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A NUMERICAL APPROACH FOR THE SPREAD OF GONORRHEA IN HOMOSEXUALS

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ABSTRACT

This paper presents a simple mathematical model of gonorrhea in homosexual population. This model is characterized by a pair of non-linear first order ordinary differential equations reflecting the growth rates of promiscuous and infective in a homosexual population and here cured infectives are separated from the main stream for further investigation. Numerical examples are given to explain the effect of cure rate and infective rate on the spread and control of the disease.

Keywords: model, gonorrhea, homosexuals, disease, growth, promiscuous, cure rate, infective rate.

1. INTRODUCTION

A mathematical modeling of epidemic diseases has been a subject of intensive study of the last half a century by both theoretical and experimental scientists working in the areas of population dynamics, social medicine and applied mathematics. A mathematical model of gonorrhea was given by cook and York [1] in the year 1973 in the form of a set of first ordered ordinary coupled nonlinear differential equations. Later Braun [2] described a model of gonorrhea which is identical with the model given by cook and York [1]. Hethkoote, York [3] provided an exhausted bibliography on mathematical modeling of gonorrhea. Beretra and Capasso [4] established the stability criteria of gonorrhea employed a suitably constructed liapanov functional. Srinivas and Pattabhiramacharyulu [5] investigated stability of time delay gonorrhea. Ramakishore and Pattabhiramacharyulu [6] have given stability criteria for gonorrhea in homosexual population by considering the population as variable.

In the present investigation we have discussed the effect of cure rate and infective rate on the spread and control of the disease. For this we have solved a pair of nonlinear equations which represents the growth rates of promiscuous population and infective by employing the Runge-kutta method of fourth order. And some of the solution graphs are presented whenever necessary.

2. BASIC EQUATIONS

Equations for this model are given by a pair of non linear ordinary differential equations which represents the growth rates of promiscuous and infective population. In this model cured infectives are eliminated from the main stream. Following notations are used in this model:

P = Total number of promiscuous individuals in the population.
I = Number of infectives in population.
S = Number of susceptibles in population (P-I).
a_1 = Natural growth rate in population.
a_{11} = Natural self inhibition coefficient.
c = Cure rate in infectives.
b = Infective rate in promiscuous population.
f = Specific cure rate (c / b).
k = Carrying capacity (a_1/a_{11}).
P_0 = Initial number of population.
I_0 = Initial number of infectives.
S_0 = Initial susceptible (P_0 - I_0).
a_1, a_{11}, c, b are non negative constants.

Equation for growth rate of promiscuous population (P)

$$\frac{dP}{dt} = (a_1 - a_{11}P)P \tag{1}$$

Equation for growth rate of Infective population (I)

$$\frac{dI}{dt} = b(P - I)I - cI \tag{2}$$

3. EQUILIBRIUM POINTS

The system has three equilibrium points:

a) $\bar{P}$= 0, $\bar{I}$= 0. (Fully washed out stage)

b) $\bar{P} = \frac{a_1}{a_{11}}$, $\bar{I}$= 0. (This state is the perfectly healthy state without any infection)

c) $\bar{P} = \frac{a_1}{a_{11}}$, $\bar{I} = k - f$. (Coexistence state)

this would exist only when $k > \frac{c}{b}$. i.e., when the carrying capacity is greater than the specific cure rate.

Criteria for Stability of each equilibrium points and possible solution curves together with trajectories of perturbed equations are given by the present authors [6]. This paper presents a numerical solution of the equations (1), (2).

4. A NUMERICAL SOLUTION OF THE BASIC NON LINEAR COUPLED DIFFERENTIAL EQUATIONS

Solving the equation (1) and substituted in the equation (2) then we get the nonlinear differential equation representing the growth rate of infective.

$$\frac{dI}{dt} = -bI^2 + \left[\frac{bP_0 k}{P_0 + (k - P_0)e^{-a_{11}kt}} - c\right] I \quad (3)$$

Numerical solutions of this equation is obtained by employing Runge-Kutta method of fourth order with the initial condition I $(t_0) = I_0$. The interval is assumed to range over (0, 100) for investigate the behaviour of the infectives of this model. Graphs are presented for possible cases.

4.1 Case (A): Specific cure rate f >1

4.1.1 Case (1): when f > 1 and P_0 > k

i.e., Cure rate is greater than infective rate when initial population greater then carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-1 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-1.

Table-1.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.2	0.1	10	4	2	0.01
2	0.4	0.1	10	4	2	0.01
3	0.6	0.1	10	4	2	0.01
4	0.8	0.1	10	4	2	0.01
5	1.0	0.1	10	4	2	0.01

4.1.2 Case (2): when f > 1 and P_0 = k

i.e., Cure rate is greater than infective rate when initial population is equal to the carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-2 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-2.

Table-2.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.2	0.1	10	4	10	0.01
2	0.4	0.1	10	4	10	0.01
3	0.6	0.1	10	4	10	0.01
4	0.8	0.1	10	4	10	0.01
5	1.0	0.1	10	4	10	0.01

4.1.3 Case (3): when f > 1 and P_0 < k

i.e., Cure rate is greater than infective rate when initial population is less than the carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-3 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-3.

Table-3.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.2	0.1	6	4	10	0.01
2	0.4	0.1	6	4	10	0.01
3	0.6	0.1	6	4	10	0.01
4	0.8	0.1	6	4	10	0.01
5	1.0	0.1	6	4	10	0.01

4.2 Case (B): specific cure rate f = 1

4.2.1 Case (4): when f = 1 and P_0 > k

i.e., Cure rate is equal to infective rate when initial population greater then carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-4 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-4.

Table-4.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.1	0.1	10	4	2	0.01

4.2.2 Case (5): when f =1 and P_0 = k

i.e., Cure rate is equal to infective rate when initial population equal to carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-5 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-5.

Table-5.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.1	0.1	10	4	10	0.01

4.2.3 Case (6): when f = 1 and P_0 < k

i.e., Cure rate is equal to infective rate when initial population is less than the carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-6 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-6.

Table-6.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.1	0.1	6	4	10	0.01

4.3 CASE (C): specific cure rate f < 1

4.3.1 Case (7): when f < 1 and P_0 > k

i.e., Cure rate is less than infective rate when initial population greater then carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-7 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-7.

Table-7.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.1	0.2	10	4	2	0.01
2	0.1	0.4	10	4	2	0.01
3	0.1	0.6	10	4	2	0.01
4	0.1	0.8	10	4	2	0.01
5	0.1	1.0	10	4	2	0.01

4.3.2 Case (8): when f < 1 and P_0 = k

i.e., Cure rate is less than infective rate when initial population equal to carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-8 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-8.

Table-8.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.1	0.2	10	4	10	0.01
2	0.1	0.4	10	4	10	0.01
3	0.1	0.6	10	4	10	0.01
4	0.1	0.8	10	4	10	0.01
5	0.1	1.0	10	4	10	0.01

4.3.3 Case (9): when f < 1 and P_0 < k

i.e., Cure rate is less than infective rate when initial population less than the carrying capacity. Computations have been carried for diverse spectrum of values for the system parameters as shown in the Table-9 to estimate the strength of the infective (I). The variations of infective verses time is illustrated in the Figure-9.

Table-9.

S. No.	c	b	P_0	I_0	k	a_{11}
1	0.1	0.2	6	4	10	0.01
2	0.1	0.4	6	4	10	0.01
3	0.1	0.6	6	4	10	0.01
4	0.1	0.8	6	4	10	0.01
5	0.1	1.0	6	4	10	0.01

4.4. Solution curves corresponding parameter values of Table-1 to Table-9

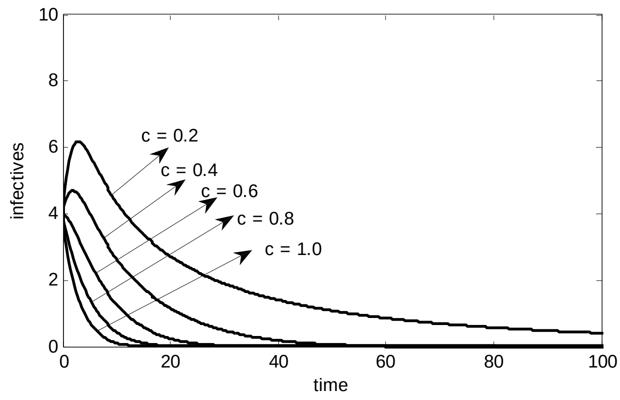

Figure-1. (c > b, P_0 > k) for Table-1.

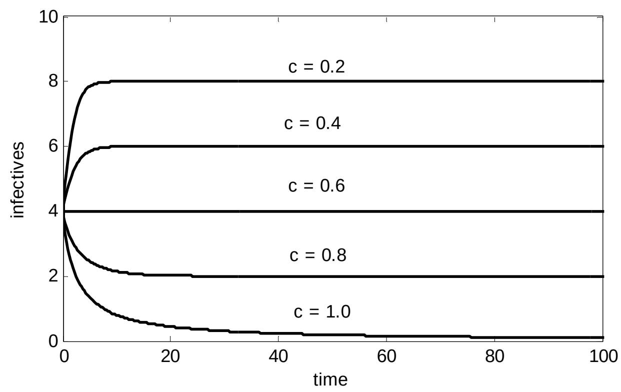

Figure-2. (c > b, P0 = k) for Table-2.

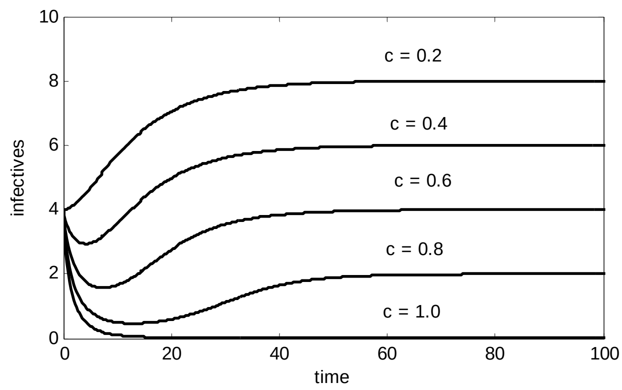

Figure-3. (c > b, P_0 < k) for Table-3.

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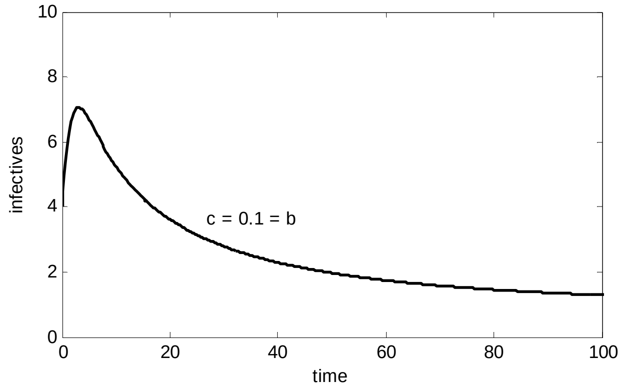

Figure-4. (c = b, $P_0 > k$) for Table-4.

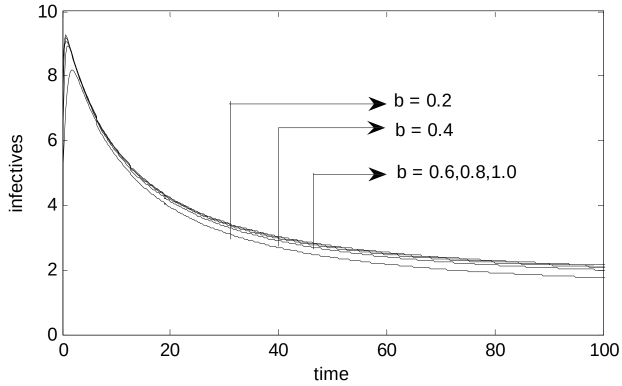

Figure-7. (c < b, $P_0 > k$) for Table-7.

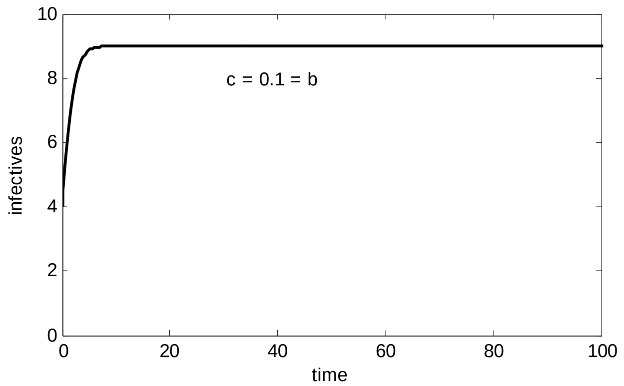

Figure-5. (c = b, $P_0 = k$) for Table-5.

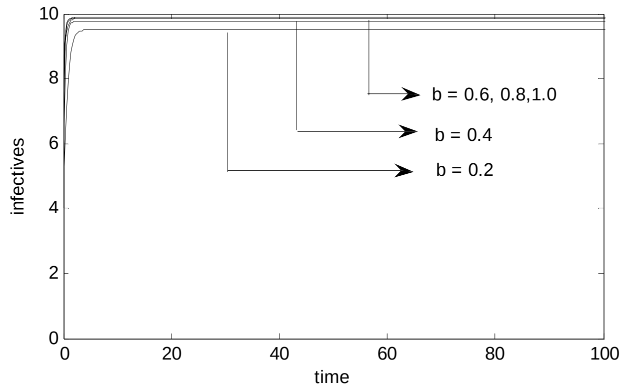

Figure-8. (c < b, $P_0 = k$) for Table-8.

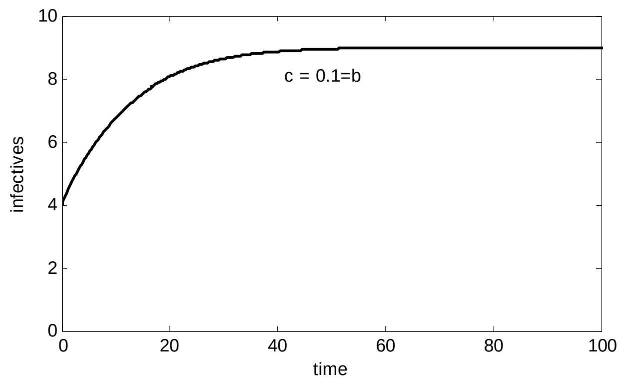

Figure-6. (c = b, $P_0 < k$) for Table-6.

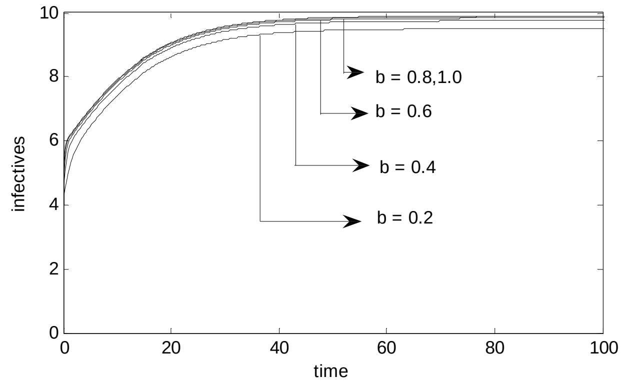

Figure-9. (c < b, $P_0 < k$) for Table-9.

4.5 Conclusions

		Specific cure rate f = c / b		
		> 1	**= 1**	**< 1**
Initial Population / Carrying Capacity (P_0/k)	> 1	Cure rate is greater than infective rate and initial population is greater than carrying capacity: In this case we observed that the infective are decreasing for increasing values of cure rates. Here only healthy population would survive the infective will wash out. Shown in the Figure-1.	Cure rate and infective rate are equal and the initial population greater than carrying capacity: Here infective are gradually increasing for some time after that all Population becomes infected and starts falling down as shown in Figure-4.	Cure rate is less then Infective rate and initial population is greater than carrying capacity: In this case we can see that the infective are steeply increasing for some time and they are gradually decreasing as shown in the Figure-7.
	= 1	Cure rate is greater than infective rate and initial population equals to carrying capacity: In this case infective are increasing for some time after that they are constant for C = 0.2, C = 0.4. For the C = 0.6 no change in the infective population throughout. Infective are falling down for bigger values of cure rate as shown in Figure-2.	Cure rate and infective rate are equal and also initial population is equal to the carrying capacity: In this case infective has steep raise with in very less time, after that they maintain constant number in total population .shown in Figure-5.	Cure rate is less then Infective rate and initial population is equal to carrying capacity: Here the infective are increasing with larger growth rate and after some time all population will be infected i.e., only infective can survive and healthy population washed out as shown in Figure-8
	< 1	Cure rate is greater than infective rate and initial population is less than carrying capacity: In this case we observed that infective are gradually increasing to some extent for small cure rates and these are falling down when cure rate is increasing as shown in the Figure-3.	Cure rate and infective rate are equal and the initial population less than carrying capacity: In this case we noticed that the infective are gradually increasing and after some time they maintain constant number in the total population as shown in Figure-6.	Cure rate is less then Infective rate and initial population is less than carrying capacity: In this case we observed that the infective are gradually increasing and all the population becomes infective for higher values of infective ate as shown in the Figure-9

Here numerical work is extended to examine the variations of the infective and susceptible. Here Susceptible = promiscuous population - infective (P-I). Here we have considered values for all parameters of this model. Eighteen cases are arises for different values of parameters, in those eight interesting cases (S. No: 1, 2, 5, 7, 10, 17, 22, 27 in the Table-10) are graphically illustrated in below from Figure-10 to Figure-17. Time interval assumed to range over (0,100).

Table-10.

S. No.	C	b	P_0	I_0	S_0	k	A_{11}
1	0.2	0.1	10	7	3	2	0.01
2	0.4	0.1	10	7	3	2	0.01
3	0.6	0.1	10	7	3	2	0.01
4	0.4	0.1	10	5	5	2	0.01
5	0.6	0.1	10	5	5	2	0.01
6	0.2	0.1	10	3	7	2	0.01
7	0.8	0.1	10	3	7	2	0.01
8	0.1	0.8	10	3	7	2	0.01
9	0.2	0.1	4	1	3	10	0.01
10	0.4	0.1	4	1	3	10	0.01
11	0.4	0.1	6	2	4	6	0.01
12	0.2	0.1	6	3	3	5	0.01
13	0.4	0.1	6	3	3	6	0.01
14	0.6	0.1	6	3	3	6	0.01
15	0.1	0.2	6	3	3	6	0.01
16	0.1	0.2	10	7	3	2	0.01
17	0.4	0.1	10	3	7	2	0.01
18	0.1	0.6	10	7	3	2	0.01
19	0.1	0.4	10	5	5	2	0.01
20	0.1	0.6	10	5	5	2	0.01
21	0.1	0.2	10	3	7	2	0.01
22	0.1	0.4	10	7	3	2	0.01
23	0.1	1.0	10	3	7	2	0.01
24	0.1	0.2	4	1	3	10	0.01
25	0.1	0.4	4	1	3	10	0.01
26	0.1	1.0	4	1	3	10	0.01
27	1.0	0.1	4	1	3	10	0.01
28	0.1	0.4	6	3	3	6	0.01
29	0.1	0.6	6	3	3	6	0.01
30	0.1	0.4	6	2	4	6	0.01

5. SOLUTION CURVES CORRESPONDING PARAMETER VALUES OF S. NO. 1, 2, 5, 7, 10, 17, 22, 27 IN THE TABLE-10.

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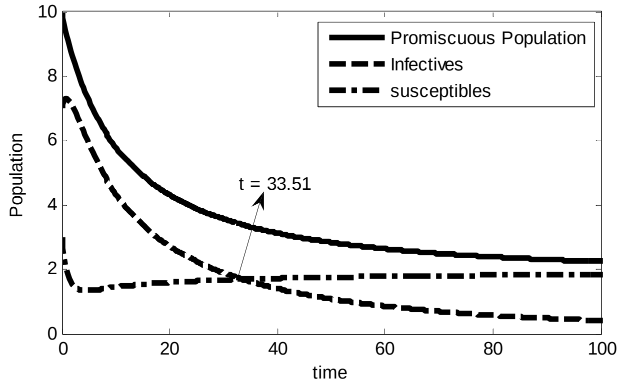

Figure-10. (c > b, P_0 > k, I_0 > S_0) S. No. 1 in Table-10.

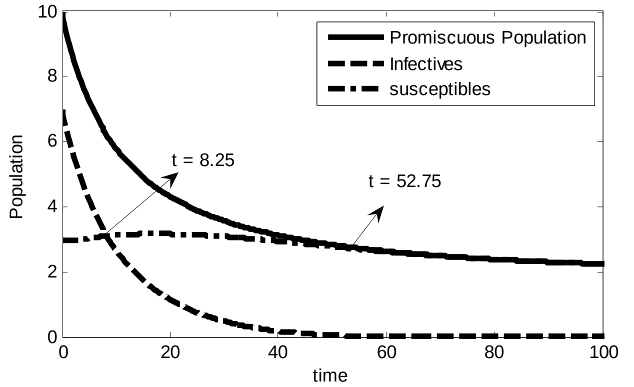

Figure-11. (c > b, P_0 > k, I_0 > S_0) S. No. 2 in Table-10.

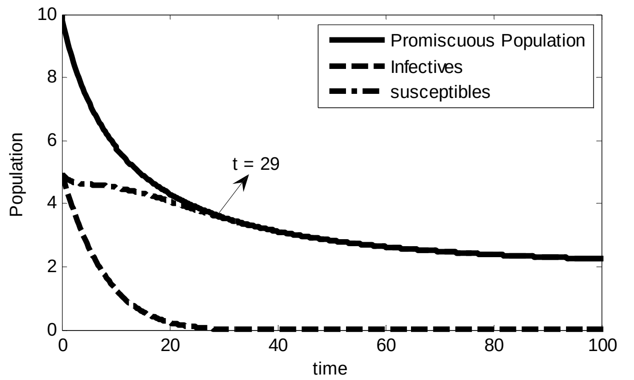

Figure-12. (c > b, P_0 > k, I_0 = S_0) S. No. 5 in Table-10.

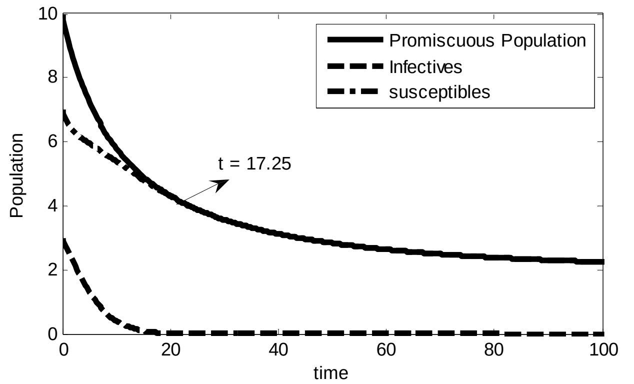

Figure-13. (c > b, P_0 > k, I_0 < S_0) S. No. 7 in Table-10.

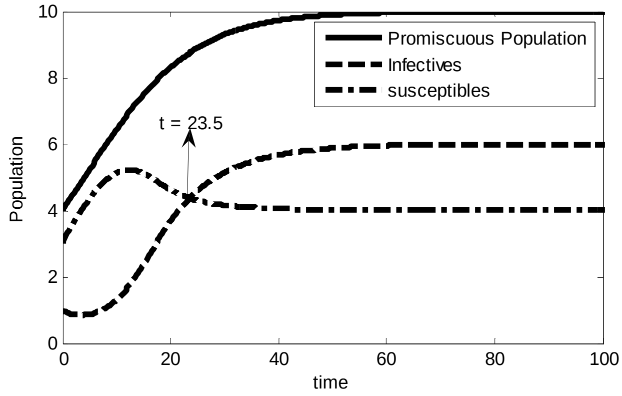

Figure-14. (c > b, P_0 < k, I_0 > S_0) S. No. 10 in Table-10.

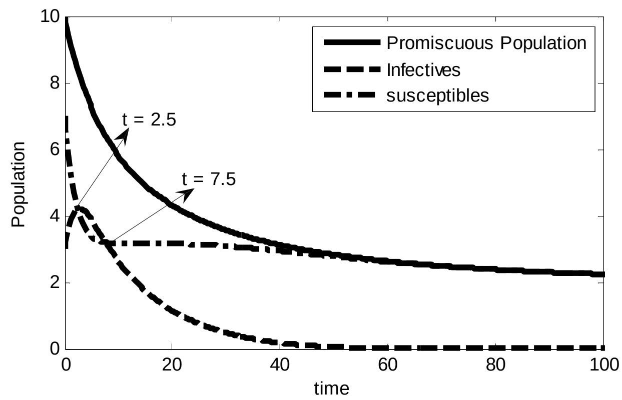

Figure-15. (c > b, P_0 > k, I_0 < S_0) S. No. 17 in Table-10.

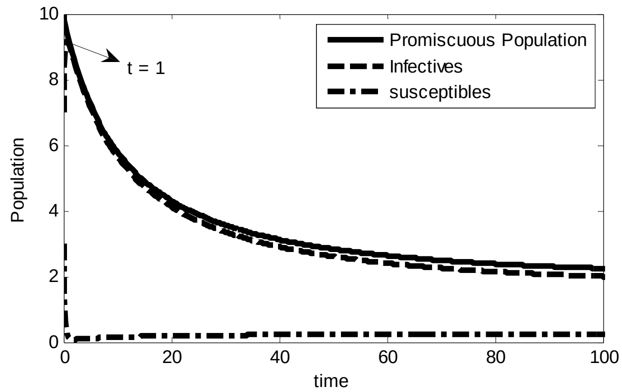

Figure-16. (c < b, P_0 > k, I_0 < S_0) S. No. 22 in Table-10.

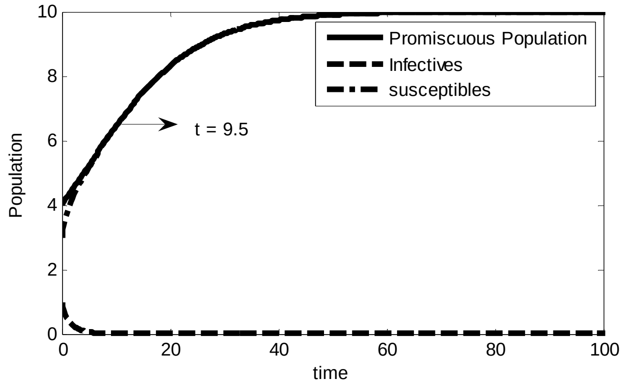

Figure-17. (c > b, P_0 < k, I_0 < S_0) S. No. 27 in Table-10.

6. CONCLUSIONS

a) The cure rate is greater than the infective rate and the initial strength of infective are greater than that of the susceptible:

In this case infective dominates the susceptible till the time of reversal dominance t = 33.51 after that the susceptible dominates over the infective. i.e., only healthy population would survive after t = 33.51 shown in the Figure-10.
(for parameter values C = 0.2, b = 0.1, P_0 = 10, I_0 = 7, S_0 = 3, k = 2, a_{11} = 0.01).

b) The cure rate is greater than infective rate and initial strength of infective are greater than susceptible:

Here we can see that infective dominates the susceptible till t = 33.51 after that susceptible dominate the infective and also after t = 52.5 infective declaim to zero. i.e., only the susceptible would survive and infective will be washed out as shown in the Figure-11.
(For parameter values C = 0.4, b = 0.1, P_0 = 10, I_0 = 7, S_0 = 3, k = 2, a_{11} = 0.01)

c) The cure rate is greater than the infective rate and the initial strength of both the infective and susceptible are equal:

For this case it is observed that the susceptible dominate over the infective throughout. i.e., in this case only healthy population would survive and the infective washed out after t = 29 as shown in the Figure-12.
(For parameter values C = 0.6, b = 0.1, P_0 = 10, I_0 = 5, S_0 = 5, k = 2, a_{11} = 0.01)

d) Cure rate is greater than infective rate and Strength of the susceptible is greater than infective initially:

Here we noticed that the susceptible are greater than infective in their strength throughout the infective are washed out. As shown in the Figure-13.
(For parameter values C = 0.8, b = 0.1, P_0 = 10, I_0 = 3, S_0 = 7, k = 2, a_{11} = 0.01)

e) Cure rate is greater than that of the infective rate and the susceptible are greater than the infective initially:

In this case we observed that susceptible dominates the infective till the time of reversal dominance t=23.5, after which invective dominate over the susceptible. i.e., infective and susceptible both co-exist throughout as shown in the Figure-14.
(For parameter values C = 0.4, b = 0.1, P_0 = 4, I_0 = 1, S_0 = 3, k = 10, a_{11} = 0.01)

f) Cure rate is greater than the infective rate and the initial strength of infective are less than susceptible:

Here the susceptible dominate over the infective up to t = 2.5 beyond that the infective dominate over the susceptible up to t = 7.5 after that the dominance of the susceptible is restored. i.e., infective dominates susceptible in the time interval t = 2.5 to t = 7.5 and beyond that time the susceptible are dominating infective. It is also observed that after t = 58.5, the susceptible only would survive and infective are washed out as shown in the Figure-15.
(For parameter values C = 0.4, b = 0.1, P_0 = 10, I_0 = 3, S_0 = 7, k = 2, a_{11} = 0.01)

g) Cure rate is less the infective rate and the infective are greater than susceptible initially:

In this case the infective dominates the susceptible all throughout. i.e., only infective are survived and susceptible are washed out. Here the infective have steep rise at t=1 as shown in the Figure-16.
(For parameter values C = 0.1, b = 0.4, P_0 = 10, I_0 = 7, S_0 = 3, k = 2, a_{11} = 0.01)

h) Cure rate is greater than infective rate and the initial Strength of the susceptible is greater than infective initially:

This situation is opposite to the previous case i.e., susceptible increase with a large growth rate with a fast approach to the total population so this is the case devoid of gonorrhea shown in the Figure-17.
(For parameter values C = 1.0, b = 0.1, P_0 = 4, I_0 = 1, S_0 = 3, k = 10, a_{11} = 0.01)

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