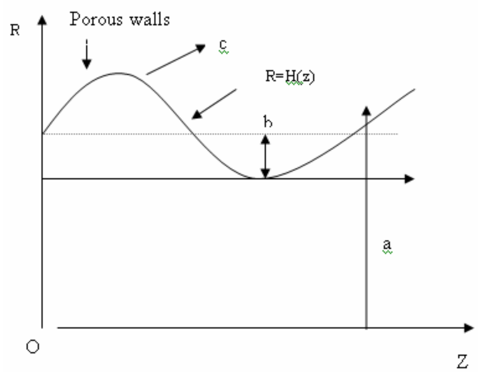


**Figure-1.** Physical model.

The mean radius of the tube is "a". The wall of the tube is flexible, which is an interface of the fluid and porous medium. The wall is subjected to a periodic peristaltic wave movement with wave speed "c", wavelength $\lambda$ and amplitude "b" given by

$$R = \bar{H}(\bar{Z},\bar{t}) = a + b.Sin[\frac{2\pi}{\lambda}(\bar{Z} - c\bar{t}) \qquad (1)$$

The constitutive equations for an incompressible Jeffrey fluid are

$$\bar{T} = -\bar{p}\,\bar{I} + \bar{S}$$

$$\bar{S} = \frac{\mu}{1+\lambda_1}(\frac{\partial\bar{\gamma}}{\partial t} + \lambda_2\frac{\partial^2\bar{\gamma}}{\partial t^2}) \qquad (2)$$

where $\bar{T}$ and $\bar{S}$ are Cauchy stress tensor and extra stress tensor respectively, $\bar{P}$ is the pressure, $\bar{I}$ is the identity tensor, $\lambda_1$ is the ratio of the relaxation to retardation times, $\lambda_2$ is the retardation time, $\mu$ is the dynamic viscosity and $\bar{\gamma}$ is the shear rate.

In the fixed frame of reference ($\bar{R},\bar{Z}$) the flow is unsteady. However, in a coordinate frame moving with the wave speed c, ($\bar{r},\bar{z}$) is stationary. The transformation from fixed frame to wave frame is given by

$$\bar{z} = \bar{Z} - c\bar{t}),\ \bar{r} = \bar{R},\ \bar{w}(\bar{r},\bar{z}) = \bar{W} - c,\ \bar{u}(\bar{r},\bar{z}) = \bar{U}$$

The governing equations in the wave frame are given as

$$\frac{\partial\bar{u}}{\partial\bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial\bar{w}}{\partial\bar{r}} = 0 \qquad (3)$$

$$\rho(\bar{u}\frac{\partial\bar{u}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}}) = -\frac{\partial\bar{p}}{\partial\bar{r}} + \frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{r}\bar{S}_{\bar{r}\bar{r}}) + \frac{\partial}{\partial\bar{z}}(\bar{S}_{\bar{r}\bar{z}}) \qquad (4)$$

$$\rho(\bar{u}\frac{\partial\bar{w}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}}) = -\frac{\partial\bar{p}}{\partial\bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{r}\bar{S}_{\bar{r}\bar{z}}) + \frac{\partial}{\partial\bar{z}}(\bar{S}_{\bar{z}\bar{z}}) \qquad (5)$$

Now introducing the non-dimensional quantities,

$$r = \frac{\bar{r}}{a},\ u = \frac{\bar{u}}{a},\ z = \frac{2\pi\bar{z}}{\lambda},\ p = \frac{2\pi a^2}{\mu c\lambda}\bar{p},\ t = \frac{2\pi c\bar{t}}{\lambda},\ k = \frac{\bar{k}}{a},\ S = \frac{Sa}{\mu}$$

and defining the Reynolds number and wave number as

$$\mathrm{Re} = \frac{\rho ca}{\mu},\ \delta = \frac{2\pi a}{\lambda}$$

And the stream function

$$u(r,z) = \frac{-\delta}{r}\frac{\partial\psi}{\partial z},\ w(r,z) = \frac{1}{r}\frac{\partial\psi}{\partial r} \qquad (6)$$

The equations of motion reduces to

$$\delta^3\,\mathrm{Re}[\frac{1}{r}\frac{\partial\psi}{\partial z}\frac{\partial}{\partial r} - \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial z}]\frac{1}{r}\frac{\partial\psi}{\partial z} = \frac{-\partial p}{\partial r} + \frac{\delta}{r}\frac{\partial}{\partial r}(rS_{rr}) + \delta^2\frac{\partial}{\partial z}(S_{rz}) \qquad (7)$$

reduces to

$$\delta\mathrm{Re}[\frac{1}{r}\frac{\partial\psi}{\partial z}\frac{\partial}{\partial r} - \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial z}]\frac{1}{r}\frac{\partial\psi}{\partial r} = \frac{-\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \delta\frac{\partial}{\partial z}(S_{zz}) \qquad (8)$$

in which

$$S_{rr} = \frac{-2\delta}{1+\lambda_1}[1 - \frac{\delta\lambda_2 c}{a}(\frac{1}{r}\frac{\partial\psi}{\partial z}\frac{\partial}{\partial r} - \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial z})]\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\psi}{\partial z}) \qquad (9)$$

$$S_{rz} = \frac{1}{1+\lambda_1}[1 - \frac{\delta\lambda_2 c}{a}(\frac{1}{r}\frac{\partial\psi}{\partial z}\frac{\partial}{\partial r} - \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial z})]\{\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\psi}{\partial z})[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial\psi}{\partial z}) - \delta^2\frac{\partial}{\partial z}(\frac{1}{r}\frac{\partial\psi}{\partial z}) \qquad (10)$$

$$S_{zz} = \frac{-2\delta}{1+\lambda_1}[1 - \frac{\delta\lambda_2 c}{a}(\frac{1}{r}\frac{\partial\psi}{\partial z}\frac{\partial}{\partial r} - \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial z})]\frac{\partial}{\partial z}(\frac{1}{r}\frac{\partial\psi}{\partial r}) \qquad (11)$$

Using the long wave length approximation and neglecting the wave number along with low Reynolds number, we get

$$\frac{\partial p}{\partial r} = 0 \qquad (12)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(\frac{r}{1+\lambda_1}\{\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\psi}{\partial r})\}] \qquad (13)$$

Or

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(\frac{r}{1+\lambda_1}\frac{\partial w}{\partial r}) \qquad (14)$$

**Boundary conditions**

The boundary is an interface which separates the Jeffrey fluid and the porous medium. The problem is solved by using two types of slip boundary conditions and tries to compare the effects of these conditions on the flow characteristics.

**Beavers - Joseph boundary conditions**

Beavers-Joseph boundary conditions for Jeffrey fluid are given by (in a wave frame of reference)

w = -1 + wB at r = h (z) (15)

$$\frac{\partial w}{\partial r} = -\frac{\alpha}{\sqrt{Da}}(w_B - R) \quad \text{at } r = h(z) \qquad (16)$$

$$\frac{\partial w}{\partial r} = 0 \quad \text{at } r = 0 \qquad (17)$$

where Q = - ( Da) $\frac{\partial p}{\partial z}$

where wB is the slip velocity at the boundary r = h (z), α is dimensionless Beavers - Joseph constant which depends on the nature of the porous medium but not the fluid viscosity.

**Saffman boundary conditions**

The boundary conditions corresponding to Saffman are given by

$$w=-\frac{\sqrt{Da}}{\alpha}\frac{\partial w}{\partial r}-1 \quad \text{at } r = h(z) \tag{18}$$

$$\frac{\partial w}{\partial r}=0 \quad \text{at } r = 0 \tag{19}$$

where 'Da' is the Darcy number given by $Da=\frac{k}{a^2}$.

and $h=1+\phi.Sinz$ where $\phi.=\frac{b}{a}$ is the amplitude ratio.

## 3. SOLUTION OF THE PROBLEM

**Beavers-Joseph solutions**

Solution of equations (12) and (14) together with boundary conditions (15) - (17) is given by

$$w=\frac{\partial p}{\partial z}\frac{(1+\lambda_1)}{4}(r^2-h^2)+w_B-1 \tag{20}$$

where

$$w_B=-\frac{\partial p}{\partial z}[Da+\frac{\sqrt{Da}}{\alpha}\frac{(1+\lambda_1)}{2}h] \tag{21}$$

The solutions in terms of stream function is obtained from equation (20) by using the condition $\psi=0$ at r = 0 as

$$\psi=\frac{\partial p}{\partial z}\frac{(1+\lambda_1)}{4}r^2(r^2-h^2)+w_B\frac{r^2}{2}-\frac{r^2}{2} \tag{22}$$

The non dimensional flow rate q across any cross section of the tube is independent of z is

$$q=2\int_0^h rwdr$$

$$=-\frac{\partial p}{\partial z}[(\frac{1+\lambda_1)h^4}{8}+h^2(D\ a)+\frac{\sqrt{Da}}{\alpha}\frac{h^3}{2}(1+\lambda_1)]-h^2 \tag{23}$$

and this can be written as

$$\frac{\partial p}{\partial z}=\frac{-(q+h^2)}{[(\frac{1+\lambda_1)h^4}{8}+h^2(D\ a)+\frac{\sqrt{Da}}{\alpha}\frac{h^3}{2}(1+\lambda_1)]} \tag{24}$$

**Saffman solutions**

Solving equations (12) and (14), together with boundary conditions (18) and (19), we get

$$w=(\frac{\partial p}{\partial z})[(\frac{1+\lambda_1}{4})(r^2-h^2)-\frac{\sqrt{Da}}{\alpha}\frac{h}{2}(1+\lambda_1)]-1 \tag{25}$$

and the solution in terms of the stream function is

$$\psi=(\frac{\partial p}{\partial z})[\frac{(1+\lambda_1)}{4}r^2(\frac{r^2}{4}-\frac{h^2}{2})-\frac{\sqrt{Da}}{\alpha}\frac{h}{2}(1+\lambda_1)\frac{r^2}{2}]-\frac{r^2}{2} \tag{26}$$

Corresponding flow rate q is given by

$$q=2\int_0^h r\,w\,d\,r$$

$$=-\frac{\partial p}{\partial z}[(\frac{1+\lambda_1)h^4}{4}+\frac{\sqrt{Da}}{\alpha}h^3(1+\lambda_1)]-h^2 \tag{27}$$

which implies

$$\frac{\partial p}{\partial z}=\frac{-(q+h^2)}{[(\frac{1+\lambda_1}{4})h^4+\frac{\sqrt{Da}}{\alpha}\frac{h^3}{2}(1+\lambda_1)]} \tag{28}$$

The dimensionless time averaged flux

$$\bar{Q}=\frac{2}{T}\int_0^T\int_0^H RUdRdt=2\int_0^1\int_0^h r(u+1)drdz=$$

$$q+\int_0^1 h^2dz \tag{29}$$

## 4. RESULTS AND DISCUSSIONS

The variation of $\bar{Q}$ with $\Delta p$ is given by evaluating the integral (29) for both Beavers - Joseph or Saffman models for different parameters.

In Figures 2 and 3, the variation of $\bar{Q}$ with $\Delta p$ is shown for different values of Jeffrey parameter of λ1 by fixing the other parameters for Beavers - Joseph and Saffman models. It is observed that the pumping rate decreases with the increase of λ1 for pumping ($\Delta p$ >0) and as well as for free pumping A ($\Delta p$ =0). Pumping is more for a Jeffrey fluid when compared with a Newtonian fluid.

The variation of $\bar{Q}$ with $\Delta p$ for different amplitude ratios is shown in Figures 4 and 5. We observe that the larger the amplitude ratio, the greater the pressure rise against which the pump works. For a given $\Delta p$, the flux $\bar{Q}$ for a Jeffrey fluid in a tube depends on $\varphi$ and it increases with increase in $\varphi$ and also it is observed that

Beavers - Joseph model gives a better pumping performance than Saffmann model.

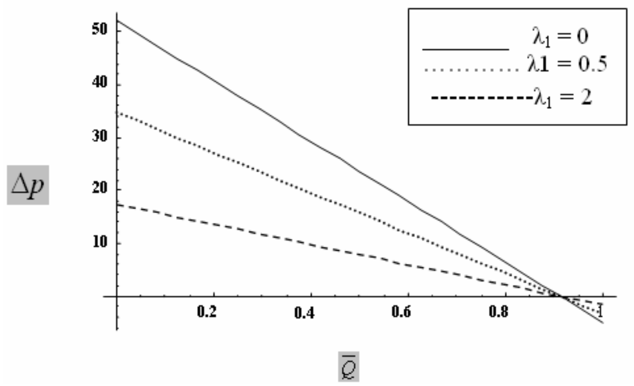


**Figure-2.** The variation of $\Delta p$ with $\bar{Q}$ for different values of $\lambda_1$ with φ = 0.6, d = 2, α = 1 Da = 0.00001 (Beavers and Joseph solutions).

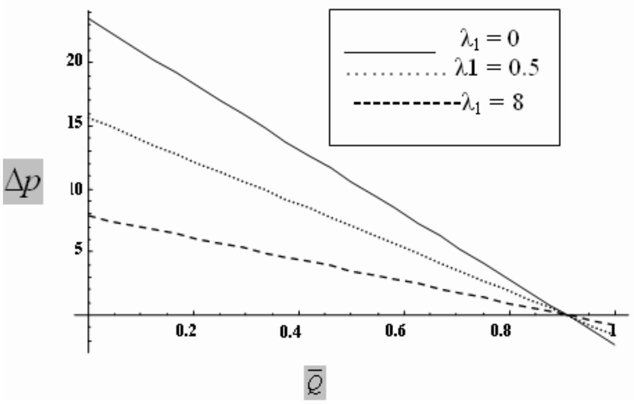


**Figure-3.** The variation of $\Delta p$ with $\bar{Q}$ for different values of $\lambda_1$ with φ = 0.6, d = 2, α = 0.1, Da = 0.00001 (Saffmann solutions).

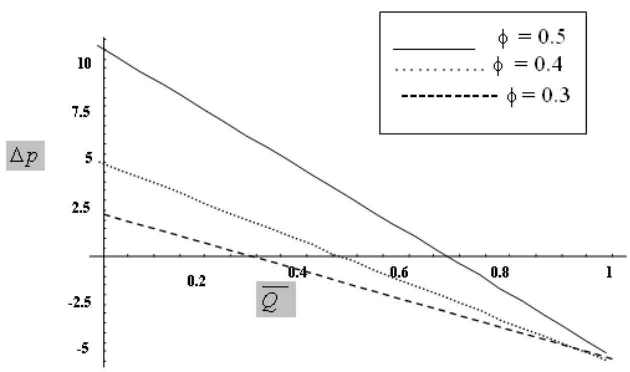


**Figure-4.** The variation of $\Delta p$ with $\bar{Q}$ for different values of φ with $\lambda_1$ = 0.1, d = 2, α = 0.1, Da = 0.0001 (Beavers and Joseph solutions).

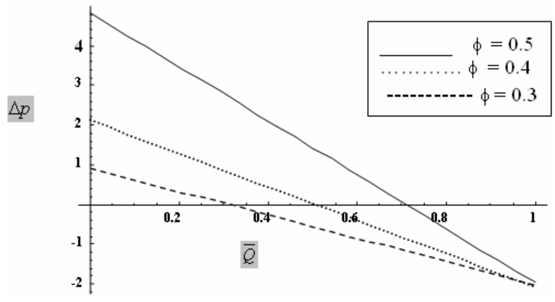


**Figure-5.** The variation of $\Delta p$ with $\bar{Q}$ for different values of α with $\lambda_1$ = 0.1, d = 2, α = 0.1, Da = 0.0001 (Saffman solutions).

The effect of Darcy number 'Da' on the pumping performance is shown in Figures (6) and (7). It is observed that the smaller the Darcy number, the greater the pressure raise against which the pump works. For a given $\Delta p$, the flux $\bar{Q}$ with $\Delta p$ depends on 'Da' and it decreases with the increase in 'Da'.

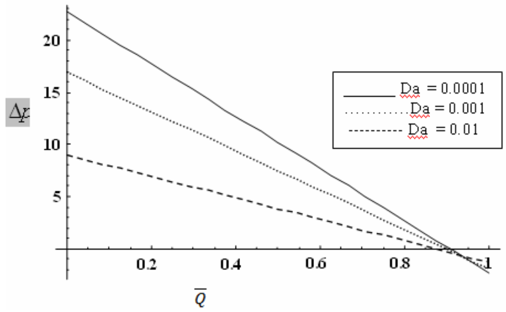


**Figure-6.** The variation of $\Delta p$ with $\bar{Q}$ for different values of Da with $\lambda_1$ = 1, φ = 0.60, α = 0.5. ( Beavers and Joseph solutions).

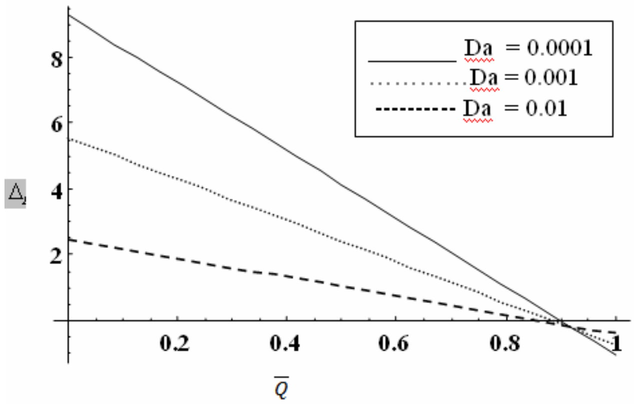


**Figure 7.** The variation of $\Delta p$ with $\bar{Q}$ for different values of Da with $\lambda_1$ = 1, φ = 0.60, α = 0.1. (Saffmann solutions).

Figures 8 and 9 are drawn to study the effect of the parameter $\alpha$ on the pumping characteristics. It is observed that the increasing $\alpha$ increases the pumping as well as free pumping.

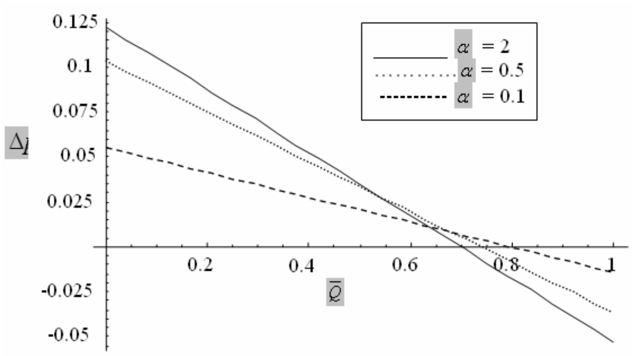


**Figure-8.** The variation of $\Delta p$ with $\overline{Q}$ for different values of α with $\lambda_1 = 1$, φ = 0.60, Da = 10

(Beavers and Joseph solutions).

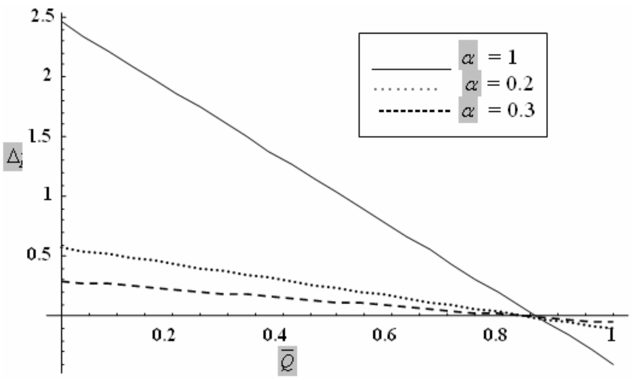


**Figure-9.** The variation of $\Delta p$ with $\overline{Q}$ for different values of α with $\lambda_1 = 1$, φ = 0.60, Da = 10

(Saffman solutions).

The variation of flux $\Delta p$ with $\overline{Q}$ for the following wave forms (in non-dimensional form) is presented graphically in Figure-10.

1. Sinusoidal wave: $h(x) = 1 + \phi.Sin(x)$

2. Triangular wave:

$$h(x) = 1 + \phi\{\frac{8}{\pi^3}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{(2m-1)^2}.Sin((2m-1)x)).$$

3. Trapezoidal wave:

$$h(x) = 1 + \phi\{\frac{32}{\pi^2}\sum_{m=1}^{\infty}\frac{Sin\frac{\pi}{8}(2m-1)}{(2m-1)^2}.Sin((2m-1)x)).$$

It is observed that the trapezoidal wave gives the best pumping characteristics among the three wave forms whereas the triangular wave has the worst pumping characteristics.

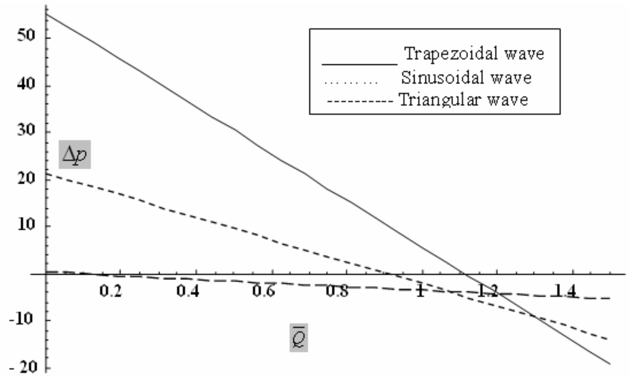


**Figure-10.** The variation of $\Delta p$ with $\overline{Q}$ for different wave forms with $\lambda_1 = 0.1$, φ = 0.60, Da = 0.001, α = 1.

**ACKNOWLEDGEMENTS**

One of the authors Dr. S. Sreenadh, thanks DST, New Delhi, India for providing financial support through a major research project with No: SR / S4 / MS;503/07.

**REFERENCES**

[1] J.R. Keener, J. Sneyd. 1998. Mathematical Physiology. Springer, New York. Chapter 21.

[2] G. Bohme and R. Friedrich. 1983. R. Peristaltic flow of viscoelastic liquids. J. Fluid Mech. 128: 109-122.

[3] G. Radhakrishnamacharya. 1982. Long wavelength approximation to peristaltic motion of a power law fluid. Rheol. Acta. 21: 30-35.

[4] L. M.Srivastava and V.P. Srivastava. 1984. Peristaltic transport of blood: Casson model II. J. Biomech. 17: 821-829.

[5] D. Srinivasacharya, M. Mishra and A. Ramachandra Rao. 2003. Peristaltic pumping of a micro polar fluid in a tube. Acta Mechanica. 161: 165-178.

[6] L. M. Srivastava and V.P. Srivastava. 1988. Peristaltic transport of power law fluid: application to the ductus of efferuntus of the reproductive tract. Rheol. Acta. 27: 428-433.

[7] D. Tsiklauri and I. Beresnev. 2001. Non-Newtonian effects in the peristaltic flow of a Maxwell fluid. Phys Rev E Stat Nonlin Soft Matter Phys. p. 64.

[8] Abd El Hakeem, Abd El Naby, El Misiery, A.E.M and Abd El Kareem, M.F. 2004. Separation in flow through peristaltic motion of a Carreau fluid in uniform tube. Physica A. 343: 1-15.

ARPN Journal of Engineering and Applied Sciences



[9] T. Hayat, Y. Wang, A.M. Siddiqui, K. Hutter. 2003. Peristaltic motion of Johnson-Segalman fluid in a planar channel. Math Problems Eng. 1: 1-23.

[10] G.S. Beavers and D.D. Joseph. 1967. Boundary conditions at a naturally permeable wall. J. Fluid. Mech. 30: 197-207.

[11] P.G. Saffman. 1971. On the boundary conditions at the surface of a porous medium. Stud. Appl. Math. 1(2): 93-101.