VOL. 6, NO. 5, MAY 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

REGRESSION MODELING FOR STRENGTH AND TOUGHNESS EVALUATION OF HYBRID FIBRE REINFORCED CONCRETE

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ABSTRACT

This paper presents regression equations for evaluating the strength and toughness of hybrid fibre reinforced concrete (HFRC). A total of 102 concrete specimens were tested to study the effect of hybrid fibre reinforcement on the strength and toughness of fibre reinforced concrete. The fibre content dosage V_f ranged from 0.0 to 2.0 percent. Steel and Polyolefin fibres were combined in different proportions and their impact on strength and toughness studied. Addition of 2.0 percent by volume of hooked-end steel fibres increases the modulus of rupture and toughness by about 58.78% and 19.27% respectively, when compared to the plain concrete. When the fibres were used in a hybrid form, the increase in above study parameters was about 81.0% and 31.42% respectively, when compared to the plain concrete. Empirical expressions for predicting the strength and toughness of hybrid fibre reinforced concrete (HFRC) are proposed based on regression analysis. A close agreement has been obtained between the predicted and experimental results.

Keywords: model, hybrid fibre reinforced concrete, strength evaluation, toughness, rupture, polyolefin, steel, regression analysis.

1. INTRODUCTION

Concrete is a relatively brittle material. Addition of fibres to concrete makes it more homogeneous and isotropic and transforms it from a brittle to a more ductile material [1-3]. The function of short-cut fibres as secondary reinforcement in concrete is mainly to inhibit crack initiation and propagation [4]. The basic purpose of using hybrid fibres is to control cracks at different size levels, in different zones of concrete (cement paste or interface zone between paste and aggregate), at different curing ages and at different loading stages. The large and strong fibres control large cracks. The small and soft fibres control crack initiation and propagation of small cracks [5,6]. This research work focuses on the polyolefin - steel hybrid fibre reinforced system. It has been shown recently that the hybrid composite can offer more attractive engineering properties because the presence of one fibre enables more efficient utilization of the potential properties of the other fibre [7,8]. However, the hybrid composites studied by previous researchers were focused on cement paste or mortar. Information available pertaining to the strength and toughness of hybrid fibre reinforced concrete is found to be limited. Hence an attempt has been made to study the strength and toughness of hybrid fibre reinforced concrete. A total of 102 specimens were cast and tested to determine the impact of hybrid fibres on the strength and toughness of fibre reinforced concrete (FRC).

The development of a mathematical equation to represent real world phenomena is required for predicting the behaviour of systems. Regression helps in the formation of mathematical equations for observed phenomena using statistical principles. The objective of regression is to apply the least squared errors principle to minimize the error between the experimental results and the predicted results. The procedure involves assuming a suitable initial form for the equation, which approximately resembles the form of relationship between the independent and dependent parameters. The schematic diagram of regression equation system is shown in Figure-1.

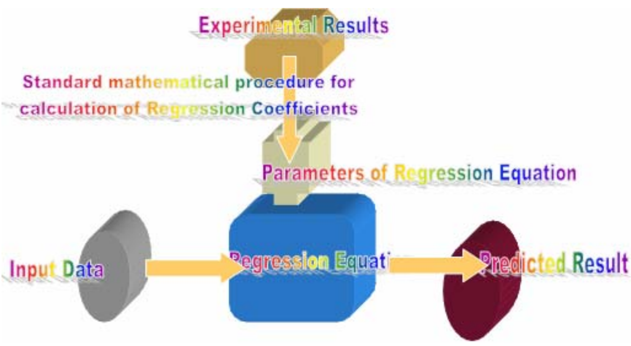

Figure-1. Schematic diagram of regression equation system.

involves choosing the basic form of the equation, with a number of unknown coefficients, called regression coefficients. The data obtained from experimental study on real world phenomena are used for calibrating the values of unknown regression coefficients in such a way that the difference between predicted values and experimental values remains a minimum. In a regression equation, the variables like compressive strength, modulus of rupture and toughness are called independent parameters. The parameters which are known only after prediction like the total fibre volume fraction, percentage of polyolefin fibre and percentage of steel fibre are called the dependent parameters. Hence, regression is simply an effort to predict dependent parameters using known independent parameters.

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MATERIALS AND METHODS

Experimental program

This study is part of a research program on evaluating the performance of hybrid fibre reinforced concrete beams. In this paper, strength and toughness properties of steel fibre reinforced concrete and hybrid fibre reinforced concrete are assessed and the applicability of empirical expressions for predicting the above properties is explored.

Materials

The cement used in the concrete mix was Portland Pozzolana Cement (PPC), which confirms to IS: 1489 (1991) [9]. The sand used was local natural sand with a specific gravity of 2.54. The coarse aggregate was crushed granite with a maximum size of 20 mm and specific gravity of 2.67 which confirms to IS: 383-1970 [10]. Properties of steel and polyolefin fibres are shown in Table-1. The polyolefin fibres used were straight, while the steel fibres were with hooked ends.

Table-1. Properties of fibres.

Fibre properties	Fibre details	
	Polyolefin	Steel
Length (mm)	48	30
Shape	Straight	Hooked at ends
Size/diameter(mm)	1.22×0.732 mm	0.5 mm
Aspect Ratio	39.34	60
Density (kg/m^3)	920	7850
Young's modulus (GPa)	6	210
Tensile strength (MPa)	550	532

Table-2. Concrete mix proportions.

Material	Quantity
Cement (kg/m^3)	400
Sand (kg/m^3)	745
Coarse aggregate (kg/m^3)	960
Water (kg/m^3)	200
Slump (mm)	100

In this study, M20 grade of concrete was used. The mix design for the above grade of concrete was done using ACI method [11]. The mix proportion adopted was 1: 1.82: 2.64: with a water-cement ratio of 0.50. Table-2 presents the control concrete mix proportions used in the testing program. For concrete with fibres, superplastizer was used in appropriate dosage to maintain a slump of about 100 mm.

Test specimens

150 mm cubes were used for determining the compressive strength of steel fibre reinforced concrete and hybrid fibre reinforced concrete. 100 mm x 100 mm x 500 mm prisms were used for determining the modulus of rupture as well as the toughness. The details of control specimens are presented in Table-3.

Testing procedure

150 mm cubes were tested in a standard manner in a 2000kN capacity compression testing machine. The prisms were tested in a loading frame. The specimens were subjected to four-point bending. Deflection measurements were made using dial gauges of 0.01 mm accuracy. The above tests were conducted as per the relevant Indian Standard specifications [12-14].

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Table-3. Details of control specimens.

S. No.	Specimen designation	Fibre content, V_f (%)	Type of fibre	
			Polyolefin	Steel
1	H0 - P0 S0	0	0	0
2	H0.5 - P0 S100	0.5	0	100
3	H0.5 - P20 S80	0.5	20	80
4	H0.5 - P30 S70	0.5	30	70
5	H0.5 - P40 S60	0.5	40	60
6	H1 - P0 S100	1.0	0	100
7	H1 - P20 S80	1.0	20	80
8	H1 - P30 S70	1.0	30	70
9	H1 - P40 S60	1.0	40	60
10	H1.5 - P0 S100	1.5	0	100
11	H1.5 - P20 S80	1.5	20	80
12	H1.5 - P30 S70	1.5	30	70
13	H1.5 - P40 S60	1.5	40	60
14	H2 - P0 S100	2.0	0	100
15	H2 - P20 S80	2.0	20	80
16	H2 - P30 S70	2.0	30	70
17	H2 - P40 S60	2.0	40	60

Table-4. Principal test results of HFRC.

S. No	Specimen designation	Compressive strength, (MPa)	Modulus of rupture (MPa)	Toughness indices		Toughness ratio
				I_5	I_{10}	I_{10} / I_5
1	H0 - P0 S0	26.65	7.06	1.0	1.0	1.0
2	H0.5 - P0 S100	27.96	7.16	5.15	11.10	2.15
3	H0.5 - P20 S80	28.34	9.20	5.72	12.65	2.21
4	H0.5 - P30 S70	27.86	9.60	6.36	15.60	2.45
5	H0.5 - P40 S60	31.61	9.70	7.02	17.69	2.52
6	H1 - P0 S100	28.18	10.18	6.47	13.73	2.12
7	H1 - P20 S80	29.65	10.58	7.0	15.59	2.22
8	H1 - P30 S70	28.98	11.50	7.43	18.62	2.51
9	H1 - P40 S60	27.00	11.53	8.0	20.69	2.59
10	H1.5 - P0 S100	31.10	10.52	7.05	16.51	2.34
11	H1.5 - P20 S80	33.13	10.64	7.65	19.86	2.60
12	H1.5 - P30 S70	31.16	11.84	8.36	24.13	2.89
13	H1.5 - P40 S60	27.07	11.94	9.35	27.29	2.92
14	H2 - P0 S100	30.74	11.21	8.03	19.27	2.40
15	H2 - P20 S80	34.75	11.54	8.98	24.18	2.69
16	H2 - P30 S70	30.34	12.18	9.20	26.27	2.85
17	H2 - P40 S60	30.69	12.78	10.58	31.42	2.97

MULTIVARIATE LINEAR REGRESSION

Multivariate linear regression involves more than one independent variable to predict the value of the dependent variable. The basic formulation of multivariate linear regression equation is,

$$\begin{Bmatrix} \frac{\partial}{\partial a_0} \\ \frac{\partial}{\partial a_1} \\ \frac{\partial}{\partial a_2} \\ \frac{\partial}{\partial a_3} \\ \vdots \\ \frac{\partial}{\partial a_n} \end{Bmatrix} \sum_{i=1}^{K} \left(P_i - \left(a_0 + a_1 x_{1i} + a_2 x_{2i} a_3 x_{3i} + \cdots + a_n x_{ni}\right)\right) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{1}$$

where, $a_0 \cdots a_n$ are the coefficients to be determined, $x_1 \cdots x_n$ are the independent variables, P is the dependent variable or the actual result value for the set of i^{th} input data and K is the number data sets available for regression. On executing the partial derivative operators, equation 1 reduces to,

$$\sum_{i=1}^{K} \begin{bmatrix} 1 & x_{1i} & x_{2i} & x_{3i} & \cdots & x_{ni} \\ x_{1i} & x_{1i}^2 & x_{1i} x_{2i} & x_{1i} x_{3i} & \cdots & x_{1i} x_{ni} \\ x_{2i} & x_{2i} x_{1i} & x_{2i}^2 & x_{2i} x_{3i} & \cdots & x_{2i} x_{ni} \\ x_{3i} & x_{3i} x_{1i} & x_{3i} x_{2i} & x_{3i}^2 & \cdots & x_{3i} x_{ni} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{ni} & x_{ni} x_{1i} & x_{ni} x_{2i} & x_{ni} x_{3i} & \cdots & x_{ni} x_{ni} \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{Bmatrix} = \sum_{i=1}^{K} \begin{Bmatrix} P_i \\ P_1 P_i \\ P_2 P_i \\ P_3 P_i \\ \vdots \\ P_n P_i \end{Bmatrix} \tag{2}$$

The above equation can be solved by summing up the values of independent and dependent variables after carrying out the required operations.

RESULTS AND DISCUSSIONS

The principal test results are presented in Table-4. Each strength value presented is the average of three specimens. A total of 102 specimens were tested in this investigation.

Fibres do not significantly improve the properties of concrete or strength prior to the peak (concrete cracks) as they do on the post-peak response. Hence it would be more practical to evaluate the performance of FRC based on the energy absorption (area under the load-deflection curve). The area under the load-deflection curve is usually referred to as the toughness obtained from a static test of a beam specimen up to a specified deformation.

The test results show that modulus of rupture and toughness increases with all fibre contents. It was noticed that once the maximum tensile stress was reached, the beams without micro-reinforcement failed suddenly. For beams with fibres the failure was not sudden. The randomly oriented fibres crossing the cracked section resisted the propagation of cracks and separation of the section. This caused an increase in the load-carrying capacity beyond the first cracking[15].

Addition of 2.0 percent by volume of hooked-end steel fibres increases the modulus of rupture and toughness by about 58.78% and 19.27% respectively, when compared to plain concrete. When the fibres were used in a hybrid form, the increase in above study parameters was about 81.0% and 31.42% respectively, when compared to plain concrete.

Regression equations have been proposed for predicting the study parameters. The data used for regression analysis is presented in Table-4 and the regression equations are presented in Table-5. Predictions from the regression equations were compared against experimental results and are presented in Figures 2 to 6. It has been found that the multivariate linear regression can

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make a reasonable estimate on the prediction values for compressive strength, modulus of rupture and toughness. A close agreement has been obtained between the predicted and experimental results.

Table-5. Regression equations for the study parameters.

S. No	Parameter	Equation	Fitness	RMS error
1.	Compressive strength (MPa)	$27.27+137.36V_f-85.10V_p+119.80V_s$	0.41	1.69
2.	Modulus of rupture (MPa)	$8.40-250.51V_f+629.72V_p+387.11V_s$	0.89	0.53
3.	Toughness index I_5	$3.93+244.86V_f+308.80V_p-41.81V_s$	0.81	0.90
4.	Toughness index I_{10}	$7.55+711.12V_f+1516.84V_p-154.65V_s$	0.90	2.19
5.	Toughness ratio (I_{10}/I_5)	$1.80+70.07V_f+57.50V_p-38.54V_s$	0.68	0.253

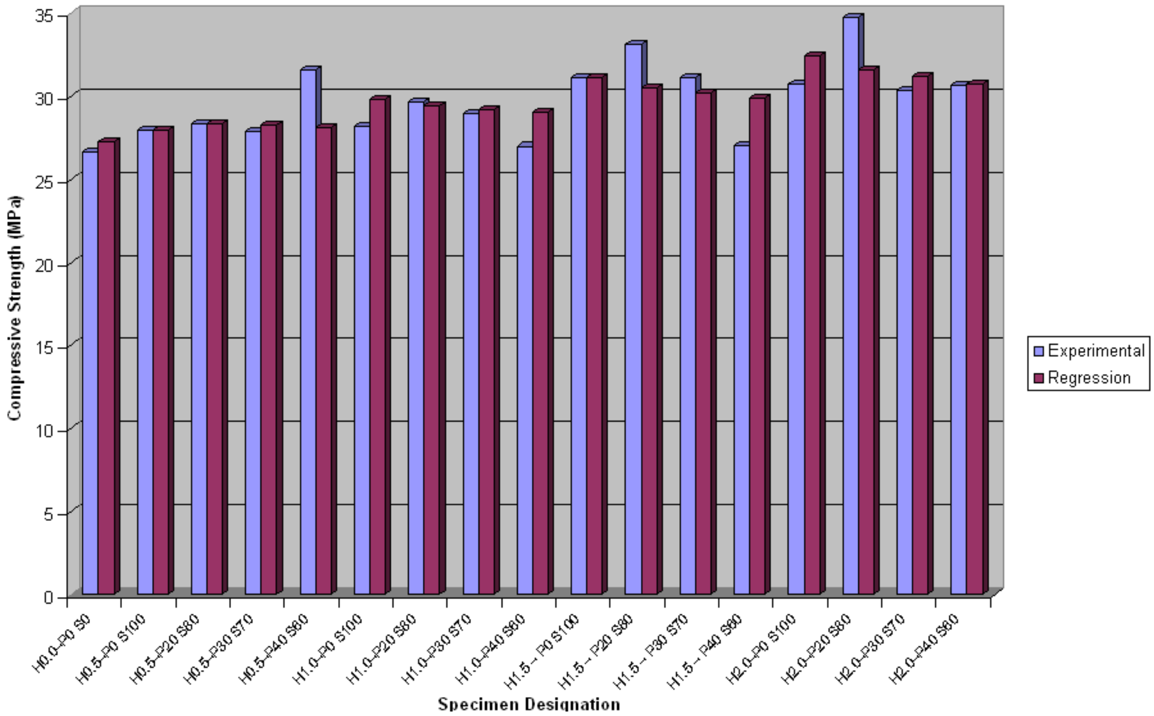

Figure-2. Comparison of predicted and experimental results - compressive strength.

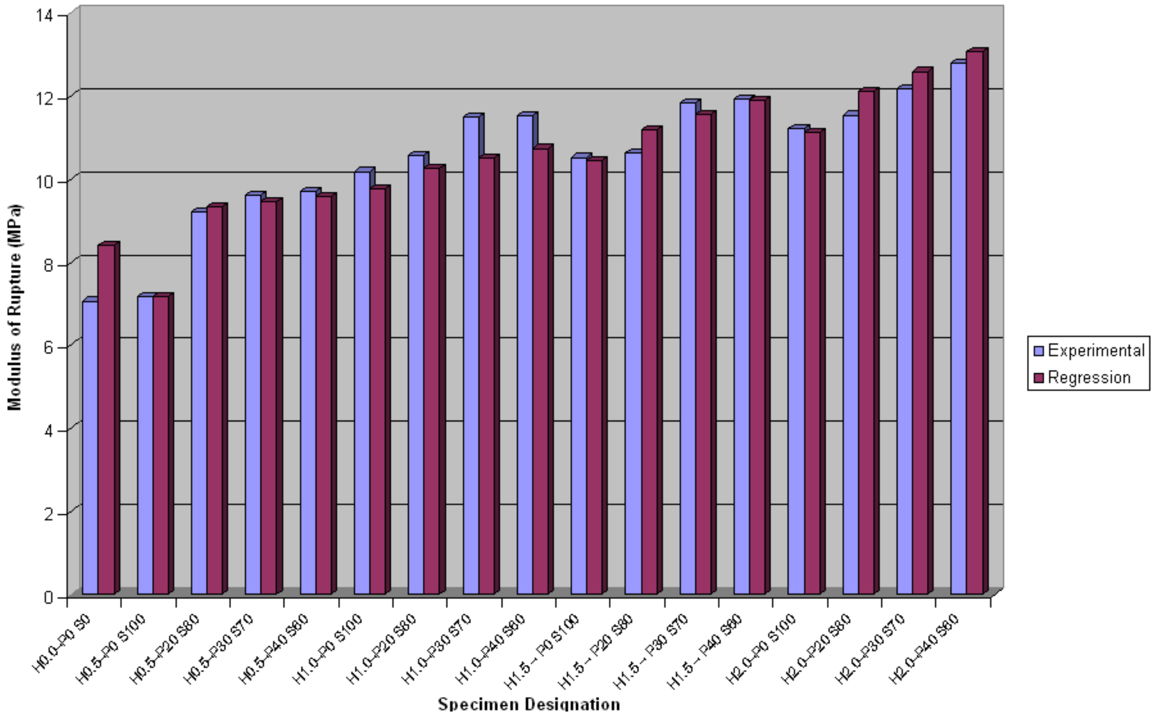

Figure-3. Comparison of predicted and experimental results - modulus of rupture.

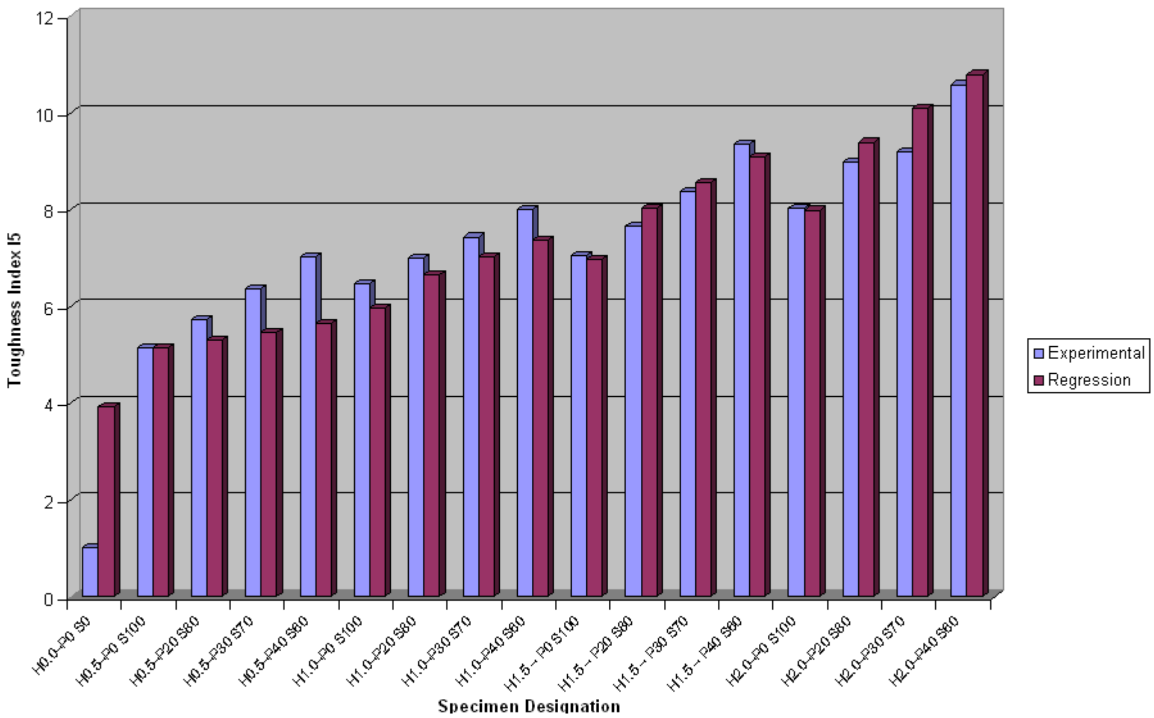

Figure-4. Comparison of predicted and experimental results - toughness index $I_{5.}$

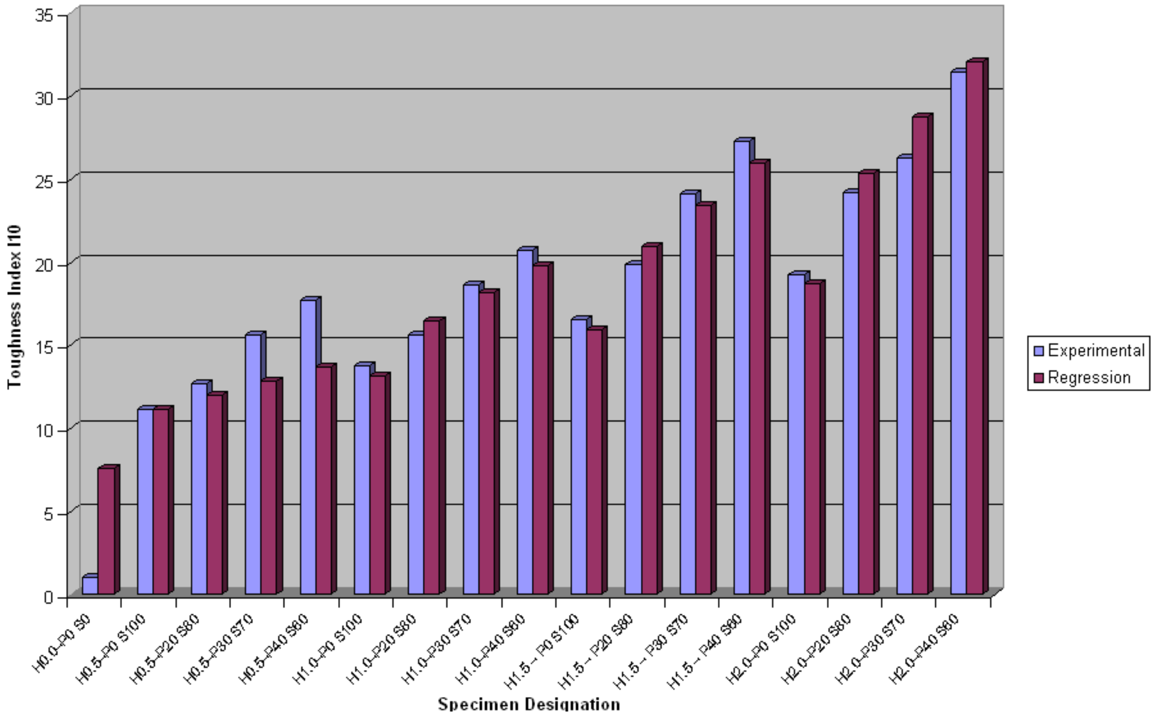

Figure-5. Comparison of predicted and experimental results - toughness index I_5.

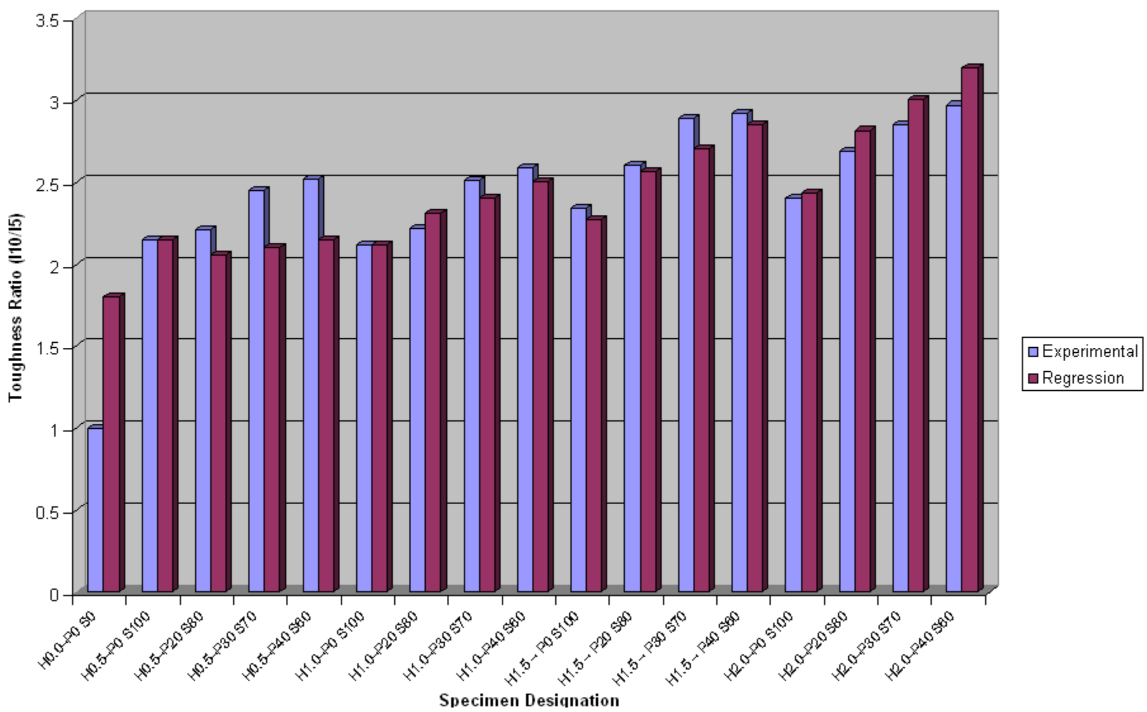

Figure-6. Comparison of predicted and experimental results - toughness ratios (I_{10}/I_5).

CONCLUSIONS

Based on the test results of this investigation, the following conclusions are drawn:

- Addition of fibres has marginal improvement on the compressive strength when compared to the plain concrete;
- HFRC beams exhibit enhanced strength in flexure. The modulus of rupture increased up to 81% compared to their plain concrete counterparts;
- HFRC beams provide toughness ratio up to 2.97 compared to the plain concrete specimens; and
- Regression equations for predicting the strength and toughness of hybrid fibre reinforced concrete are

proposed. A close agreement has been obtained between the predicted and experimental results.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the financial support offered by Tamilnadu State Council for Science and Technology (TNSCST), Chennai, for carrying out this research investigation. The authors also wish to thank Elasto-Plastic Concrete (Europe) Ltd., for their support by way of providing polyolefin fibres required for carrying out the research work.

Notations

V_f - Fibre content (%)
V_p - Volume of Polyolefin Fibre (%)
V_s - Volume of Steel Fibre (%)

REFERENCES

[1] ACI Committee 544. 1989. Measurement of Properties of Fibre Reinforced Concrete, (ACI 544.2R-889). American Concrete Institute, Detroit, Michigan, USA.

[2] ACI Committee 544. 1982. State of the Report on Fibre Reinforced Concrete, (ACI 544.1R-82), Concrete International: Design and Construction. 4(5): 9-30. American Concrete Institute, Detroit, Michigan, USA.

[3] Naaman A.E. 1985. Fibre Reinforcement for Concretes. Concrete International: Design and Construction. 7(3): 21-25.

[4] Ramakrishnan V. 2000. Construction of Highway Structures with synthetic Structural Fibre Reinforced Concrete. ICFRC, Proceedings of the High Performance Concrete Composites, India. pp. 1-17.

[5] Banthia N. and Nandakuma. N. 2003. Crack Growth Resistance of Hybrid Fibre Reinforced Cement Composites. Cement and Concrete Composites. 25: 3-9.

[6] Surendra P. Shah. 1987. Strength Evaluation and Failure Mechanisms of Fibre Reinforced Concrete. Proceedings of the International Symposium on Fibre Reinforced Concrete, India. pp. 1-18.

[7] Wu Yao, Jie Li and Kern Wu. 2002. Mechanical Properties of Hybrid Fibre-Reinforced Concrete at Low Fibre Volume Fraction. Cement and Concrete Research. 33: 27-30.

[8] Soroushian P., Tlili A., Alhozaimy A. and Khan A. 1993. Development and Characterization of Hybrid Polypropylene Fibre Reinforced Cement Composites. ACI Journal. 90(2): 182-189.

[9] IS 1489: 1991. Specification for Portland Pozzolana Cement (PPC). BIS, New Delhi.

[10] IS 383: 1970. Specification for Coarse and Fine Aggregates from Natural Sources for Concrete. BIS, New Delhi.

[11] ACI Committee 2111.1 - 91. 1994. Standard Practice for Selecting Proportions for Normal, Heavy weight, and Mass Concrete. Part-I, ACI Manual of Concrete Practice.

[12] IS 516. 1959. Method of test for strength of concrete. BIS, New Delhi.

[13] IS: 456. 2000. Plain and Reinforced Concrete - Code of Practice. BIS, New Delhi.

[14] ACI Committee 318. 1989. Building Code Requirements for Reinforced Concrete (ACI 318-89). American Concrete Institute.

[15] Kaushik S. K., Kumar V. and Bhargava V. P. 2001. Mechanical Properties of High-Strength Concrete: A Review. The Indian Concrete Journal. pp. 515-521.