VOL. 6, NO. 7, JULY 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

INFORMATIVE BINARIZATION BASED ON UNSHARP MASKING

Yumnam Kirani Singh
Centre for Development of Advanced Computing, Plot-E2/1, Block GP, Sector V, Salt Lake Electronics Complex, Kolkata, India
E-Mail: Yumnam.singh@cdackolkata.in

ABSTRACT

In the existing methods of binarization, most of the edge information is lost and the resulted binary image gets blurred. Edges carry most of the information in an image. If we can preserve or retain most of the edge information in a binary image, the image will be informative and look natural. To preserve edge information in a binary image, the edges in the grey image must be enhanced using some effective unsharp masks. We propose a new method of generating effective unsharp masks, which can be used to enhance the edges in an image. Once an effective unsharp mask is obtained, we apply it on an input grey image to enhance the edges in the image. The edge enhanced grey image is binarized to give informative binary image. The informative binary images look better and more informative as compared with binary images obtained using by popular Otshu's method.

Keywords: image binarization, informative binarization, edge enhancement, unsharp masks, edge detection.

1. INTRODUCTION

A Binary image requires less storage space, less processing time and is used in many application areas such as optical character recognition, Object recognition, face recognition, image segmentation, scene analysis etc. Binarization of an image is a process for representing an image using only two different pixel values. It is generally performed by classifying a grey scale image into two groups of pixels based on certain threshold value. Those pixel values greater than or equal to the threshold is set to a particular grey value and those below the threshold to another grey value. There are several binarization techniques but Otshu's [3] based on the thresholding techniques using discrimnant analysis and Kwon's [2] based on thresholding techniques using cluster analysis are the popular ones. A survey on some earlier thresholding techniques for image binarization is given in [4]. It is found that the quality of a binary image depends not only on the proper choice of threshold value but also on the quality of the input image. Moreover, in most of the existing binarization techniques, the resulting binary image is blurry as it loses most of the edge information in the original image. If we can preserve or retain edge information in a binary image, it would be more or less similar to the original image as regards information contents. Such a binary image where most of the possible edges are retained will be referred to as informative binary image and the process as informative binarization.

A study has been made in [6] to see whether we could generate binary image with most of its edges preserved in a binarized image. In [6], a modified form of Laplacian edge detector [5, 7] was used to retain edge information in binary image. This method has the difficulty to obtain good edge preserving Laplacian mask for informative binarization. In this paper, a simpler approach for informative binarization is proposed. Here, the gray level image is first enhanced its edges applying a suitable unsharp mask. The edge-enhanced image is then binarized to get the informative binary image where most of the informative features are retained in it. The quality of the generated informative binary image depends much on the effectiveness of the unsharp masks applied. To study the effectiveness of unsharp masks, we use the various unsharp masks that can be obtained from the MATLAB function fspecial for various values of the parameter alpha. It is found that the unsharp masks obtained from the MATLAB are not effective to generate informative binary images. So, we propose a new unsharp mask generator that can generate unsharp masks, which would be effective for informative binary images. We compare the quality of the binary images obtained with the various unsharp masks. It is found that the unsharp masks obtained from the proposed generator are more effective to obtain better informative binary image than those unsharp masks obtained from MATLAB using fspecial function. Also, we could get much better binary image than that of the binary image obtained directly by Otshu's method [3].
The remainder of the paper is organized as follows: Section (2) discusses on unsharp mask generators to be used for informative binarization, which is discussed in section (3). Experimental results are given in Section (4) and Conclusions in Section (5).

2. UNSHARP MASK GENERATORS

It is known that unsharp masking enhances the edge information in an image. There are various ways to perform unsharp masking. One way is to subtract a fraction of a blurred or low pass filtered image from the original image so that the resulting image contains most of high pass components. In most of the image editing tools, such as Photoshop, unsharp masking is done in adaptive way by using three parameters amount, radius and threshold. More information on use of these parameters for sharpening an image can be found in [8, 9]. Another approach to enhance edges in an image is to apply a suitably designed high pass filter or mask to the image. Edges carry significant information of an image. So, if we binarize an image after applying with suitable unsharp mask, the resulted binary image would contain most of the edge information. The unsharp masks generated by fspecial function of MATLAB are such edge enhancement masks, which can be directly applied to an image to enhance edges. Such enhancement mask can be easily generated and the edge enhancement using such a mask is also simple. We will consider such edge enhancement technique based on unsharp masks to obtain informative binary image.

Unsharp masks of fspecial functions

The unsharp masks in MATLAB are generated using the function fspecial. This function takes two inputs, a string constant ‘unsharp’ and an input parameter alpha to generate an unsharp mask, a matrix of size 3x3. For different values of the parameter alpha, different unsharp masks can be obtained. Mathematically, the unsharp mask $\mathbf{H}$ generated in MATLAB can be written as

$$\mathbf{H} = \frac{1}{1+\alpha}\begin{bmatrix} -\alpha & \alpha-1 & -\alpha \\ \alpha-1 & 5+\alpha & \alpha-1 \\ -\alpha & \alpha-1 & -\alpha \end{bmatrix}, \quad 0 \le \alpha \le 1 \qquad \text{--- 1}$$

In MATLAB, the value of the parameter α (alpha) is restricted to lie inclusively between 0 and 1. But alpha can have any value except -1. It is found that the value of alpha greater than 1 or less than 0 can also be used to generate unsharp masks.

Let us see the mask patterns of the unsharp masks for various values of α.

Case-1: When $\alpha = 0$, the unsharp mask $\mathbf{H}$ is given below.

$$\mathbf{H} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \text{----M1}$$

Case-2: When $0 < \alpha < 1$, the unsharp mask has the pattern

$$\mathbf{H} = \begin{bmatrix} - & - & - \\ - & + & - \\ - & - & - \end{bmatrix} \qquad \text{----M2}$$

$$\text{e,g, } \mathbf{H} = \begin{bmatrix} -0.33 & -0.33 & -0.33 \\ -0.33 & 3.67 & -0.33 \\ -0.33 & -0.33 & -0.33 \end{bmatrix} \text{ for } \alpha = 0.5$$

Case-3: When $\alpha = 1$, the unsharp mask is

$$\mathbf{H} = \begin{bmatrix} -0.50 & 0 & -0.50 \\ 0 & 3.0 & 0 \\ -0.50 & 0 & -0.50 \end{bmatrix} \qquad \text{---M3}$$

Case-4: When $-1 < \alpha < 0$, the unsharp mask has the pattern

$$\mathbf{H} = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \qquad \text{---M4}$$

$$\text{e.g., } \mathbf{H} = \begin{bmatrix} 1 & -3 & 1 \\ -3 & 9 & -3 \\ 1 & -3 & 1 \end{bmatrix} \text{ for } \alpha = -0.5$$

Case-5: When $-5 \le \alpha < 1$, the mask has the following pattern

$$\mathbf{H} = \begin{bmatrix} - & + & - \\ + & - & + \\ - & + & - \end{bmatrix} \qquad \text{--- M5}$$

$$\text{e.g., } \mathbf{H} = \begin{bmatrix} -2 & 3 & -2 \\ 3 & -3 & 3 \\ -2 & 3 & -2 \end{bmatrix} \text{ for } \alpha = -2$$

Case-6: When $\alpha < -5$ or $\alpha > 1$, the unsharp mask has the following pattern

$$\mathbf{H} = \begin{bmatrix} - & + & - \\ + & + & + \\ - & + & - \end{bmatrix} \qquad \text{---M6}$$

$$\text{e,g, } \mathbf{H} = \begin{bmatrix} -0.833 & 0.667 & -0.833 \\ 0.667 & 1.667 & 0.667 \\ -0.8333 & 0.667 & -0.833 \end{bmatrix} \text{ for } \alpha = 5$$

Of these six possible unsharp mask types that can be obtained from equation (1); M4 and M5 are not suitable for informative binarization as they produce distorted edges. The remaining unsharp mask types, i.e., M1, M2. M3 and M-6 do not generate any distortion in the edges and hence can be considered for informative binarization. It is found that the unsharp masks with higher central value gives more edge information in the generated binary image. However, because of the division by a factor of $(1+\alpha)$, the central value of the unsharp mask is less than 3 when $\alpha > 1$ and less than 1 when $\alpha < -5$. This has the limiting factor on the performance of the resulting mask for edge enhancement. In other words, in the unsharp masks generated by (1) i.e., fspecial function of MATLAB, we do not have the direct control on the central value of the generated mask. We want the central value of the mask to increase or decrease arbitrarily as we wish.

Proposed unsharp masks

We propose a generalized way of generating unsharp mask where the performance of the masks can be controlled effectively. Let us consider two 3x3 matrices $\mathbf{I}_1$ and $\mathbf{I}_2$ based on any real number k.

$$\mathbf{I}_1 = \begin{bmatrix} 0 & -(k-1) & 0 \\ -(k-1) & 4k & -(k-1) \\ 0 & -(k-1) & 0 \end{bmatrix}$$

$$\mathbf{I}_2 = \begin{bmatrix} -(k-1) & 0 & -(k-1) \\ 0 & 4k & 0 \\ -(k-1) & 0 & -(k-1) \end{bmatrix}$$

Any unsharp mask $\mathbf{H}$ can be obtained as linear combination of $\mathbf{I}_1$ and $\mathbf{I}_2$ as follows

$$\mathbf{H} = \frac{a\mathbf{I}_1 + b\mathbf{I}_2}{4(a+b)}, \quad a+b \ne 0 \qquad \text{----- (2)}$$

for any real values of a and b.
Simplifying, we get

$$\mathbf{H} = \frac{1}{4(a+b)}\begin{bmatrix} -b(k-1) & -a(k-1) & -b(k-1) \\ -a(k-1) & 4k(a+b) & -a(k-1) \\ -b(k-1) & -a(k-1) & -b(k-1) \end{bmatrix} \quad \text{------ (3)}$$

From (3), we can get various unsharp masks for different values of a, b and k. We will consider the simple unsharp mask generators, which depend only on the value of k.

Case-1: when $a = 0, b = 1$, we obtain

$$\mathbf{H} = \frac{1}{4}\begin{bmatrix} 0 & -(k-1) & 0 \\ -(k-1) & 4k & -(k-1) \\ 0 & -(k-1) & 0 \end{bmatrix} \quad \text{---- P1}$$

The unsharp mask generated using this mask for different values of k will have similar pattern as that of M1. This mask will enhance the edges in the horizontal and the vertical directions.

Case-2: when $a = 1, b = 0$, we obtain

$$\mathbf{H} = \frac{1}{4}\begin{bmatrix} -(k-1) & 0 & -(k-1) \\ 0 & 4k & 0 \\ -(k-1) & 0 & -(k-1) \end{bmatrix} \quad \text{---- P3}$$

This unsharp mask has the pattern similar to that of M3 and hence is marked as P3. This mask will enhance the edges in the two diagonal directions.

Case-3: when $a = 1, b = 1$, we get

$$\mathbf{H} = \frac{1}{8}\begin{bmatrix} -(k-1) & -(k-1) & -(k-1) \\ -(k-1) & 8k & -(k-1) \\ -(k-1) & -(k-1) & -(k-1) \end{bmatrix} \quad \text{-----P2}$$

This mask having a pattern similar to that of M2 is marked as P2. This mask will enhance the edges in all possible directions.

Case-4: when $a = 2, b = -1$, we get

$$\mathbf{H} = \frac{1}{4}\begin{bmatrix} (k-1) & -2(k-1) & (k-1) \\ -2(k-1) & 4k & -2(k-1) \\ (k-1) & -2(k-1) & (k-1) \end{bmatrix} \quad \text{----P4, for } k > 1$$

This mask will have a pattern similar to that of M4 when $k > 1$ is marked as P4.

Case-5: When $a = -1, b = 2$, we get

$$\mathbf{H} = \frac{1}{4}\begin{bmatrix} -2(k-1) & (k-1) & -2(k-1) \\ (k-1) & 4k & (k-1) \\ -2(k-1) & (k-1) & -2(k-1) \end{bmatrix} \quad \text{---P6, for } k > 1$$

This mask will have a pattern similar to that of M6 when $k > 1$ is marked as P6.

The performance of the proposed unsharp masks depends on the value k. The more the value of k, the more is the central value of the unsharped mask and more edge information can be retained in the binary image.

Larger unsharp mask can be generated from the original 3x3 unsharp mask. This can be done by 2-D convolution of the original unsharp mask with an appropriate low-pass or smoothing filter. The process is well described in [6] and [7]. Note here that the larger unsharp mask introduces coarser edges and reduces noise speckles in the generated informative binary image.

3. INFORMATIVE BINARIZATION

In this binarization technique, we first enhance the edges of an image using an effective unsharp mask. The edge-enhanced image is then binarized to get the informative binary image. The unsharp masks can be generated either equation (1) or from equation (3). To generate the unsharp masks using equation (1), we need to select a proper value for the parameter $\alpha \neq -1$, which is a real number. The masks obtained for various values α, do not have varying degrees of effects on the resulting images. It is difficult to control the effectiveness of these masks to enhance the edges in an image. To generate unsharp masks from equation (3), we require three input parameters a, b and k, which are all real numbers where $a \neq b$. For a given value of a,b, we can control the effectiveness of the unsharp mask to enhance the edges in an image by varying the values of k. It is found that larger values of k gives sharper edges. However, the value of k should not be very large to avoid over-sharpened images.

Once an unsharp mask is obtained, we apply it on the image to get an edge-enhanced image. Applying the unsharp mask on the image is nothing but a filtering operation of the image with the unsharp mask. This can be done in Matlab using the conv2 or imfilter function. But the resulting filtered image is different for the same unsharp mask. The conv2 function is based on the direct 2-D convolution of the image and the unsharp mask, the output of which may contain positive and negative real numbers. The process of 2-D convolution is given in detail in [6]. The imfilter function is based on the correlation operation of the input image with the unsharp mask. The result is then converted into an 8-bit unsigned number to correspond to the range of pixel values of an 8-bit image after some post processing operations. The edges of the image get sharpened when filtered with an effective unsharp mask. When such an edge-sharpened image is binarized, most of the edges contained in the image are retained in the resulting binary image. Such binary images are more informative than ordinary binary images, which are generally blurred and distorted.

If $\mathbf{X}$ is an image matrix, and $\mathbf{H}$ an unsharp mask then sharpened image $\mathbf{Y} = \mathbf{X} \otimes \mathbf{H}$, where $\otimes$ denotes size preserving 2-d convolution operation. If $\mathbf{Y}_S$ denotes the scaled edge enhanced image obtained by scaling the values of $\mathbf{Y}$ in the range 0 to 255, then the informative binary image is $\mathbf{Y}_b = \mathbf{Y}_S > T$, where T is the threshold for binarization.

The steps involved in informative binarization are given blow.

a) Read the original image
b) Convert it into gray if necessary
c) Generate an unsharp mask
d) Filter the image with the unsharp mask
e) If neccessay, scale the values of the filtered image between 0 and 255
f) Select a proper threshold and binarize it.

The quality of a binary image depends on the proper choise of threshold for binarirization. This is true for informative binarization as well. But the threshold selection is somewhat easier in the case of informative binarirization. This is because when an image is edge sharpened by filtering the image with an unsharp mask, the resulted image usually has a unimodel histogram. In most of the cases the threshold value (T) to binarize an edge-sharpened image is about 10 to 15 % more or less than the position of the peak value in the histogram (Tp). The threshold value corresponding to the position of peak value of the histogram can be thought of demarcation value for white and black background. If T<Tp, the resulted binary image has white background and if T>Tp, the resulted binary image has black background. In other words, the number of white pixels is more in the resulted binary image if the threshold T is less than Tp and if T>Tp, the number of black pixels is more in the resulted binary image.

Figure-1 Shows the histogram of an edge enhanced image obtained after filtering with an unsharped mask.

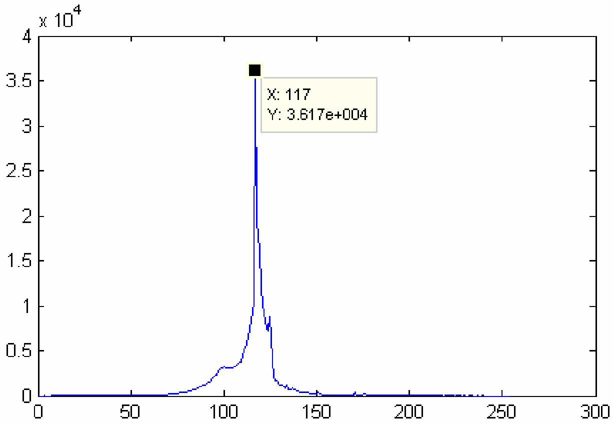


Figure-1. Histogram of an edge-sharpened image where peak occurs at 117, i.e., Tp=117.

When the image is binarized at the threshold T=107, less than Tp=117, the resulted binary image has white background. When the same edge sharpened image is binaized at the threshold value T=127 greater than Tp, the resulted binary image has black background. The two binary images at thresholds 107 and 127 are shown in Figures 2(a) and 2(b), respectively.

Figure-2(a). Binary image with white background

Figure-2(b). Binary image with black background.

The binary image obtained at the peak value Tp is quite noisy having almost same number of black and white pixels. The binary image with white background looks more pleasant and natural than the binary image with black background. But this is not always the case. In some binary images, images with black background look more natural and meaningful. The threshold values for such binary images are more than the Tp values of the corresponding edge enhanced images, which can be seen from the binary images in Figure-4.

It may be noted that the quality of the binary image is also dependent on the size of the unsharp mask used. If the image is not very small or very fine edges in the image are not important, larger size unsharp masks can be used. It is found that the unsharp masks of size 5x5 to 9x9 generate better informative binary images than the original 3x3 masks obtained from equation (1) or (3). If the image is large, even larger masks than 9x9 may be effective for generating informative binary image.

Larger unsharp masks could be obtained by 2-d convolution of original 3x3 masks with a suitable smoothing filter of appropriate size. Two simple but effective smoothing filters that can be used for generating larger unsharp masks are

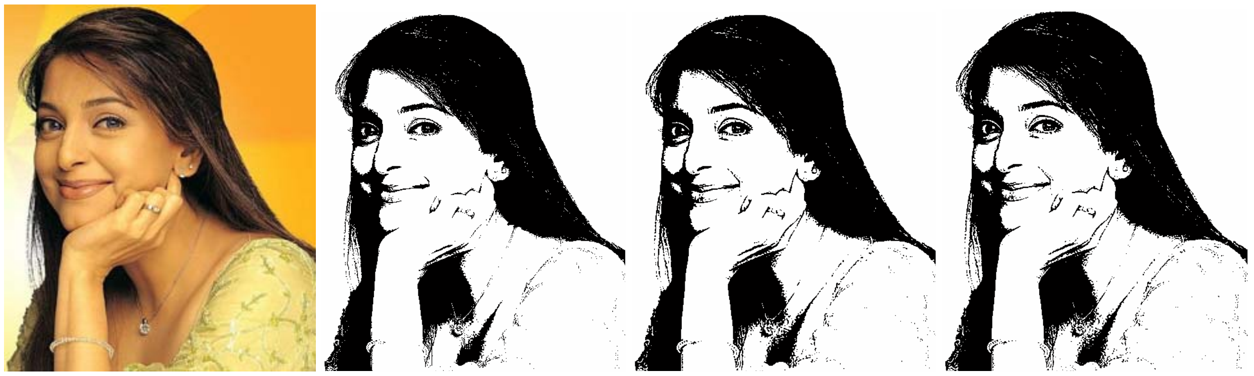
$$\mathbf{F} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{G} = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Applying 2-d convolution on an unsharp masks of 3x3 with $\mathbf{F}$ or $\mathbf{G}$, results an unsharp mask of size 5x5. Each time an unsharp mask or smoothing filter is applied 2-d convolution with $\mathbf{F}$ or $\mathbf{G}$, the size of the resulting unsharp mask or smoothing filter increases by 2 in rows and columns. Larger smoothing filters can be obtained by repetitive 2-d convolution of $\mathbf{F}$ with $\mathbf{F}$ or $\mathbf{F}$ with $\mathbf{G}$ or $\mathbf{G}$ with $\mathbf{G}$.

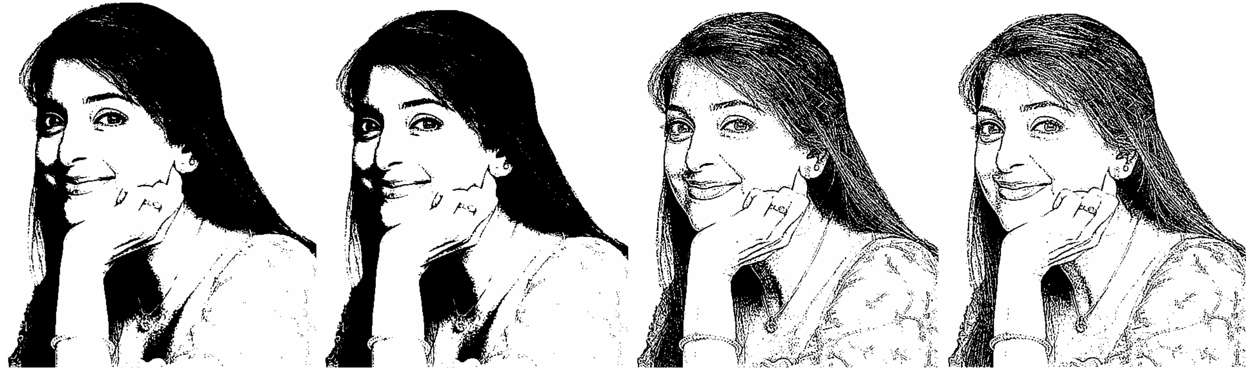
4. EXPERIMENTAL RESULTS

We consider two images labelled Original Image-1 and Original Image-2 in Figures 3 and 4 for our experiment on informative binarization using different unsharp masks.

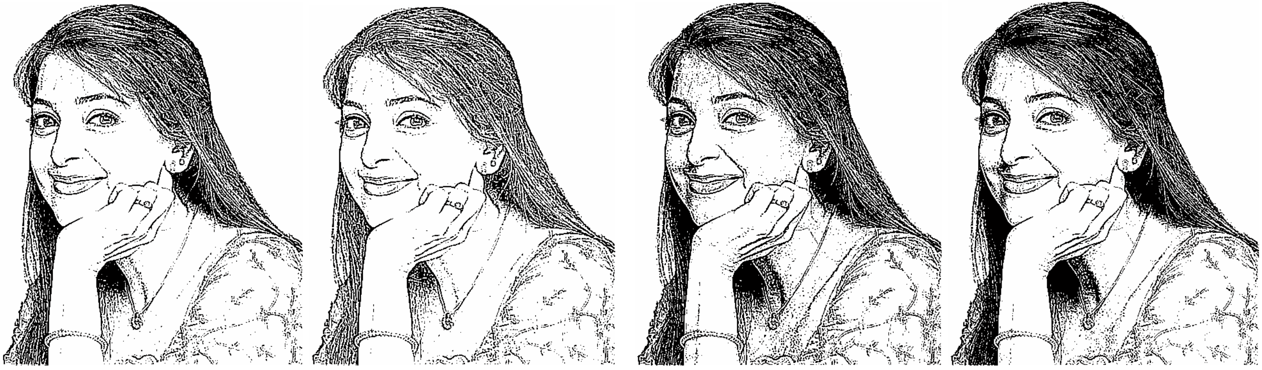
Image-1 has some finer details of embroidery designs in the dress, which we are interested to preserve in the binary image. The binary image of this image looks more natural in white background. Image-2 is visually quite clear but when binarized, it gives quite blurred image. We are interested to get a better binary image of Image-2. Another reason for choosing this image is that its binary image looks quite natural in black background. We consider here four main types of unsharp masks (M1, M2, M3, and M6 obtained from the equation (1) corresponding to Matlab function 'fspecial') and P1, P2, P3 and P6 (obtained from the proposed generator equation (3)). Also, we use unsharp masks of size 5x5 for all types of masks. The 5x5 masks are generated from the original 3x3 masks by 2-D convolution with the smoothing filter $\mathbf{G}$. The parameters of the unsharp masks M1, M2, M3, M6 and P1, P2, P3, P6 given in section 2 are used for generating the unsharp masks.



Original Image-2 Binary Image-2 (Otshu) (M1: Tp=150, T=127) (M2, 0.125: Tp=147, T=125)



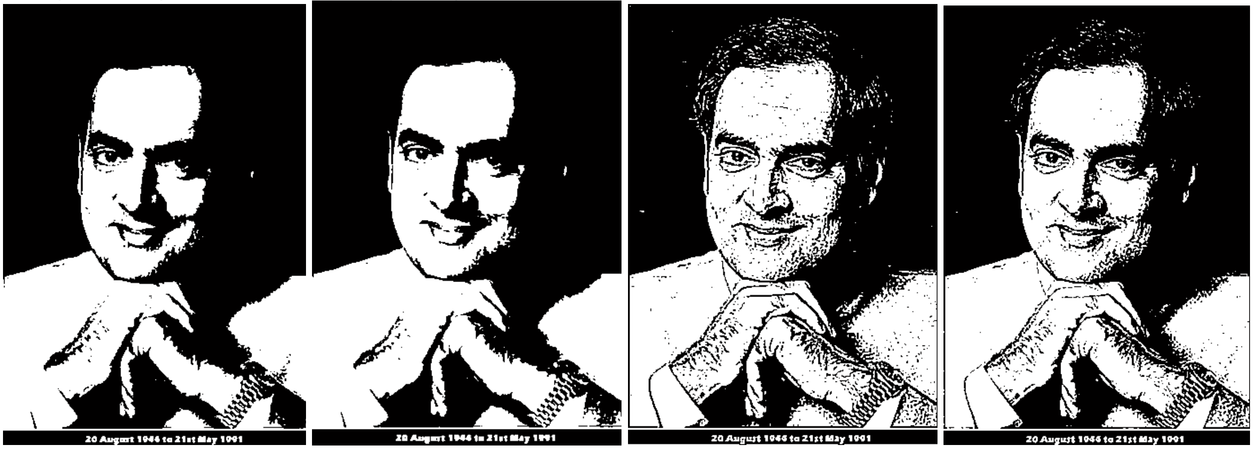
(M3: Tp=147, T=125) (M6, 1.25: Tp=148, T=125) (P1, 25: Tp=137, T=128) (P2, 25: Tp=138, T=128)



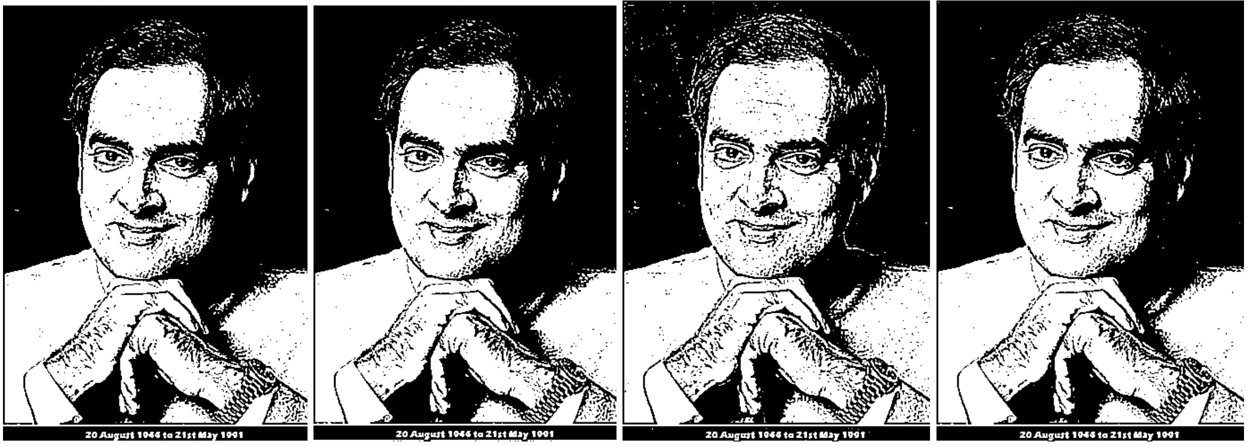
(P3, 25: Tp=141, T=130) (P6, 25: Tp=136, T=129) (-2, 13, 25: Tp=137, T=130) (13, 5, 25: Tp=141, T=131)

Figure-3. Original Image-1 and its different binary images (with white backgrounds).

Original Image-1 Binary Image-1 (Otshu) (M1: Tp=55, T=70) (M2, 0.75: Tp=53, T=68)



(M3: Tp=52, T=100) (M6, -20: Tp=51, T=95) (P1, 20: Tp=97, T=102) (P2, 20: Tp=96, T=107)



(P3, 20: Tp=94, T=101) (P6, 20: Tp=95, T=103) (-2, 13, 25: Tp=102, T=104) (13, 5, 25: Tp=99, T=102)

Figure-4. Original Image-2 and its different binary images (with black backgrounds).

Once an unsharp mask of any particular type of size 5x5 is obtained, it is applied on an image to get the edge-enhanced image. The edge-enhanced image is then binarized at threshold T to give the informative binary image. Figure-3 shows the Image-1 and different binary images obtained from it using ordinary binarization and informative binarization methods. The binary images for this image have white as background and the threshold value T is less than the corresponding Tp values. Each binary image is labelled separately at the bottom with the parameters or method used in generating it. The first binary image is obtained by Otshu method and the remaining binary images are generated after applying various unsharp masks. The label of a binary image generated using an unsharp mask is divided to two parts by a colon. The part on left of the colon signifies the unsharp mask used and the part on the right signifies the Tp value of the edge-enhanced image and threshold value T used for getting the binary image. For example, M3: denotes the unsharp mask M3 with the default parameter, M2, 0.75: denotes the unsharp mask M2 corresponding to the parameter alpha=0.75. Similarly P1, 20: denotes the P1 unsharp mask with parameter k=20 and -2, 3, 25: signifies the unsharp masks generated directly from equation (3), with parameters a=-2, b=3 and k=25.

The binary images obtained from edge-enhanced images using the unsharp masks derived from fspecial function such as M1, M2, M3 and M6 are not very effective as most of the edge information could not be retained in the resulting binary images and hence much of the information of embroidery parts is lost in the binarized images. The embroidery and other edge-information in the original image are retained well in the binary image obtained using the proposed unsharp masks P1, P2, P3 and P6. In other words, the binary images obtained from the edge-enhanced images obtained using the unsharp masks P1, P2, P3 and P6 are informative as compared to those of M1, M2, M3 and M6. From the binary images of the proposed unsharp masks with parameters (-2, 13, 25) and (13, 5, 25), we see that they are also effective in generating informative binarization. In short, the proposed unsharp masks are more effective in generating informative binary images than those unsharp masks obtained from the fspecial function of MATLAB. The same is true for the Image-2, which can be seen from the original and the binary images in Figure-4. The binary images here have black as background and the threshold value T is more than corresponding Tp values. Most of the informative edges are lost in the binary images obtained by using the masks M1, M2, M3 and M6 and hence looks more similar to the ordinary binary image obtained using the Otsu’s method. The binary images obtained by using the unsharp masks of the proposed generator are more informative than those binary images obtained using unsharp masks M1, M2, M3 and M6. It may be noted that the parameter k of the proposed unsharp masks used for Image-1 is higher (25) than those for Image-2 (20). This is because values of k greater than 20 introduce more noise speckles in the binary images of Image-2 as can be seen from the last two images in Figure-4. The informative binary images for higher values of k seem to be a bit noisy as more and more edges are retained in the binary image. To reduce noise level, higher unsharp masks may be considered and lower threshold value may be chosen. By and large, observing the binary images in Figure-3 and Figure-4, the binary images obtained by using the unsharp masks of the proposed unsharp mask generator are more informative and more natural than ordinary binary image of Otsu and those obtained by using the unsharp masks obtained from ‘fspecial’ function of MATLAB.

5. CONCLUSIONS

A generalized way of generating effective unsharp masks has been proposed. Various unsharp masks can be generated easily using the proposed method, which will allow sharpening of image in varying degrees. The unsharp masks generated by the proposed method are found to be more effective for generating informative binary images. The informative binary images look more natural than the ordinary binary images, which are blurry in nature. The proposed unsharp masks can also be used for edge detection purpose by choosing larger values of k, which will retain mostly the edge information after binarization.

REFERENCES

[1] Canny J. 1986. A Computational Approach for Edge Detection. IEEE Trans. Pattern Anal. Machine Intell. 8(6): 679-698.

[2] S.H. Kwon. 1979. Threshold selection based on cluster analysis. Patten Recognition Lett. 25(9): 1045-1050. 2004.

[3] N. Otsu. 1979. A threshold selection method from gray-level histograms. IEEE Trans. Syst. Man Cybern. 9(1): 62-66.

[4] P.K. Sahoo, S. Soltani, A.K.C. Wong and Y.C. Chen. 1988. A survey of the thresholding techniques. Comput. Vision Graphics Image Process. 41(2): 233-260.

[5] R. C. Gonzalez, R. E. Woods. 2003. Digital Image Processing, Pearson Education.

[6] Y. K. Singh. 2008. Preserving Edge Information in Binary Images. Proccedings 9th International Conference on Signal Processing. pp. 824-827.

[7] Y.K. Singh. 2011. Multi-level edge detectors based on the convolution matrices of base lengths 2 and 3. ARPN Journal of Engineering and Applied Sciences. 6(1): 29-37.

[8] http://en.wikipedia.org/wiki/Unsharp_masking, Unsharp masking technique.

[9] http://www.scantips.com/simple6.html, Sharpening using unsharp mask.