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SELECTIVE MODAL SPACE CONTROL APPROACH FOR SMART COMPOSITE CYLINDRICAL SHELL STRUCTURE

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ABSTRACT

This paper presents selective modal space control approach (SMSC) to address the vibration suppression of critical elastic modes of composite cylindrical shell, using collocated/non collocated sensor/actuator. A four node, electro-mechanically coupled field consistent facet shell finite element is used to model the shell vibration. Subsequently, a linear quadratic regulator (LQR) is designed in modal domain to conduct the vibration control simulation studies. The results of active vibration control (AVC) of a laminated composite cylindrical shell, equipped with piezoelectric composite actuators i.e., Macro Fiber Composite (MFC) and PZT sensors are presented. The effectiveness of MFC in controlling shell vibration is evaluated. An effort has been made to addresses the question of whether performance is affected by the collocation/non-collocation of the sensors and actuator.

Keywords: selective modal space control, composite cylindrical shell, sensors, actuators, vibration control, MFC actuator.

1. INTRODUCTION

Aerospace structures (curved/ flat/ thin walled) may experience adverse aerodynamic environments, which induce random vibrations. The concept of active vibration control in fact is very much useful to enhance the performance in such structures. Electro-mechanically coupled PZT materials are now considered as actuators and sensors in active vibration control application. Since the discovery of piezoelectric phenomena in 1880 by the Curie brothers, there has been a great amount of research activity in the area of piezoelectricity theories, its applications in various engineering problems, and the development of new kinds of materials and technologies for better actuation and sensing. A significant amount of research works has been carried out on modeling and active vibration control of beam and plate structure with actuators and sensors based on the piezoelectricity. However vibration control of cylindrical shell structures with piezoelectric actuators has received less attention. Aircraft fuselage and submarine structures can be modeled as cylindrical shells for active vibration and noise control studies.

Active vibration control of smart shells using distributed piezoelectric sensors and actuators was studied by V. Balamurugan, S. Narayanan [1], Theoretical analysis of active control of vibration in a cylindrical shell was presented by Xu, M. B [2]. Kwak proposed dynamic modeling, active vibration controller design and experiments for a cylindrical shell equipped with piezoelectric sensors and actuators [3]. Tzou H. S. and D. W. Wang presented Vibration Control of Toroidal Shells with Parallel and Diagonal Piezoelectric Actuators [4]. Del Rosario and Smith applied a linear quadratic regulator (LQR) scheme for the vibration control of a circular cylinder using distributed piezoceramic patches [5]. Raja *et al*., [6] studied the influence of active stiffness on the dynamic behaviour of piezo-hygro-thermo-elastic laminated plates and shells. Use of MFC, a conformable actuator on shell structures is not examined to find out its directional actuation ability on the geometrically curved surfaces. Piezoceramic patch, widely used as actuator and sensor, has inherent brittle nature and cannot be bonded to curved surface of the cylindrical shell structure in practical applications. Recently, Macro Fiber Composite (MFC) actuator, based on piezoelectric ceramic fiber, was developed at NASA Langley Research Center [7]. MFC is flexible and therefore, applicable to the curved structures. In-plane poling property can be achieved by interdigitated electrode, which produces more induced actuating strain than monolithic piezoelectric ceramic patch. Flat facet shell elements are popular and are integral parts of any general purpose finite element software (e.g., ABAQUS ANSYS, MSC/NASTRAN etc.). With the advent of high-speed computers, it is also possible now a day to employ a large number of elements, to approximate even a deep curved shell by flat facet elements. A simple four-noded quadrilateral shell element (called QUAD4) based on isoparametric formulation with a reduced order of integration for shear terms was first presented by MacNeal [8]. Tarun Kant later presented a higher order Flat facet composite shell element [9].

Present work deals with active vibration control of composite cylindrical shell with flat facet shell element with electro-mechanical coupling, using surface bonded MFC actuators with collocated/non collocated sensor/actuator. For this purpose, a CFRP cylindrical shell with surface bonded Macro Fiber Composite (MFC, Smart Materials®) actuators and PZT sensors is taken as a model problem to assess the vibration control performance. The structural model is simulated using a four-noded composite facet element based on coordinate transformation with drilling degree of freedom [10, 11]. Further the coupled finite element formulations are coded in MATLAB. The developed FE model is then validated with the help of ANSYS® (ver.11). Two popularly known approaches for vibration control include independent modal space control (IMSC) and selective modal space control (SMSC). Partha Bhattacharya *et al.*, studied distributed control of laminated composite shells using LQR/IMSC approach [12]. Efficient modal control strategies for active control of vibrations were proposed by S.P. Singh [13]. Y. Y. Lee and J. Yao studied Structural Vibration Suppression Using the Piezoelectric Sensors and Actuators using the independent Modal Space Control (IMSC) approach is employed for the controller design [14]. Optimal control of active structures with piezoelectric actuators and sensors was studied extensively [15-17].Gary J. Balas *et al.*, studied Collocated versus Non-collocated Multivariable Control for Flexible Structure [18]. An-Chen Lee and Song-Tsuen Chen presented Collocated Sensor/Actuator Positioning and Feedback Design in the Control of Flexible Structure System [19]. From The literature study it is clear that IMSC technique has been employed by many authors but SMSC with collocated/non collocated sensor/actuator on shell structure has received less attention. Hence selective modal space control (SMSC) approach is proposed in this paper with collocated/non collocated sensor/actuator.

2. FINITE ELEMENT FORMULATION

A four node flat shell element integrated with active layers is being used in the present analysis with five mechanical degrees of freedom u_0, v_0, w_0, θ_x, θ_y and three electrical degrees of freedom ϕ_{1a}, ϕ_{2a}, ϕ_{1s} per node. The linear displacement relations are given by

$$\begin{aligned} u(x,y,t) &= u_0(x,y,t) + z\theta_y(x,y,t) \\ v(x,y,t) &= v_0(x,y,t) - z\theta_x(x,y,t) \\ w(x,y,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

Where, u_0, v_0, w_0, are the middle plane displacements along x, y and z directions, respectively and θ_x, θ_y, are the rotations of the cross sections in YZ and XZ planes, respectively.

2.1 Electric field equations

The formulation includes three multi-functional layers, which can be placed anywhere along the thickness direction of the laminated composite shell. The active layers integrated to the shell structure may act as actuator or sensor in the distributed active control. The total electric potential in each active layer is given by,

$$\phi_a(x,y,z) = \phi_{0a}(x,y) + \frac{(z - h_{k-1})}{(h_k - h_{k-1})}\phi_{1a}(x,y),$$

$$\phi_s(x,y,z) = \phi_{0s}(x,y) + \frac{(z - h_{k-1})}{(h_k - h_{k-1})}\phi_{1s}(x,y) \tag{2}$$

where, ϕ_0 is the mean electric potential defined at the middle plane of the active layer, ϕ_1 is the difference of potential between top and bottom surfaces of the active layer and subscripts 'a' and 's' in the above equations denote actuator and sensor, respectively.

2.2 Gradient relations

The linear gradient relations are described for membrane, bending and shear strain fields as follows:

$$\{\varepsilon_{mb}\} = \{\varepsilon_m\} + z\{\varepsilon_b\}$$

$$\{\varepsilon_m\} = \begin{Bmatrix} \varepsilon_x{}^m \\ \varepsilon_y{}^m \\ \gamma_{xy}{}^m \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} \quad \{\varepsilon_b\} = \begin{Bmatrix} \varepsilon_x{}^b \\ \varepsilon_y{}^b \\ \gamma_{xy}{}^b \end{Bmatrix} = \begin{Bmatrix} \left(\dfrac{\partial \theta y}{\partial x}\right) \\ -\left(\dfrac{\partial \theta x}{\partial y}\right) \\ \left(\dfrac{\partial \theta y}{\partial y} - \dfrac{\partial \theta x}{\partial y}\right) \end{Bmatrix}$$

$$\{\varepsilon_s\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \left(\dfrac{\partial w}{\partial x} + \theta y\right) \\ \left(\dfrac{\partial w}{\partial y} - \theta x\right) \end{Bmatrix} \tag{3}$$

The linear gradient relations for the electric field are defined as:

$$\begin{aligned} \{E\}_i &= \{E_x\, E_y\, E_z\}^T \\ &= \left\{ -\frac{\partial \phi_i}{\partial x} \;\; -\frac{\partial \phi_i}{\partial y} \;\; -\frac{\partial \phi_i}{\partial z} \right\} \end{aligned} \tag{4}$$

The element geometry and displacement-fields are expressed in terms of shape functions N_i. The shape functions for standard four noded isoparametric elements are employed the strains and electrical potential gradients are obtained as

$$\begin{aligned} \{\varepsilon\} &= \sum_{i=1,4} [J]^{-1}[B_u]_i\, \{\bar{u}\}_i \\ \{E\} &= \sum_{i=1,4} [J]^{-1}[B_\phi]_i\, \{\bar{\phi}\}_i \end{aligned} \tag{5}$$

where, $[B_u]$ and $[B_\phi]$ are the spatial derivatives of shape function of elastic and electric fields respectively. With these approximate fields and the constitutive material properties are, in conjunction with the equilibrium and conservation equations, using the kinematic and the coupled constitutive relations, the energy equation is minimized to obtain the stationary condition.

$$\delta U_a = \int_v (\rho \ddot{u}_j\, \delta u_j + \sigma_{ij}\, \delta\varepsilon_{ij})\, dv - \int_v (f_{bj}\, \delta u_j\, dv) - \int_s (f_{sj}\, \delta u_j\, ds)$$

$$\delta U_s = \int_v (D_j\, \delta\phi_{,j})\, dv - \int_s (q\, \delta\phi)\, ds \tag{6}$$

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Following system of equations are derived in terms of nodal displacements and nodal voltages.

$$[M_{uu}]\{\ddot{\bar{u}}\}+[K_{uu}]\{\bar{u}\}+[K_{u\phi}]\{\bar{\phi}\}=\{F_m\} \quad (7)$$

$$[K_{\phi u}]\{\bar{u}\}+[K_{\phi\phi}]\{\bar{\phi}\}=\{F_{el}\} \quad (8)$$

Where, $[M_{uu}]=\iint [N_u]^T[\bar{\rho}][N_u]|J|d\xi\, d\eta$ is the mass matrix and $\bar{\rho}$ is the mass density.

$$[K_{uu}]=\iint [B_u]^T[\bar{c}][B_u]|J|d\xi\, d\eta$$

is the displacement stiffness matrix.

$$[K_{u\phi}]=\iint [B_u]^T[\bar{d}][B_\phi]|J|d\xi\, d\eta$$

is the piezoelectric coupling stiffness matrix.

$[K_{\phi u}]=[K_{u\phi}]^T$ is the piezoelectric coupling stiffness matrix.

$[K_{\phi\phi}]=\iint [B_\phi]^T[\bar{\kappa}][B_\phi]|J|d\xi\, d\eta$ is the dielectric stiffness matrix. F_m is the mechanical force vector.

$\{F_{el}\}=\dfrac{\kappa_{33}}{(h_k-h_{k-1})}\iint [N_\phi]^T\phi\,|J|d\xi\, d\eta$ is the electric charge vector.

3. MODELING AND VALIDATION OF MACRO FIBER COMPOSITE

Macro Fiber Composite (MFC), is a layered, planar actuation device that employs rectangular cross-section, unidirectional piezoceramic fibers (PZT 5A - lead zirconate-titanate), embedded in a thermosetting polymer matrix. This active layer is further sandwiched with electrode and environment protection layers. The d_{33} piezoelectric effect makes MFC to induce higher strain levels into the host structure.

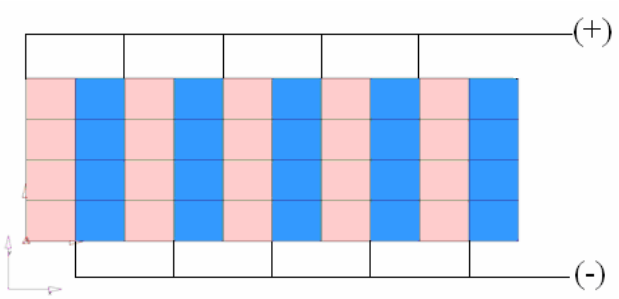

Figure-1. MFC actuator modeling.

A simple and efficient modeling scheme has been developed in the general purpose FE software ANSYS® (ver.11) for validation purpose. The active area of the actuator is segmented in to equal specified number of parts as shown in Figure-1; and parallel electric configuration is further assumed alternatively. The MFC type M4010-P1 of smart materials® is considered. The actuator is modeled as a 3-Dimensional structure using solid-5 elements in commercial finite element software ANSYS®. Solid 5 elements have eight nodes with up to six degrees of freedom at each node for electro mechanical coupling. Further the coupled finite element formulation is coded in MATLAB. The structural model is simulated using a four-noded composite facet element based on coordinate transformation with drilling degree of freedom [20].

3.1 Static induced deflection analysis

The ability of the four noded facet shell element to analyze shell structure is validated here. The cylindrical CFRP shell is considered, which has the following dimension: radius 200 mm, width 200 mm and length 300 mm. The composite shell structure is fabricated with the BDC fabric of total 4 layers, each having the thickness of 0.3 mm. A cantilevered boundary condition is simulated using the appropriate fixtures (Figure-2). Static induced deflection analysis was carried out to test the deflection of shell attached with MFC actuators, when voltage was applied to actuators.

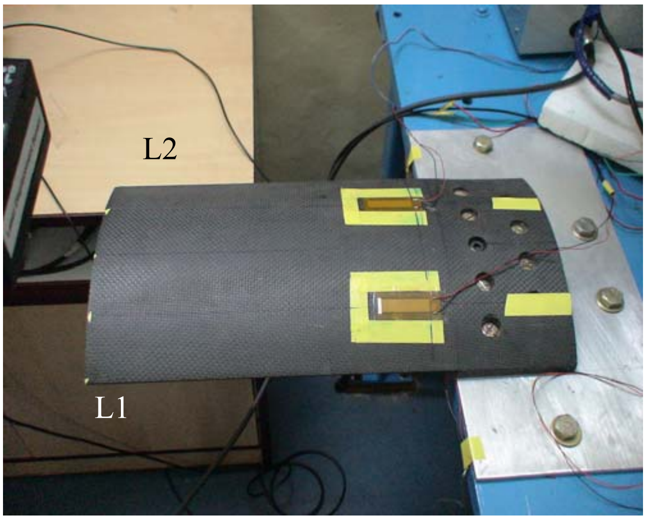

Figure-2. Shell with surface bonded MFC actuators.

Two actuators were surface bonded to the composite laminate cylindrical shell. The deflection was measured at the two locations (extreme ends of the shell marked as location 1(L1) and location 2 (L2) using optical measurement device, by applying 1000v to each actuator as shown in the Figure-2. Deflections of shell when both actuators were active are presented Table-1.

Table-1. Induced defection of shell.

Experiment		MATLAB (FEM)		ANSYS	
L1 (mm)	L2 (mm)	L1 (mm)	L2 (mm)	L1 (mm)	L2 (mm)
0.025	0.028	0.027	0.027	0.026	0.026

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3.2 Free vibration analysis

Free vibration analysis of cantilever cylindrical shell was conducted by impulse testing with two PZT sensors, of size 15mm x 15mm x 0.5 mm thickness were bonded on the bottom side of the shell. First four mode frequencies were recorded. The shell was modeled in ANSYS and Modal analysis was performed using block lanczos method. A MATLAB Finite Element program was developed based on the formulation presented in section 2. It's evident from the Table-2, that the formulated facet shell element captures the dynamics and a good agreement are seen between the experimental and ANSYS results. Figure-3 shows the first four mode shapes extracted from the free vibration analysis in MATLAB.

Table-2. Free vibration analysis of shell.

Mode	Present FEM (Hz)	ANSYS (Hz)	Experiment (Hz)
1	51.85	51.23	50.86
2	107.03	106.49	103.44
3	171.7	174.07	165.99
4	186.56	189.81	185.88

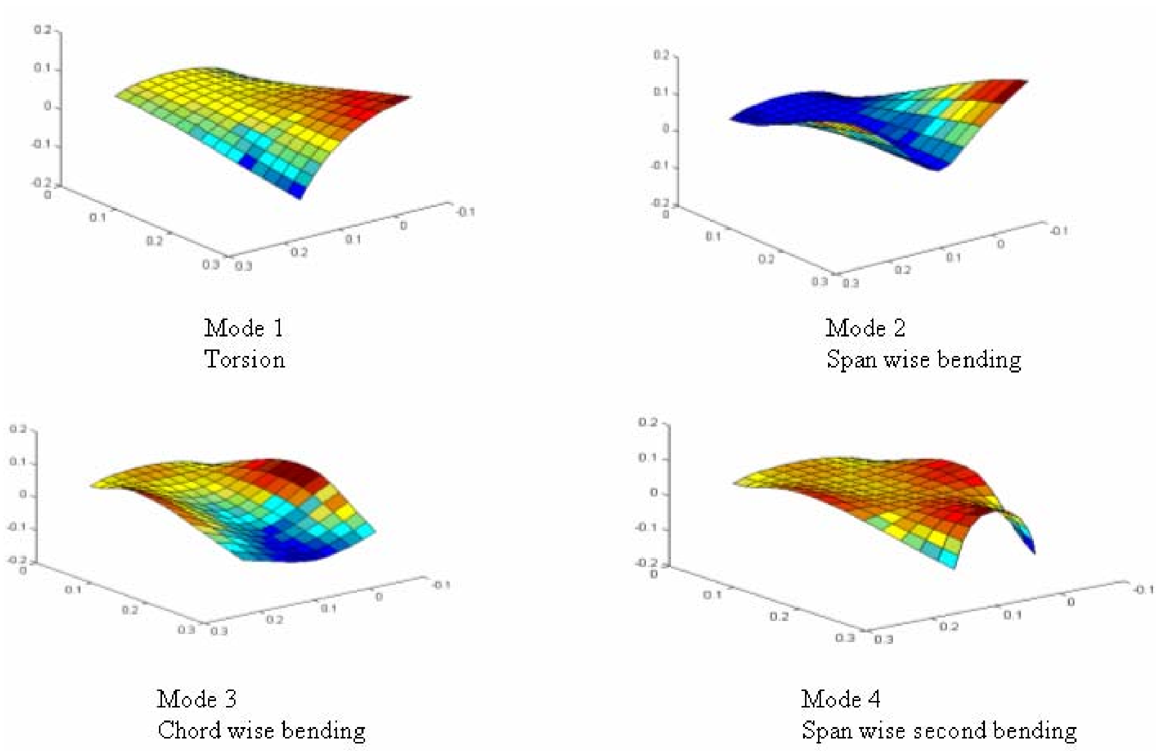

Figure-3. First four mode shapes.

4. VIBRATION CONTROL

Linear quadratic regulator (LQR), an optimal control theory based scheme is used to determine the control gains. In this, the feedback control system is designed to minimize a cost function or a performance index, which is proportional to the required measure of the system's response. A state feedback rather than output feedback is adopted to enhance the control performance.
The cost function used in this case is given by

$$J = \int_0^\infty \left(\{\chi\}^T [Q] \{\chi\}^T + \{\phi_a\}^T [R] \{\phi_a\} \right) dt \qquad (9)$$

Where, [Q] and [R] are the semi-positive-definite and positive-definite weighting matrices on the states $\{\chi\}$ and control inputs $\{\phi_a\}$, respectively. Larger (relatively) elements in [Q] mean that we demand more vibration suppression ability from the controller. The MATLAB software has inbuilt functions for estimating the control gains using LQR strategy. In the present work, these functions are used for solving the associated Riccati equation and to obtain the control gains.

4.1 Active control analysis- state space model

The shell vibration control is attempted to evaluate the performance of PZT composite actuators. The structural system has been represented in modal domain, after decoupling its elastic models. This has facilitated to target a single mode in the control design. Simple resonance control (single mode) and combination resonance control (selective modal control) procedures are developed and analytically studied. The shell (plant) dynamics with feedback control have been defined in state variable form in order to estimate the controller gain matrix. A state feedback control is applied based on linear quadratic regulator technique (LQR), in which the

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parameters Q (state variable weighting), R (control input weighting) are appropriately tuned to have asymptotic stability in the system.

$$\dot{\chi}_i = A_i \chi_i + B_i \phi_{ai} \tag{10}$$

Where, χ =state vector $= \left\{ \eta \; \dot{\eta} \right\}$,

η = modal displacement,

$\dot{\eta}$ = modal velocity, and

$$A_i = \begin{bmatrix} 0 & 1 \\ -\omega^2{}_i & -2s_i\omega_i \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & 0 \\ f(i) & b(i,a) \end{bmatrix}$$

The following linear feedback law is adopted

$$\phi_a = -G\chi, \tag{11}$$

Where, the modal gain G is calculated by solving the algebraic Reccati equation. The actuator voltage (ϕ_a) is estimated for the ith mode and the nth patch using the following relation.

$$\phi_{ai}^n = -G_i^d \frac{1}{C_p^n} K_{\phi u}^n \phi_i^n \eta - G_i^v R_e K_{\phi u}^n \phi_i \dot{\eta} \tag{12}$$

Where,

G_i^d : Displacement feedback gain

G_i^v : Velocity feedback gain

C_p^n : Equivalent capacitance of nth patch in loop on condition

R_e : Equivalent circuit resistance

$K_{\phi u}$: Sensor patch influence coefficient

$\eta, \dot{\eta}$: Modal displacement, modal velocity, respectively

The composite deep shell vibrations are controlled by two MFC actuators. The description of shell structure and the actuator details are presented in section 3. The developed modal control procedures are adopted and control simulation studies are carried out independently for single and combination resonances control. Results are obtained as Bode plots (frequency response analysis) as shown in Figures 4, 5 and 6. Figure-4 shows independent modal space control of first mode and Figure-5 and Figure-6 show the combination or selective modal space control of first two modes and first three modes, respectively.

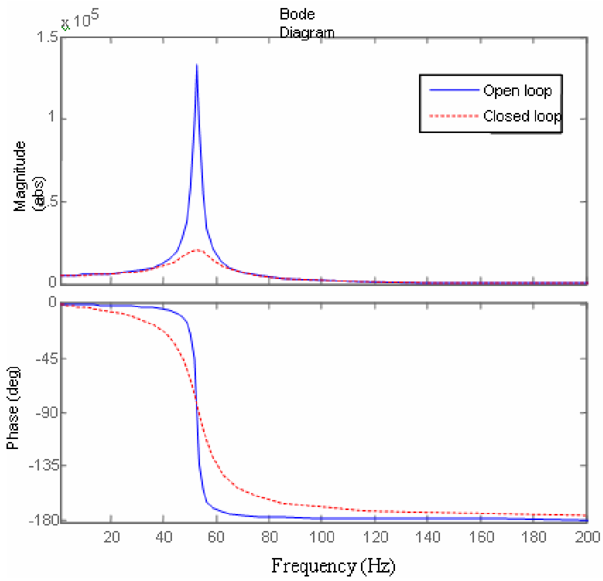

Figure-4. 1st mode independent resonance control.

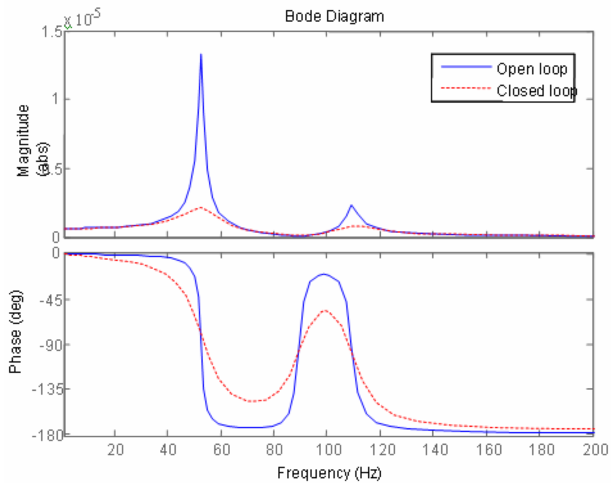

Figure-5. Combination resonance control of 1st and 2nd mode.

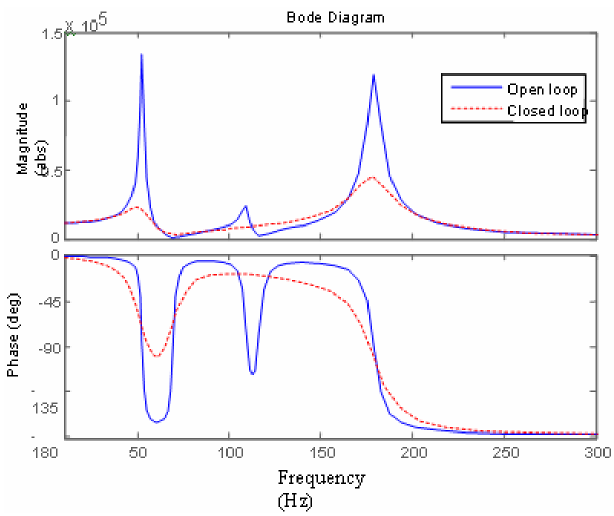

Figure-6. Combination resonance control of 1st, 2nd and 3rd mode.

5. COLLOCATED AND NON-COLLOCATED CONTROL

Control design using non-collocated and collocated sensors and actuators are formulated for the composite cylindrical shell structure. In vibration attenuation in large space structures, Collocated control is often considered to be the solution. Benefit of this approach is that single-input/single-output (SISO) control laws can be synthesized that are robust and attenuate vibration at the collocation point. collocated control is limited by the placement of the sensor/actuator pairs, the amount of force the actuators are able to be exerted on the structure and its ability to achieve performance objectives at other locations, besides the point of collocation, on the structure. Non-collocated control, on the other hand, takes advantage of measuring the exact quantity at the sensor locations to be controlled, provided the sensors are placed at locations where performance is desired. Non-collocated control is constrained by the actuators having to attenuate vibration at sensor locations through a flexible structure.

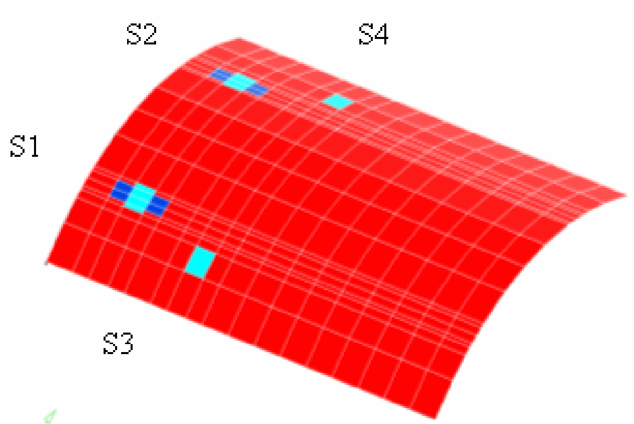

Figure-7. Location of collocated and non collocated sensors on shell structure.

In this section an effort has been made to addresses the question of whether performance is affected by the non-collocation of the sensors and actuators. Control designs using non-collocated and collocated sensors and actuators are formulated for the composite cylindrical shell structure to investigate the benefits and limitations of each approach. The design uses four sensors where $S1$, S_2 are collocated sensors and S_3, S_4 are non collocated sensors as shown in Figure-7. The results are compared to that of collocated sensors and actuators for vibration suppression. Both theoretical and experimental results are included.

5.1 Stability analysis- phase margin

Gain margin and phase margin are measures of relative stability of a system. The phase margin is the difference in phase between the phase curve and -180° at the point corresponding to the frequency that gives us a gain of 0dB (the gain cross over frequency). In other words the phase margin can be defined as the change in open loop phase shift required to make a closed loop system unstable (Figure-8). The stability analysis in terms of phase margin is conducted on the shell structure with sensors in collocated and Non-collocated case and the results are presented in Tables 3 and 4. Table-3 presents the IMSC approach, in which each mode is considered separately and phase margin is computed for the first four modes. Table-4 presents the SMSC approach. This analysis was carried out for both collocated and non collocated configuration of the sensors.

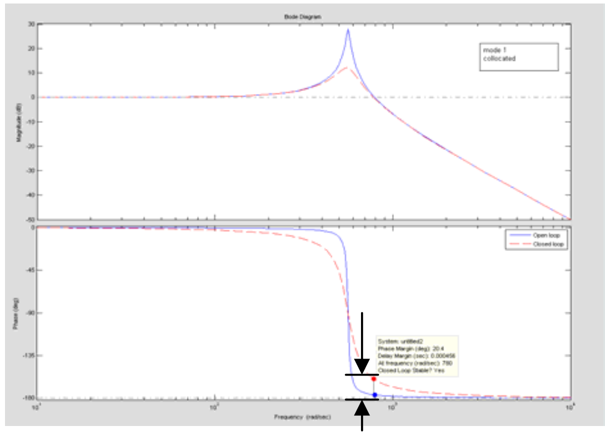

Figure-8. Phase margin plot with collocated sensor in first mode.

Table-3. Phase margin for collocated and non-collocated sensor in IMSC

Modes	**Phase margin** (deg)	
	Collocated	Non collocated
Mode 1	30.4	18.2
Mode 2	25.8	16.3
Mode 3	30.8	15.2
Mode 4	23.5	15.4

Table-4. Phase margin for collocated and non-collocated sensor in SMSC.

Modes	**Phase margin** (deg)	
	Collocated	Non collocated
Modes 1-3	15.1	12.9
Modes 2-4	15.7	16.5
Modes 2-3-4	15.0	12.6

5.2 Power spectral density

Power spectral density (PSD) shows the strength of the variations (energy) as a function of frequency. In other words, it shows at which frequency variations are strong and at which frequency variations are weak. Form PSD plot one can obtain energy within a specific frequency range by integrating PSD within that range. MATLAB software has an inbuilt function “avgpower”, which uses rectangle approximation to integrate under the curve to calculate the average power within the specified frequency band. In the collocated configuration the sensors S1 and S2 are located as shown in the Figure-7. An impulse disturbance is used to generate the time signal. This time signal was employed to construct the PSD plot of a particular mode in both open and closed loop systems. In the non collocated configuration S3 and S4 sensors are used and PSD plot is constructed with the impulse time signal. Table-5 shows the power in the first four modes for collocated and non collocated case. Figures 9 to 11 show the power spectral densities for independent and selective modal control. The average power calculations were carried out for both independent and selective modal space control techniques. Table-6 presents the average power for both collocated and non-collocated cases in open and closed loop condition.

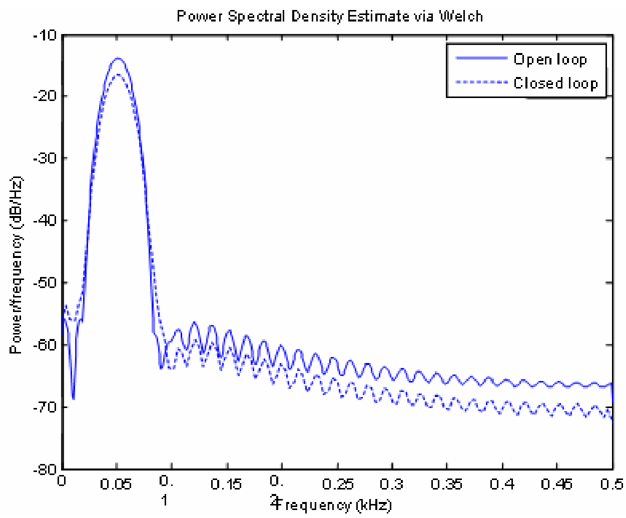

Figure-9. PSD plot for mode 1.

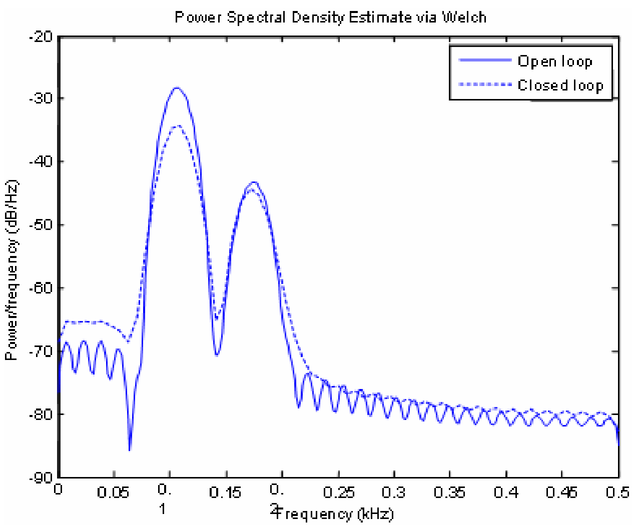

Figure-10. PSD plot for modes 1-3.

PSD analysis shows that all the modes can be controlled effectively in independent modal space domain and selective modal space control can be applied to specific set of modes. Collocated actuator/sensor configuration has shown more average power compared to that of non-collocated configuration.

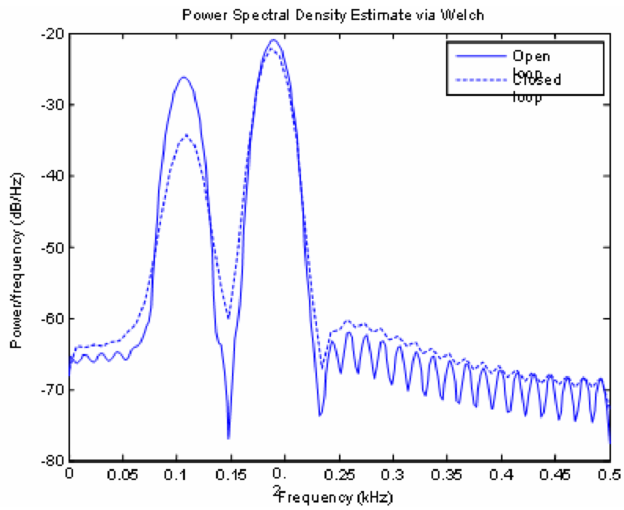

Figure-11. PSD plot for modes 2-4.

Independent modal space control

Table-5. Power for collocated and non collocated case in IMSC.

PSD analysis	Average power (g^2)		
Mode	Open loop	Closed loop	
		Collocated	Non-Collocated
Mode 1	2.7851	2.5323	1.9897
Mode 2	0.2370	0.2325	0.1986
Mode 3	0.0265	0.0196	0.0165
Mode 4	2.2651	2.2321	1.968

Selective modal space control

Table-6. Power for collocated and non collocated case in SMSC.

PSD analysis	Average power (g^2)		
Modes	Open loop	Closed loop	
		Collocated	Non-collocated
Modes 1-3	0.216	0.198	0.186
Modes 2-4	0.256	0.1945	0.1892
Modes 2-3- 4	4.245	3.784	3.562

6. CONCLUSIONS

A four noded composite facet shell element formulated with electro-mechanical coupling is used in active vibration control study of the CFRP cylindrical shell structure. Two PZT composite MFC actuators are used as feedback actuators. Independent modal space control (IMSC) and selective modal space control (SMSC) approaches are employed. It has been shown that proper utilization of actuators, control energy and power are possible through selective modal space control (SMSC) and Collocated configuration is suggested for effective control of shell structure.

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