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# BICONICAL LINEAR ARRAY ANALYSIS FOR NON-UNIFORM AMPLITUDE EXCITATION METHODS

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**ABSTRACT**

Five element linear biconical array is proposed in this paper. The response of the array for standard non uniform amplitude excitations such as Binomial and Dolph-Tschebyscheff distributions are considered. Variation of radiation patterns in horizontal plane, vertical plane, Gain, beam width, side lobe levels and directivity are critically investigated. Behavior of Impedance, scattering parameters of the biconical dipole with frequency in array operation are elucidated. The response of the array and levels of various parameters for binomial and Dolph-Tschebyscheff amplitude excitations are critically compared and contrasted. Plots for various parameters are presented. Simple techniques for controlling side lobe levels and beam width of linear biconical dipole array are introduced.

**Keywords:** array, biconical, excitation, cone angle, dipole.

## 1. INTRODUCTION

The beam of an array can be shaped to become narrow and control the level of the side lobes by adjusting the current amplitudes in an array. The array excitation current distribution can be used to control the shape of the radiation pattern. A biconical antenna consists of two cones aligned along a common axis with their points together at the feed point. Conventional biconical antennas have only three parameters that can be adjusted. The upper and lower cone angles $\theta_1$ and $\theta_2$ and the length of the upper and lower cones $R_1$ and $R_2$. A biconical antenna is used in a system that requires 360 degree coverage in the azimuthal plane with a particular coverage in the elevation plane [1]. Due to the frequency independent nature of its construction, the biconical antenna is well-suited for use in ultra wideband systems [2, 3]. The radiation pattern of a single element is relatively wide and each element provides low values of gain [4-6]. In many applications it is necessary to design antennas with very directive characteristics and very high gains to meet the demands of long distance communication [7, 8]. An assembly of radiation elements in an electrical and geometrical configuration is formed. The total field of the array is determined by the vector addition of the fields radiated by the individual elements [9]. To provide very directive patterns, it is necessary that the fields from the elements of the array interfere constructively (add) in the desired direction and interfere destructively (cancel each other) in the remaining space [9].

## 2. GENERAL ANALYSIS

The biconical antenna can be thought to represent a uniformly tapered transmission line. The analysis begins by first finding the radiated E and H fields between the cones, assuming dominant TEM mode excitation [9]. Once these are determined for any point (r, θ, Φ), the voltage 'V' and current 'I' at any point on the surface of the cone (r, $\theta = \theta_c$, Φ) will be formed. From Faraday's law

$$\nabla X\, E = -\, j\omega\mu H \tag{1}$$

Expanding in spherical coordinates and assuming that the E-field has only $E_\theta$ component independent of Φ. Since H-field has only $H_\phi$ component, necessary to form the TEM mode with $E_\theta$. Expanding (1)

$$\frac{1}{r}\frac{\partial}{\partial r}(r E_\theta) = -\, j\omega\mu H_\Phi \tag{2}$$

From Ampere's law

$$\nabla\, X\, H = j\omega\varepsilon E \tag{3}$$

Expanding in spherical coordinates

$$\frac{1}{\sin\theta}\frac{\partial}{\partial r}(r\sin\theta H_\Phi) = -\, j\omega\varepsilon E_\theta \tag{4}$$

$$H_\Phi = \frac{H_o}{\sin\theta}\frac{e^{-jkr}}{r} \tag{5}$$

$$E_\theta = \eta H_\Phi = \eta\frac{H_o}{\sin\theta}\frac{e^{-jkr}}{r} \tag{6}$$

The voltage produced between two corresponding points on the cones, a distance 'r' from the origin is

$$V(r) = \int_{\frac{\alpha}{2}}^{\pi-\frac{\alpha}{2}} E\, dl = \int_{\frac{\alpha}{2}}^{\pi-\frac{\alpha}{2}} E_\theta\, r\, d\theta \tag{7}$$

$$V(r) = 2\eta H_o e^{-jkr}\ln\left[\cot\left(\frac{\alpha}{4}\right)\right] \tag{8}$$

The current on the surface of the cones, a distance 'r' from the origin is

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$$I(r)=\int_{0}^{2\pi} H_{\phi} r\sin\theta d\phi = 2\pi H_{o} e^{-jkr} \quad (9)$$

Input impedance of single biconical antenna is

$$Z_c = \frac{V(r)}{I(r)} = \frac{\eta}{\pi}\ln\left[\cot\left(\frac{\alpha}{4}\right)\right] \quad (10)$$

The characteristic impedance is not a function of the radial distance 'r' and represents the input impedance at the antenna feed terminals.

$$Z_c = Z_{in} = 120\ln\left[\cot\left(\frac{\alpha}{4}\right)\right] \quad (11)$$

α = cone angle

## 3. ANALYSIS OF LINEAR BICONICAL ARRAY

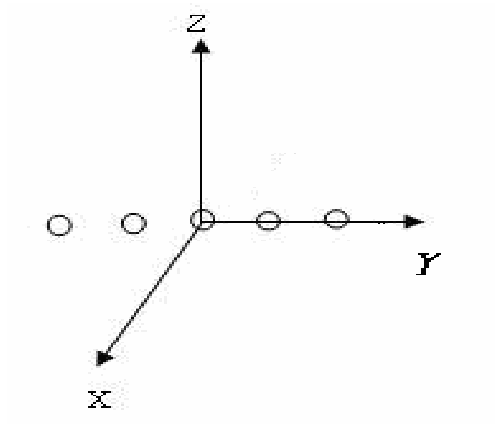


**Figure-1.** Topology of Linear Array.

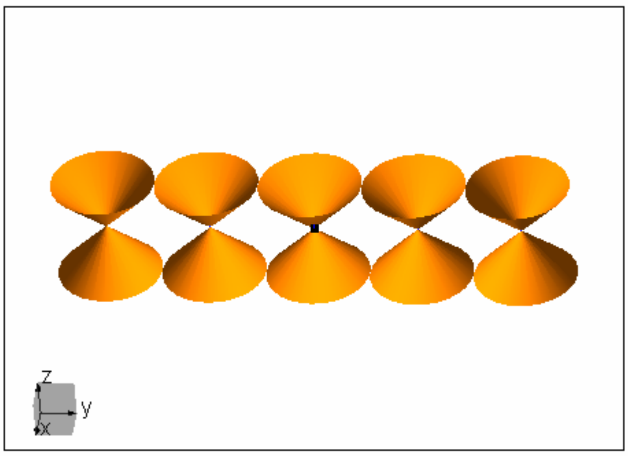


**Figure-2.** Five Element Linear Biconical Antenna Array.

### i) Binomial excitation

Although the uniform in phase current distribution is the one which produces the greatest gain for a given length, it also produces relatively high values of side-lobe levels. For many applications high side-lobe levels are undesirable since they may be responsible for unnecessary interference or in the case of radar, for false target returns. It is possible, in principle to reduce the side-lobe levels of a linear array to as low a level as desired by the choice of an appropriate current amplitude distribution along a linear array. This is obtained at the expense of reduced gain and a larger beam width. The shape of a typical current distribution is such that the current tapers from a maximum value at the center of the array to some minimum value at the edges of the array. The exact nature of the radiated field pattern is a function of the distribution. Asymmetric excitations are also used for shaped beams. One such excitation technique is the binomial amplitude distribution. A linear array with uniform spacing but non uniform amplitude distribution is considered.

The array factor of non uniform amplitude broad side linear array for odd number of elements equally spaced is

$$AF = \sum_{n=1}^{M+1} a_n \cos[(2n-1)u] \quad (12)$$

Where $u = \frac{\pi d}{\lambda}\cos\theta$ ; d = separation between elements.

M is an integer.

$a_n$'s are the excitation coefficients of the array elements.

To determine the excitation coefficients of a binomial array, the function $(1+x)^{m-1}$ be written in a series, using the binomial expansion as

$$(1+x)^{m-1} = 1 + (m-1)x + \frac{(m-1)(m-2)}{2!}x^2 + \cdots \quad (13)$$

The positive coefficients of the series expansion for different values of m are

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| m=1 | | | | | 1 | | | | | |
| m=2 | | | | 1 | | 1 | | | | |
| m=3 | | | 1 | | 2 | | 1 | | | |
| m=4 | | 1 | | 3 | | 3 | | 1 | | |
| m=5 | 1 | | 4 | | 6 | | 4 | | 1 | --- (14) |

The values of 'm' are used to represent the number of elements of the array. For 5-element array (2M+1 = 5)

$2a_1 = 6;\ a_1 = 3;\ a_2 = 4;\ a_3 = 3$ (15)

For binomial method as for any other non uniform array method, one of the requirements is the amplitude excitation coefficients for a given number of elements. This can be accomplished using the Pascal triangle for a linear array [9].

Figure-1 shows the geometrical position of the elements of the array along Y-axis. All the five elements of the array are equispaced with a separation of λ/2 between the elements. Equispaced biconical dipoles excited at the apices are depicted in Figure-2. Height of

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each element is 11.11cm., at a design frequency of 1.35 GHz, taken as center frequency for UHF band (0.3-300) GHz. The elements of the array are excited with voltages proportional to the binomial coefficients for five element array with zero phases.

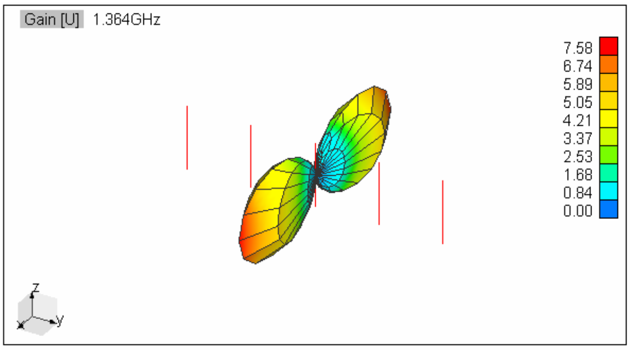


**Figure-3.** 3-D Radiation Pattern.

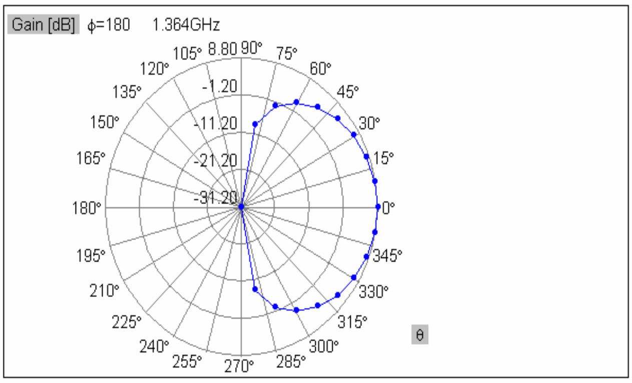


**Figure-4.** Polar Plot in Vertical Plane.

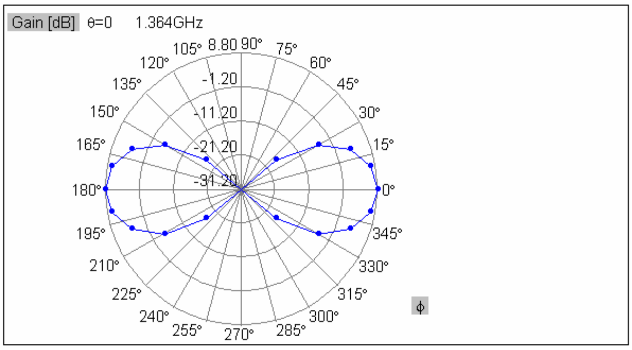


**Figure-5.** Polar Plot in Horizontal Plane.

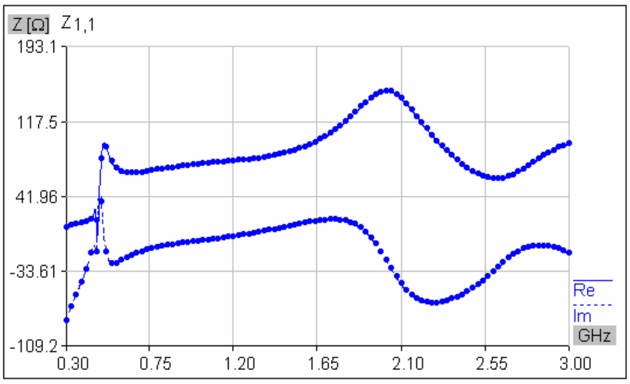


**Figure-6.** Impedance vs. Frequency.

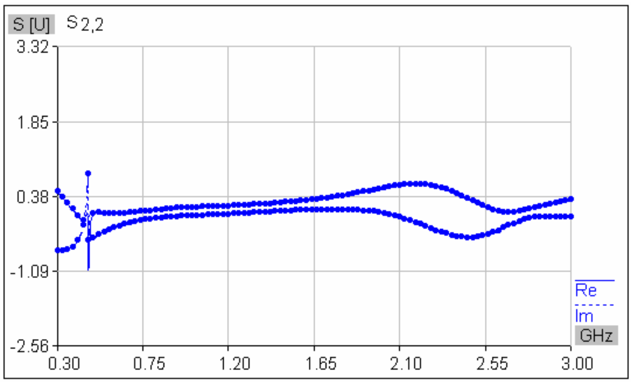


**Figure-7.** S- Parameter vs. Frequency.

Excitation Table-1.

| **Element** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| Amplitude | 1v | 4v | 6v | 4v | 1v |

Each element of the array is excited with voltage amplitudes as given in the Excitation Table-1.

The 3- dimensional radiation pattern of the linear array for binomial amplitude distribution is depicted in Figure-3. The pattern is broad side and perpendicular to the axis of the array. This pattern is bidirectional and symmetrical about positive X- axis and negative X-axis. The pattern is very narrow and highly directional with a gain of 8.8dB. At 1.364 GHz. Polar plots of the array in horizontal and vertical planes are depicted in Figures 4-5. Biconical linear array with uniform amplitude excitation has side lobes in all directions. In many practical applications the side lobes are undesirable. Hence the side lobes are completely eliminated using binomial amplitude excitation with λ/2 separation between the elements.

The directivity of linear binomial array of N - elements is

$$D = 1.77\sqrt{N} \quad (16)$$

Put N = 5 for linear array of 5-elements,hence

D = 3.96(dimension less) = 5.98dB (17)

The Half - Power Beam Width (HPBW) of the main beam for binomial excitation for separation between the elements d = λ/2 is

$$HPBW = \frac{1.06}{\sqrt{N-1}} = \frac{1.06}{\sqrt{5-1}} \quad (18)$$

N = Number of elements of the array = 5

HPBW = 0.53 radians = $30.37^0$ (19)

Directivity of the array for 5- element linear array is 5.98dB. Computed value of half-power beam width (HPBW) is $30.37^0$. Radiation pattern in polar form also exhibits beam width very close to computed value and is highly directional.

Variation of impedance with frequency is depicted in Figure-6. Impedance is fairly constant around ~ 100Ω up to 1.65GHz. And is varying within 150 Ω up to 3GHz. Plot of only one element is shown as it is not practical to show impedance plots for all the elements. All the elements in the array exhibit the same variation. S-parameter variation with frequency is illustrated in Figure-7. The frequency of excitation is varied for the entire UHF band (0.3-3GHz) and the response is plotted. The reflection coefficient parameter variation ($S_{22}$) for second element in the array is shown and response of other elements is nearly the same. For the entire band of frequencies the parameter variation is constant and less than one (~0.4). Constant and low values of impedance, reflection coefficient parameters indicate the frequency independent nature of the array. Very low value of reflection coefficient indicates correct impedance matching at the input terminals of the antenna.

**ii) Dolph-Tschebyscheff linear array**

Another major class of pattern synthesis method is that for achieving a narrow main beam accompanied by low side lobes. Patterns of this type have many applications, such as in point - to-point communications and direction finding. Dolph-Chebyschev method for linear arrays is the most important narrow beam, low side-lobe method. As the current amplitude taper from center to the edges of the array increased, the side-lobe level decreases, but with an accompanying increase in the width of the main beam. This array is a compromise between uniform and binomial arrays. Its excitation coefficients are related to Tschebyscheff polynomials. The element current amplitudes are determined such that the beam width is minimum for a specified side lobe level or conversely to specify the beam width and obtain the lowest possible side lobe level. This array is referred to as a Dolph- Tschebyscheff array and it provides a pattern with all side lobes of the same level.

The array synthesis procedure follows the principle of matching a polynomial of appropriate degree and desired properties to the array factor of the array. The procedure is based on the simple observation that every polynomial of (N-1) $^{th}$ degree can be looked at as the array factor of an N-element array. The array factor in the factored form gives the location of the roots and in the expanded form gives the excitation coefficients. Thus from the knowledge of the roots of the polynomial we can write the factored form and by expanding it in power series form, we can get the excitation coefficients.
The Chebyschev polynomial of $m^{th}$ degree is given by

$$T_m(x) = \begin{cases} (-1)^m \cosh\left(m \cosh^{-1} x\right) & x < 1 \\ \cos\left(m \cos^{-1} x\right) & -1 < x < 1 \\ \cosh\left(m \cosh^{-1} x\right) & x > 1 \end{cases} \quad (20)$$

By inspection

$T_0(x) = 1$

$T_1(x) = x$

$T_2(x) = 2x2 -1$

$T_3(x) = 4x^3 - 3x$

$T_4(x) = 8x^4 - 8x^2 + 1$ (21)

The connection between arrays and Chebyshev polynomial is established by considering a symmetrically excited, broad side array for which $i_{-m} = i_m$. Symmetrical excitation leads to a real - valued array actor that is

$$f(\psi) = \begin{cases} i_o + 2\sum_{m=1}^{N} i_m \cos(m\psi) & \text{N odd} \\ 2\sum_{m=1}^{N} i_m \cos\left[(2m-1)\frac{\psi}{2}\right] & \text{N even} \end{cases} \quad (22)$$

Where Ψ = 2π (d/λ) cosθ; m = N-1, N = number of elements.

This array factor for odd or even N is a sum of cos (m ψ /2) terms for m up to N-1. The array factor is expressible as a sum of terms with powers of cos (ψ /2) up to N-1. By choosing an appropriate transformation between x and Ψ the array factor and Chebyshev polynomial are identical. The transformation

$x = x_0 \cos(\psi/2)$ and $u(\psi) = T_{N-1}[x_0 \cos(\psi/2)]$ (23)

Equation (23) will yield a polynomial in powers of cos (ψ /2) matching that of the array factor. The main beam maximum value of ‘K’ occurs for ψ = 0 or θ = $90^0$. The main beam -to-side lobe ratio ‘K’ is the value of the array factor at the main beam maximum, since the side-lobe- level magnitude is unity. The side-lobe-level is thus 1/K or

SLL = -20 log K dB (24)

$K = T_{N-1}(x_0) = \cosh[(N-1)\cosh^{-1}(x_0)]$ (25)

Solving for $x_0$, we get

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$$x_o = \cosh\left(\frac{1}{N-1}\cosh^{-1} k\right) \qquad (26)$$

The half power beam width (HPBW) of a Dolph-Chebyshev array, is given by

$$HPBW = \pi - 2\cos^{-1}\left(\frac{\rho_h}{\beta_d}\right) \qquad (27)$$

$$\rho_h = 2\cos^{-1}\left\{\frac{\cosh\left[\frac{1}{N-1}\cosh^{-1}\left(\frac{K}{\sqrt{2}}\right)\right]}{\cosh\left[\frac{1}{N-1}\cosh^{-1}(K)\right]}\right\} \qquad (28)$$

The directivity of broad side Chebyshev array using the HPBW is

$$D \cong \frac{2K^2}{1+K^2\, HPBW} \qquad (29)$$

For 5-element linear array N=5, considering -20dB side-lobe levels SLL=-20dB.

K = $10^{-SLL/20}$ = 10; $X_0$ = 1.293, using (20)-(21) gives the amplitude distributions shown below

Excitation Table-2.

| Element | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Amplitude | 1v | 1.61v | 1.93v | 1.61v | 1v |

The half power beam width (HPBW) is computed using (26) - (27) for d = λ/2, K =10 and its value is $23.6^0$.

Substituting this in (28) with K = 10 gives the directivity of the array as

D = 4.74 (dimension less) = 10log4.74 = 6.76 dB (30)

The beam broadening relative to the uniform excitation case is given as the beam broadening factor as

$$b_{HPBW} = 0.637\sqrt{\ln(2K)} \qquad (31)$$

Substituting K = 10,

The beam broadening factor = $b_{HPBW}$ = 1.1

The biconical linear array in Figure-2 is excited with amplitude Chebyshev coefficients as in Excitation Table-2. The directivity, HPBW and beam broadening factor for the 5-element linear array are computed.

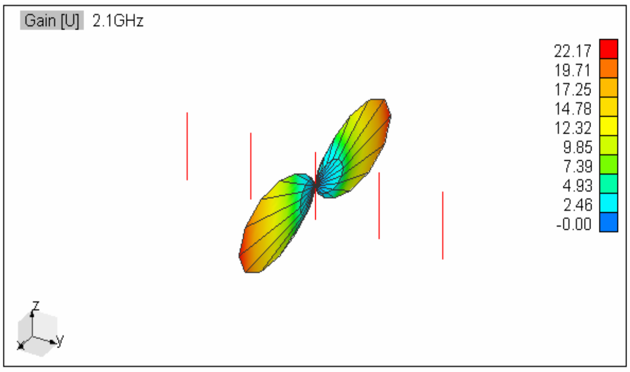


**Figure-8.** 3-D Radiation Pattern.

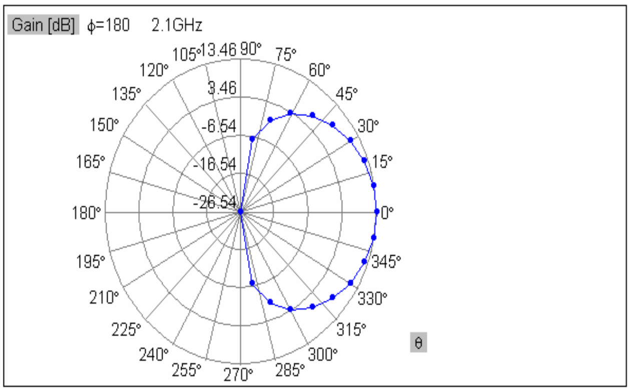


**Figure-9.** Polar Plot in Vertical Plane.

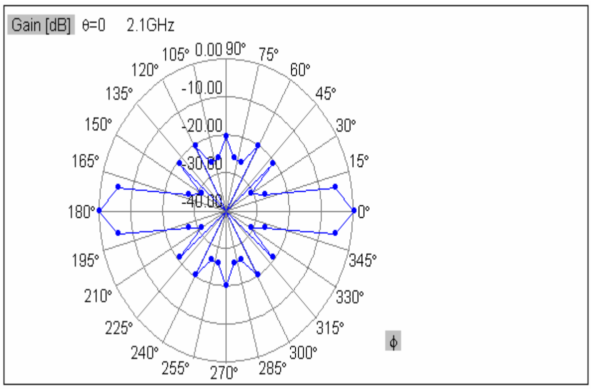


**Figure-10.** Polar Plot in Horizontal Plane.

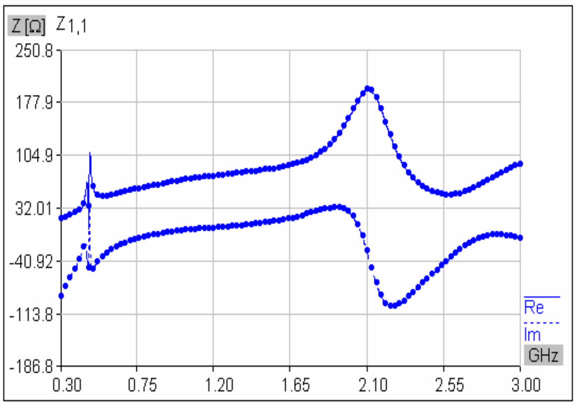


**Figure-11.** Impedance vs. Frequency.

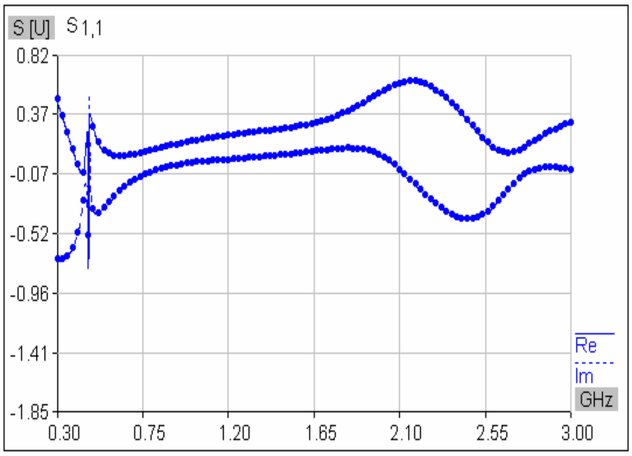


**Figure-12.** S-Parameter vs. Frequency.

The 3-dimensional radiation pattern of the array is presented in Figure-8 at a frequency of 2.1GHz as the gain of the array is a maximum of 13.46dB. The pattern is bidirectional and is highly directive. Polar plots of radiation pattern in vertical and horizontal planes are depicted in Figures 9-10, respectively. The normalized bidirectional plot in horizontal plane in Figure-10 shows a narrow main beam with equal levels of all the side lobes at -20dB. The pattern exhibits narrow beam width, high directivity and increased gain as compared to the binomial excitation.

Variation of impedance with frequency is shown in Figure-11. Impedance is less than 100Ω and is fairly constant up to 1.65GHz and varies between 35-180Ω above 1.65GHz. Variation of impedance between 1.65-3GHz. is within acceptable limits. Any improper impedance at the input of the antenna results in impedance mismatch and results in higher values of reflection coefficients (S- parameters). The S-parameter value is very low (<1) and is same for the entire frequency band considered (0.3-3GHz) as depicted in Figure-12. Low values of reflection coefficients indicate appropriate impedance match and induction of currents due to mutual coupling of radiation from adjacent elements is very less. Low value of S- parameter is an indication of fewer reflections and less mutual coupling. This also indicates strong reinforcement of fields in the desired direction and cancellation in other directions. The results of variation of impedance and S-parameters show the frequency independent nature of the biconical antenna. A plot for one element is shown for impedance and scattering parameters. Plots of other elements are also exhibiting similar variations hence omitted.

## 4. RESULTS AND CONCLUSIONS

Biconical dipole 5-element linear array, equispaced at half wavelength is formed and simulated for the UHF frequency band (0.3-3GHz). The array is excited nonuniformly using standard binomial and Chebyshev distributions. For Binomial excitation radiation pattern is broad side with maximum of the main beam perpendicular to the axis of the array. Pattern has no side lobes in any direction and is bidirectional and directive. Maximum gain of the array is ~9dB. Directivity is 5.98dB and Half-power beam width $30.37^0$. Impedance for the UHF band is 60-150Ω. Scattering parameter is ~0.4 and is constant for all the frequencies in the band.

For Chebyshev excitation the Radiation pattern is broad side and directive, maximum of the main beam is perpendicular to the axis of the array. All the side lobes of the radiation pattern are at the same level (-20dB). Maximum gain of the array is 13.46dB. Directivity of the array is 6.76dB. Half power beam width is $23.6^0$ and Beam broadening factor is 1.1. Impedance for the UHF band is between 50-178Ω. Scattering parameter is very low (<1) and is fairly constant for the entire band.

Radiation pattern of binomial array is broader, with low directivity and has no sides-lobes. As the voltage amplitude is tapered more toward the edges of the array, the side lobes tend to decrease and the beam width increases. Increased beam width is responsible for reduced directivity.

This beam width side lobe level trade off can be optimized. The element voltage or current amplitudes are determined such that the beam width is minimum for a specified side lobe level or conversely to specify the beam width and obtain the lowest possible side lobe level. The Dolph-Chebyshev array provides pattern with all side lobes of the same level. Optimum beam width, side-lobe level performance occurs when there are as many side lobes in the visible region as possible and they have the same level. Chebyshev polynomial possesses this property and is used in this synthesis. Dolph- Chebyshev array with no side lobes reduces to the binomial design. The Chebyshev array provides narrow beam width and highly directive beam from an array of fixed length. The amplitude taper of the excitation from center to edges is more in binomial array compared to Chebyshev excitation and large voltage taper is responsible for no side lobes in binomial array. The Chebyshev array presents increased directivity and narrow beam width as compared to the binomial array. The gain and directivity can be improved by increasing the number of elements in the array. Two efficient methods of non uniform amplitude excitation techniques are implemented on the linear biconical array and shown the method of controlling side lobe levels, beam width and directivity of the radiation pattern and we can conclude from the results that very less amount of mutual coupling of fields between the elements of the array.

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